Time evolution of the lepton family number of neutrino with Majorana mass

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Work based on Lepton Number Violation in a unified framework
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Unraveling of the History of the universe and Matter Evolution with underground Physics

2020/6/2 Zoom Room

Outline

- Introduction
- Lepton family number
- Numerical analysis
- Summary

Introduction

- Neutrino

- Neutrino oscillation phenomenon revealed that neutrinos have a small mass.
- Neutrino has the two mass hierarchy.

$$\begin{cases} Normal\ hierarchy : (m_1^2 < m_2^2 < m_3^2) \\ Inverted\ hierarchy : (m_3^2 < m_1^2 < m_2^2) \end{cases}$$

- Cosmic Background Neutrino(CNB)

- CNB is a low-energy neutrino whose existence is predicted from cosmology like CMB. CNB is estimated to be 1.92 K $= 1.7 \times 10^{-4}$(eV).

- Our Purpose

- Formulate the lepton family number when the neutrino is Majorana particle and calculate its time evolution.

Lepton family number

What situation do we think about?

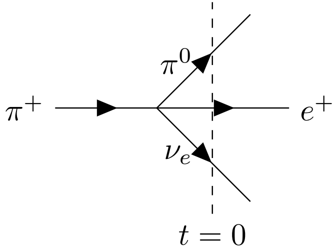


Figure: The production of the electron neutrino and its propagation. The neutrino acquires the Majorana mass term at $t = 0$.

The Lagrangian for the left-handed neutrino ν_L (Multi-flavor)

$$\mathcal{L} = \overline{\nu_{\alpha L}} i\gamma^\mu \partial_\mu \nu_{\alpha L} - \theta(t)\{\frac{m_{\alpha\beta}}{2}\overline{(\nu_{\alpha L})^c}\nu_{\beta L} + h.c.\}$$

Lepton family number

The mass term vanishes when $t_p < t < 0$.

$$\nu_{\alpha L}(\mathbf{x},t) = \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}(a_\alpha(\mathbf{p})e^{-i|\mathbf{p}|t+i\mathbf{p}\cdot\mathbf{x}}u_L(\mathbf{p}) + b_\alpha^\dagger(\mathbf{p})e^{i|\mathbf{p}|t-i\mathbf{p}\cdot\mathbf{x}}v_L(\mathbf{p}))$$

The lepton family number for $t_p < t < 0$:

$$L_\alpha(t) = \int d^3\mathbf{x} :\overline{\nu_{\alpha L}}\gamma^0\nu_{\alpha L} := \int' \frac{d^3\mathbf{p}}{2|\mathbf{p}|(2\pi)^3}\left(a_\alpha^\dagger(\mathbf{p})a_\alpha(\mathbf{p}) - b_\alpha^\dagger(\mathbf{p})b_\alpha(\mathbf{p})\right)$$

One can introduce mass eigenstates by unitary matrix V;
(V is mixing matrix = PMNS matrix)

$$\nu_{\alpha L} = V_{\alpha i}\nu_{iL}.$$

The mass matrix $m_{\alpha\beta}$ is diagonalized as,

$$m_i\delta_{ij} = (V^T)_{i\alpha}m_{\alpha\beta}V_{\beta j}$$

Lepton family number

Introducing the Majorana fields;

$$\psi_i = \nu_{iL} + (\nu_{iL})^c$$

Lagrangian can rewrite as follows;

$$\mathcal{L} = \frac{1}{2}\overline{\psi_i} i\gamma^\mu \partial_\mu \psi_i - \theta(t)\frac{m_i}{2}(\overline{\psi_i}\psi_i)$$

We consider the continunity condition at $t = 0$;

