Neutrinos from Proto-neutron Star Cooling and Nuclear Equation of State: Effects of Coherent Elastic Scattering

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Supernovae and Equation of State

- Physics in core-collapse supernovae (SNe)
 - ✓ Gravity(GR)
 ✓ (Magneto)Hydrodynamics
 - ✓ Neutrino Reaction ✓ Nuclear Equation of State
- Tables of Equation of State (EOS)
 - ✓ Skyrme type interaction
 El Eid & Hillebrandt (1980), Lattimer & Swesty (1991)
 - ✓ Relativistic mean field model

H. Shen+ (1998, 2011), Hempel & Schaffner-Bielich (2010), G. Shen+ (2011), Steiner+ (2013)

+α: Ishizuka+ (2008), Nakazato+ (2008), Sagert+ (2009)

What is EOS table?

Thermodynamical variables (e.g., free energy, entropy) should be given as functions of ρ, T and Y_e for wide range as realized in SNe.





Supernova neutrino

- Neutrinos come from deep inside supernova.
 - Interaction with nucleons.



New nuclear EOS table

Togashi, Nakazato et al. NPA 961 (2017) 78

- Constructed on the basis of the many body theory with the realistic nuclear force
 - variational method (Togashi & Takano 2013)
- EOS of finite temperature and involving the inhomogeneous phase as in Shen EOS
 - local density approximation (Oyamatsu 1993)
 - without pasta phase (e.g. Horowitz et al. arXiv:1611.10226)
- How do uniform and inhomogeneous phases of EOS affect supernova neutrino emission?

Property of new nuclear EOS Togashi, Nakazato et al. NPA 961 (2017) 78 3000 $|\chi| \le 0.05$ 2500 Consistent with NS radius 2000 measurements and tidal Less Compact $\stackrel{ m cv}{V}$ 1500 1000 deformability from GW170817. More Compact 500 -← [PRL **119** (2017), 161101] 0 -3000 5001000 15002000 25000 (Steiner et al. 2010) Λ_1 1500 2.5low-spin J0348+0432 2.0J1614-2230 1000 M/M. 1.5 90% 50% \sum_{2} 1.0 500 0.5 Variational Shen EOS 0.010 15 200 500 1000 1500 0 *R* [km] Λ_1

Comparison with other EOSs

EOS	variational	LS220	Shen
K[MeV]	245	220	281
S ₀ [MeV]	30.0	28.6	36.9
L [MeV]	35	73.8	111
$\rho_0 [10^{14} \text{g/cm}^3]$	2.66	2.57	2.41
w ₀ [MeV]	16.1	16.0	16.3

- Symmetry gradient, *L*, is lower.
 - \rightarrow large symmetry energy at low densities.
 - \rightarrow small symmetry energy at high densities.

Finite temperature phase diagram

- Inhomogeneous phase: subnuclear densities and T < 10 MeV.
- Low Y_e (elec. fraction) $\stackrel{\text{Def}}{\xrightarrow{}}$ \rightarrow low critical *T* and low transition density.
- Reflecting the property of nuclear matter model, the region of inhomogeneous phase is different among EOS.



Proto-neutron star cooling

this

phase

- Proto-neutron star (PNS) is formed at the center of SN.
 → cooled by v emission.
- EOS dependence of PNS evolution after the shock revival is studied.











- Step1: Core-collapse simulation of spherical GR v radiation hydrodynamical simulation for $15M_{\odot}$ star (Woosley & Weaver 1995) until 0.3 sec.
- Step2: After that, quasi-static evolutionary calculation of PNS ($\sim 1.5 M_{\odot}$) cooling.
 - transfer of v_e , v_e , v_{μ} (= $v_{\tau} = v_{\mu} = v_{\tau}$) is treated in Multigroup Flux Limited Diffusion scheme
- EOS: ① Variational model (Togashi), ② Shen
 ③ Mixed (Togashi for ≥ 2 × 10¹⁴ g/cc, Shen for ≤ 10¹⁴ g/cc, and they are interpolated; T+S)

Profiles of PNS



- Soft EOS makes compact PNS.
 → high density and small radius
- Low L EOS results in low Y_e due to small symmetry energy at high densities.

Neutrino light curve

- High density EOS determines luminosity.
 - Luminosity drops steeper for faster cooling
 - Variational EOS results high density / slow cooling
- Av. energy depends also on low density EOS.



Inhomogeneous region

- Variational EOS has inhomogeneous phase for higher density and temperature.
 - \rightarrow In inhomogeneous phase, cooling slows down and the surface is kept hot.





