

Leptogenesis in the minimal gauged $U(1)_{L_\mu-L_\tau}$ model and the sign of the cosmological baryon asymmetry

K. Asai, K. Hamaguchi, N. Nagata, S.-Y. Tseng, arXiv:2005.01039



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Introduction

- Baryon asymmetry of the Universe $Y_B \equiv \frac{n_B}{s} \sim 8.7 \times 10^{-11}$
- Baryogenesis
 - heavy particle decays in GUT scenarios (Sakharov 1967)
 - electroweak baryogenesis (Kuzmin et al. 1985)
 - leptogenesis (Fukugita and Yanagida 1986)
 - supersymmetric condensate baryogenesis (Affleck and Dine 1985)
 - and many others...
- In this work, we discuss the generation of Y_B through leptogenesis
 - in the framework of minimal gauged $U(1)_{L_\mu - L_\tau}$ model

Minimal gauged $U(1)_{L_\mu-L_\tau}$ model

K. Asai, K. Hamaguchi, N. Nagata, arXiv:1705.00419, Eur. Phys. J. C77 (2017) 763

- Extending the gauge sector of SM

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{G_{SM}} \times U(1)_{L_\alpha-L_\beta}$$

G_{SM}

R. Foot, Mod.Phys.Lett. A6 (1991) 527-530

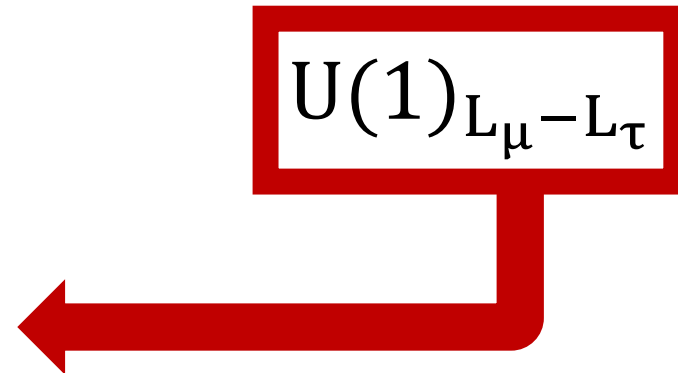
X.-G. He et al, Phys. Rev. D 43, R22

$$U(1)_{L_e-L_\mu}$$

$$U(1)_{L_e-L_\tau}$$

$$U(1)_{L_\mu-L_\tau}$$

- Less constrained by the experiments



- Charge assignment

$L_\mu - L_\tau$	$L_{e,\mu,\tau}$	e_R, μ_R, τ_R	$N_{e,\mu,\tau}$	σ	H
U(1) _Y	$-\frac{1}{2}$	-1	0	0	$+\frac{1}{2}$
U(1) _{$L_\mu - L_\tau$}	0,+1,-1	0,+1,-1	0,+1,-1	+1	0
SU(2)	2	1	1	1	2

$$\begin{aligned}
\mathcal{L}_N = & -\lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\
& - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \frac{1}{2} \sum_{\alpha,\beta=e,\mu} h_{\alpha\beta} \sigma N_\alpha^c N_\beta^c - \frac{1}{2} \sum_{\alpha,\beta=e,\tau} h_{\alpha\beta} \sigma^* N_\alpha^c N_\beta^c + \text{h.c.}
\end{aligned}$$

Neutrino physics : after the SSB of σ

- Neutrino mass matrices

$$M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \quad M_R = \begin{pmatrix} M_{ee} & \lambda_{e\mu}\langle\sigma\rangle & \lambda_{e\tau}\langle\sigma\rangle \\ \lambda_{e\mu}\langle\sigma\rangle & \boxed{0} & M_{\mu\tau} \\ \lambda_{e\tau}\langle\sigma\rangle & M_{\mu\tau} & \boxed{0} \end{pmatrix}$$

$$M_\nu^{-1} = -(M_D^{-1})^T M_R M_D^{-1} \simeq \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

two-zero minor structure

Neutrino physics : analysis of M_ν

$$\underbrace{M_\nu^{-1} = -(M_D^{-1})^T M_R M_D^{-1}}_{\text{two-zero minor structure}} = U_\nu (M_\nu^d)^{-1} U_\nu^T$$

two-zero minor structure diag(m_1, m_2, m_3)