$$\nu_{L\alpha}(t = 0_-) = V_{\alpha i} L\psi_i(t = 0_+)$$

Lepton family number

Lepton family number of $\alpha = e, \mu, \tau$ is given by,

$$
\begin{aligned}
& L_\alpha(t) \\
&= \int' \frac{d^3\mathbf{p}}{(2\pi)^3|2\mathbf{p}|} \left\{ a_\alpha^\dagger(\mathbf{p},t) a_\alpha(\mathbf{p},t) - b_\alpha^\dagger(\mathbf{p},t) b_\alpha(\mathbf{p},t) \right\} \\
&= \int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|} \sum_{\beta,\gamma,i,j} \Big[\{ V_{\alpha i}^* V_{\beta i}^* V_{\alpha j} V_{\gamma j} a_\beta(-\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) - V_{\alpha i} V_{\beta i} V_{\alpha j}^* V_{\gamma j}^* b_\beta(-\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) \} \\
&\times \frac{m_i m_j}{|\mathbf{p}|^2} \sin(E_i(\mathbf{p})t) \sin(E_j(\mathbf{p})t) \\
&+ \{ V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\gamma j}^* a_\beta^\dagger(\mathbf{p}) a_\gamma(\mathbf{p}) - V_{\alpha i} V_{\beta i}^* V_{\alpha j}^* V_{\gamma j} b_\beta^\dagger(\mathbf{p}) b_\gamma(\mathbf{p}) \} \Big\{ \cos E_i(\mathbf{p})t \cos E_j(\mathbf{p})t \\
&+ \sin E_i(\mathbf{p})t \sin E_j(\mathbf{p})t + i \frac{E_i(\mathbf{p}) \sin E_i(\mathbf{p})t \cos E_j(\mathbf{p})t - E_j(\mathbf{p}) \sin E_j(\mathbf{p})t \cos E_i(\mathbf{p})t}{|\mathbf{p}|} \Big\} \\
&+ i \{ V_{\alpha i}^* V_{\beta i} V_{\alpha j} V_{\gamma j} a_\beta^\dagger(\mathbf{p}) a_\gamma^\dagger(-\mathbf{p}) + V_{\alpha i} V_{\beta i}^* V_{\alpha j}^* V_{\gamma j}^* b_\beta^\dagger(\mathbf{p}) b_\gamma^\dagger(-\mathbf{p}) \} \frac{m_j \sin E_j(\mathbf{p})t \{ \cos E_i(\mathbf{p})t + i \frac{E_i(\mathbf{p})}{|\mathbf{p}|} \sin E_i(\mathbf{p})t \}}{|\mathbf{p}|} \\
&- i \{ V_{\alpha i}^* V_{\beta i}^* V_{\alpha j} V_{\gamma j}^* a_\beta(-\mathbf{p}) a_\gamma(\mathbf{p}) + V_{\alpha i} V_{\beta i} V_{\alpha j}^* V_{\gamma j} b_\beta(-\mathbf{p}) b_\gamma(\mathbf{p}) \} \frac{m_i \sin E_i(\mathbf{p})t \{ \cos E_j(\mathbf{p})t - i \frac{E_j(\mathbf{p})}{|\mathbf{p}|} \sin E_j(\mathbf{p})t \}}{|\mathbf{p}|} \Big]
\end{aligned}
$$

where, $E_i = \sqrt{\boldsymbol{q}^2 + m_i^2}$.

Lepton family number

We can show following relation.

$$\sum_{\alpha} L_{\alpha}(t) = \sum_{\alpha} L_{\alpha}(0)$$

Note that V satisfies $\sum_{\alpha} V_{\alpha i}^{*} V_{\alpha j} = \delta_{ij}$. From this equation, it can be seen that all lepton numbers are conserved.
Next we consider the time evolution of lepton family number for a specific flavor eigenstate.

$$|\sigma(\mathbf{q})\rangle = \frac{a_{\sigma}^{\dagger}(\mathbf{q})|0\rangle}{\sqrt{\langle 0|a_{\sigma}(\mathbf{q})a_{\sigma}^{\dagger}(\mathbf{q})|0\rangle}}, \quad |\overline{\sigma}(\mathbf{q})\rangle = \frac{b_{\sigma}^{\dagger}(\mathbf{q})|0\rangle}{\sqrt{\langle 0|b_{\sigma}(\mathbf{q})b_{\sigma}^{\dagger}(\mathbf{q})|0\rangle}}$$

We will numerically calculate the time evolution of $\langle\sigma(\mathbf{q})|L_{\alpha}(t)|\sigma(\mathbf{q})\rangle$.

Numerical analysis

Relevant elements of the PMNS matrix;

$$V_{e1} = c_{12}c_{13}, \quad V_{e2} = s_{12}c_{13}e^{i\frac{\alpha_{21}}{2}}, \quad V_{e3} = s_{13}e^{-i\delta}e^{i\frac{\alpha_{31}}{2}}$$

$$V_{\mu 1} = -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta}, \; V_{\mu 2} = (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta})e^{i\frac{\alpha_{21}}{2}}, \; V_{\mu 3} = s_{23}c_{13}e^{i\frac{\alpha_{31}}{2}}$$

δ:Dirac-KM(Kobayashi-Maskawa)phases ,
α_{21} , α_{31}:Majorana phases

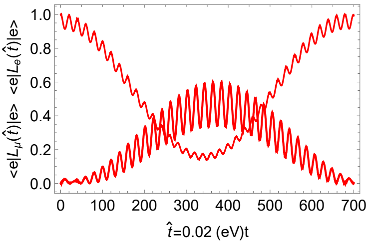
Numerical Data;

	normal mass hierarchy	inverted mass hierarchy
Δm^2_{21} (eV^2)	7.37×10^{-5}	7.37×10^{-5}
Δm^2_{21} (eV^2)	2.56×10^{-3}	2.54×10^{-3}
δ	1.38π	1.31π
s_{12}	0.545	0.545
s_{13}	0.147	0.147
s_{23}	0.652	0.767

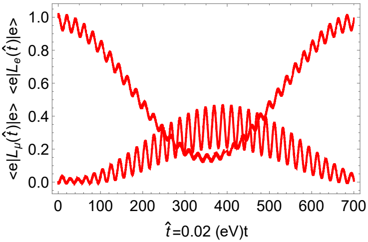
From Particle Data Group.