- Coherent elastic scattering
 - \rightarrow If nuclear size is much lower than neutrino wave length, cross section gets larger.

$$\sigma \sim \frac{G_{\rm F} E_{\nu}^2}{4\pi} [N - (1 - 4\sin^2 \theta_{\rm W})Z]^2$$

neutron rich & 1 - 4 sin² $\theta_{\rm W} \sim 0$
 $\Rightarrow \sigma \propto A^2$

Coherent effect

- Due to the scattering, neutrino wave number changed $\vec{k} \rightarrow \vec{k'}$, state of nucleus unchanged
 - \rightarrow nucleus = sum of nucleons' potentials $V(\vec{r}) = \sum_{i=1}^{A} v \delta(\vec{r} - \vec{r_i})$
- Fermi's golden rule: $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \left| \left\langle \vec{k'} \mid V(\vec{r}) \mid \vec{k} \right\rangle \right|^2$
- If neutrino wave function is a plane wave ...

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \left| \int_{i=1}^{A} v e^{-i\vec{k'}\cdot\vec{r}} \delta(\vec{r}-\vec{r_i}) e^{\vec{k}\cdot\vec{r}} \, d\vec{r}^3 \right|^2$$

$$\frac{Coherent effect (cont.)}{\frac{d\sigma}{d\Omega}} \propto \left|\sum_{i=1}^{A} v e^{-i\vec{q}\cdot\vec{r_i}}\right|^2 = v^2 \sum_{i,j}^{A} e^{i\vec{q}\cdot(\vec{r_i}-\vec{r_j})}$$
$$= v^2 A + v^2 \sum_{i\neq j}^{A} e^{i\vec{q}\cdot(\vec{r_i}-\vec{r_j})}$$
$$\stackrel{\forall}{\underset{k}{\overset{\forall}{\atop{}}}} |\vec{k}| = |\vec{k'}|$$
per nucleon coherence

- Neutrinos' wave length: $120\left(\frac{10 \text{ MeV}}{E_{y}}\right)$ fm
- Radius of nucleus: $5.6 \left(\frac{A}{100}\right)^{1/3}$ fm

$$\rightarrow \left| \overrightarrow{r_i} - \overrightarrow{r_j} \right| \ll 1/|\vec{q}| \rightarrow e^{i\vec{q}\cdot\left(\overrightarrow{r_i} - \overrightarrow{r_j}\right)} \sim 1$$

$$\rightarrow \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto v^2 A + v^2 \frac{A(A-1)}{2} \propto A^2$$

Nuclear size near PNS surface

- Large nuclear mass number is estimated for varational EOS.
 - → Cross section of the coherent elastic scattering is enhanced making cooling time scale longer.
- Thermal insulation by heavy nuclei

$$\begin{cases} \boldsymbol{\sigma} \propto A^2 \\ 1 / \boldsymbol{\lambda} \propto X_A \cdot A \end{cases}$$

A; mass number X_A ; fraction of nuclei



Neutrino Energy Sphere (1)

- Why thermalization is enhanced by coherent elastic scattering, no energy exchange?
 - neutrino mean free path for all reactions: λ_{tot}

" for energy exchange reactions: λ_{th}

- within 1 energy exchange reaction, neutrinos propagate the distance... *l*
- in this way, total reaction number... N

$$\ell = \sqrt{N\lambda_{tot}}$$

(random walk)





- In this way, neutrinos travel... $N\lambda_{tot}$ \rightarrow it is nothing but λ_{th} $N = \frac{\lambda_{th}}{\lambda_{tot}} \implies \ell = \sqrt{\lambda_{th}\lambda_{tot}}$
- Thermalization radius: R_{th}

$$\int_{R_{\rm th}}^{R_{\rm s}} \frac{\mathrm{d}r}{\ell} = \frac{2}{3} \implies \int_{R_{\rm th}}^{R_{\rm s}} \frac{\mathrm{d}r}{\sqrt{\lambda_{\rm th}\lambda_{\rm tot}}} = \frac{2}{3}$$

Neutrino Energy Sphere (3)

• Thermalization radius: R_{th}

$$\int_{\frac{R_{\rm s}}{\sqrt{\lambda_{\rm th}\lambda_{\rm tot}}}}^{R_{\rm s}} \frac{\mathrm{d}r}{\sqrt{\lambda_{\rm th}\lambda_{\rm tot}}} = \frac{2}{3}$$

 \rightarrow If λ_{tot} gets lower due to coherent scattering, thermalization radius goes outer.



Summary and Future work

- New EOS based on the variational method is constructed and applied to PNS cooling
 → EOS table: www.np.phys.waseda.ac.jp/EOS/
- Neutrino luminosity is determined by high density EOS and average energy is affected also by low density region due to coherent scattering.
- Future work: impacts on the spectrum of supernova relic neutrino should be studied.