PMNS matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\frac{\alpha_2}{2}} & \\ & & e^{i\frac{\alpha_3}{2}} \end{pmatrix}$$

$c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}, \theta_{ij}$: mixing angles, δ : Dirac CP phase, α_2, α_3 : Majorana phases

Solve the equations corresponding to the zero entries in M_ν^{-1}

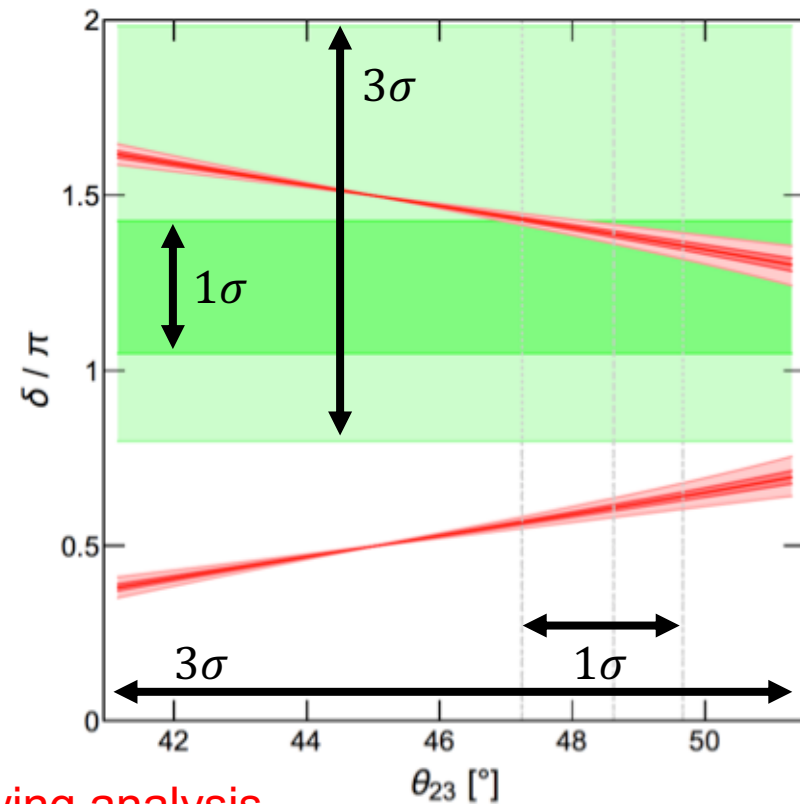
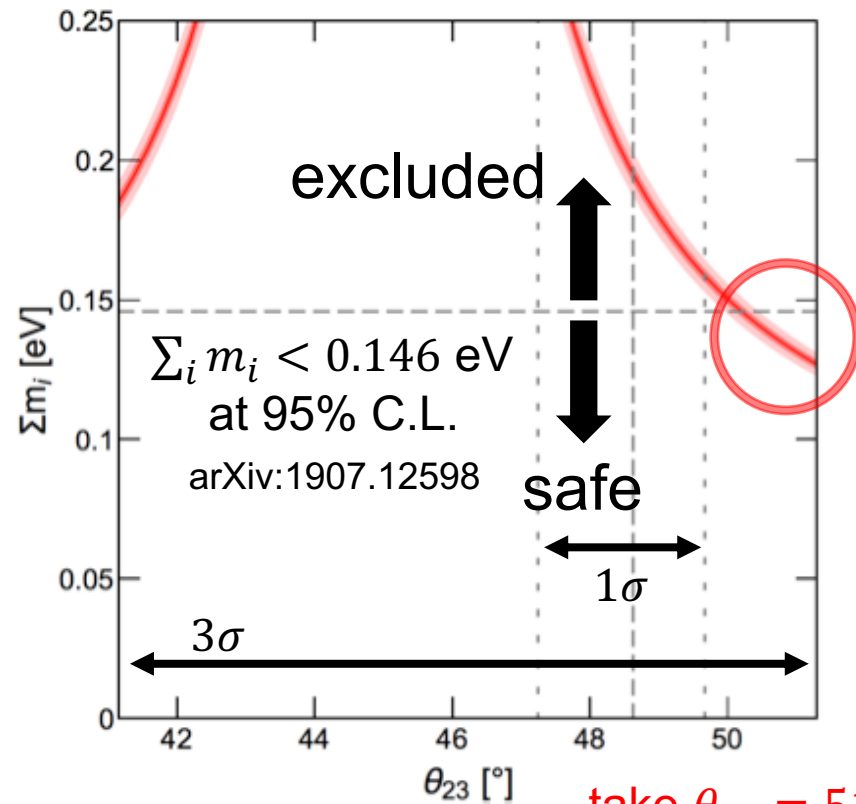
$$\hookrightarrow m_0 = m_0(\theta_{ij}, \Delta m_{ij}^2), \delta = \delta(\theta_{ij}, \Delta m_{ij}^2), \alpha_{2,3} = \alpha_{2,3}(\theta_{ij}, \Delta m_{ij}^2)$$