Numerical analysis

Numerical results for expectation value of the lepton family number with a specific initial condition $\langle e(\boldsymbol{q})|L_e(t)|e(\boldsymbol{q})\rangle$, $\langle e(\boldsymbol{q})|L_\mu(t)|e(\boldsymbol{q})\rangle$



Normal hierarchy case



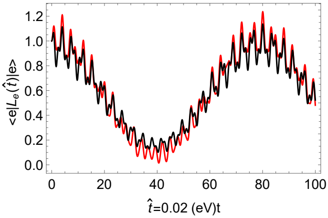
Inverted hierarchy case

Figure: The time evolution of $\langle e(\boldsymbol{q})|L_e(t)|e(\boldsymbol{q})\rangle$ and $\langle e(\boldsymbol{q})|L_\mu(t)|e(\boldsymbol{q})\rangle$ at $|\boldsymbol{q}| = 0.2$ (eV)

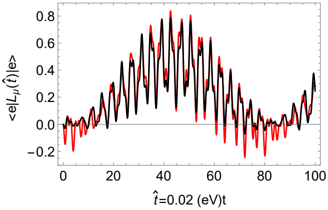
$\hat{t}$ is dimenssionless time $\hat{t} = 0.02(\text{eV})t$. We choose $(\alpha_{21}, \alpha_{31}) = (\frac{\pi}{2}, \frac{\pi}{3})$.

Numerical analysis

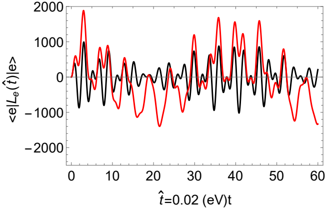
Upper panel shows $|\boldsymbol{q}| = 0.02$ (eV) case. Lower panel shows $|\boldsymbol{q}| = 0.0002$ (eV) case. Black : $(\alpha_{21}, \alpha_{31}) = (0, 0)$, Red : $(\alpha_{21}, \alpha_{31}) = (\frac{\pi}{2}, \frac{\pi}{3})$.



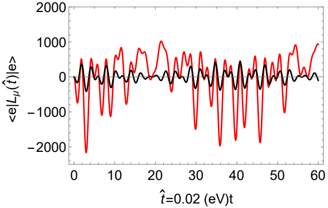
The time evolution of $\langle e(\boldsymbol{q})|L_e(t)|e(\boldsymbol{q})\rangle$



The time evolution of $\langle e(\boldsymbol{q})|L_\mu(t)|e(\boldsymbol{q})\rangle$



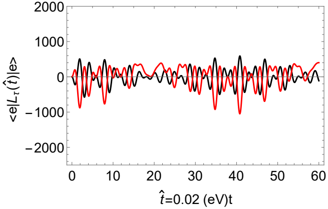
The time evolution of $\langle e(\boldsymbol{q})|L_e(t)|e(\boldsymbol{q})\rangle$



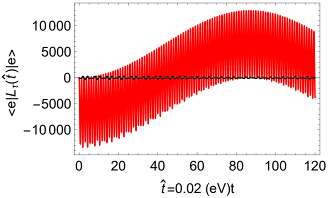
The time evolution of $\langle e(\boldsymbol{q})|L_\mu(t)|e(\boldsymbol{q})\rangle$

Numerical analysis

Tau lepton family number for normal (left) and inverted (right) case.
Black : $(\alpha_{21}, \alpha_{31}) = (0, 0)$, Red : $(\alpha_{21}, \alpha_{31}) = (\frac{\pi}{2}, \frac{\pi}{3})$.



Normal hierarchy case



Inverted hierarchy case

Figure: The time evolution of $\langle e(\boldsymbol{q})|L_\tau(t)|e(\boldsymbol{q})\rangle$ at $|\boldsymbol{q}| = 0.0002$ (eV).

Summary

- We have studied the time evolution of Lepton family number.
- It was shown that all lepton numbers are conserved.
- The lepton family number is sensitive to Majorana phases and neutrino mass hierarchy (inverted and normal).
- For instance, starting with a electron neutrino state, electron family number can change its sign. It implies neutrino to anti-neutrinos transition occurs during its time evolution.
- The abosolute value of the lepton number can be huge $>> 1$ if the momentum is smaller than their rest mass.