Neutrino physics : results

global-fitting group

$\theta_{ij}, \Delta m_{ij}^2$ from NuFIT 4.1 (2019)

Minimal gauged $U(1)_{L_\mu-L_\tau}$ model, normal mass ordering



take $\theta_{23} = 51^\circ$ in the following analysis

Leptogenesis

M. Fukugita, T. Yanagida, Phys. Lett. B174, 45 (1986)

- $Y_B \equiv n_B/s \sim 8.7 \times 10^{-11}$

- Sakharov conditions A.D. Sakharov, JETP Lett. 5 (1967) 24-27

- nonconserved baryon number
- C and CP violations
- departure from thermal equilibrium

we consider $T_R < M_1$

V. A. Kuzmin et al., Phys. Lett. 155B (1985) 36

Y_B ← sphaleron

$$\epsilon_i = \frac{\Gamma(\hat{N}_i \rightarrow LH) - \Gamma(\hat{N}_i \rightarrow \bar{L}H^*)}{\Gamma(\hat{N}_i \rightarrow LH) + \Gamma(\hat{N}_i \rightarrow \bar{L}H^*)} = \frac{1}{8\pi} \frac{1}{(\hat{\lambda}\hat{\lambda}^\dagger)_{ii}} \sum_{j \neq i} \text{Im}\{(\hat{\lambda}\hat{\lambda}^\dagger)_{ij}^2\} f\left(\frac{M_j^2}{M_i^2}\right)$$

- Nonthermal leptogenesis T. Asaka et al., Phys. Lett. B 464 (1999) 12

- N_i s are produced nonthermally in **inflaton** decays

$$f(x) = \sqrt{x} \left[1 - (1+x) \ln\left(\frac{1+x}{x}\right) + \frac{1}{1-x} \right]$$

→ $U(1)_{L\mu-L\tau}$ breaking field σ

Nonthermal leptogenesis

- Take σ as inflaton

$$\mathcal{L}_N = -\lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\ - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \frac{1}{2} \sum_{\alpha, \beta=e, \mu} h_{\alpha\beta} \sigma N_\alpha^c N_\beta^c - \frac{1}{2} \sum_{\alpha, \beta=e, \tau} h_{\alpha\beta} \sigma^* N_\alpha^c N_\beta^c + \text{h.c.}$$

$$\mathcal{L}_\sigma = \frac{|\partial_\mu \sigma|^2}{(1 - |\sigma|^2/\Lambda^2)^2} - \kappa (|\sigma|^2 - \langle \sigma \rangle^2)^2 \longrightarrow \mathcal{L}_\sigma = \frac{1}{2} (\partial \tilde{\varphi})^2 - \kappa \Lambda^4 \left[\tanh^2 \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right) - \left(\frac{\langle \sigma \rangle}{\Lambda} \right)^2 \right]^2$$

- Inflaton mass

$$m_\varphi \simeq 2\sqrt{\kappa} \langle \sigma \rangle \simeq 3 \times 10^{10} \text{ GeV} \times \left(\frac{\langle \sigma \rangle}{10^{13} \text{ GeV}} \right) \left(\frac{\Lambda}{10^{16} \text{ GeV}} \right)^{-1} \left(\frac{N_e}{50} \right)^{-1}$$

CMB normalization

$$\varphi \equiv \sqrt{2} \text{Re}(\sigma) \quad \frac{\varphi}{\sqrt{2}\Lambda} \equiv \tanh \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right)$$

Baryon asymmetry

Nonthermal leptogenesis

$$\frac{3}{4} \cdot \frac{T_R}{m_\varphi}$$

$$\frac{-28}{79}$$

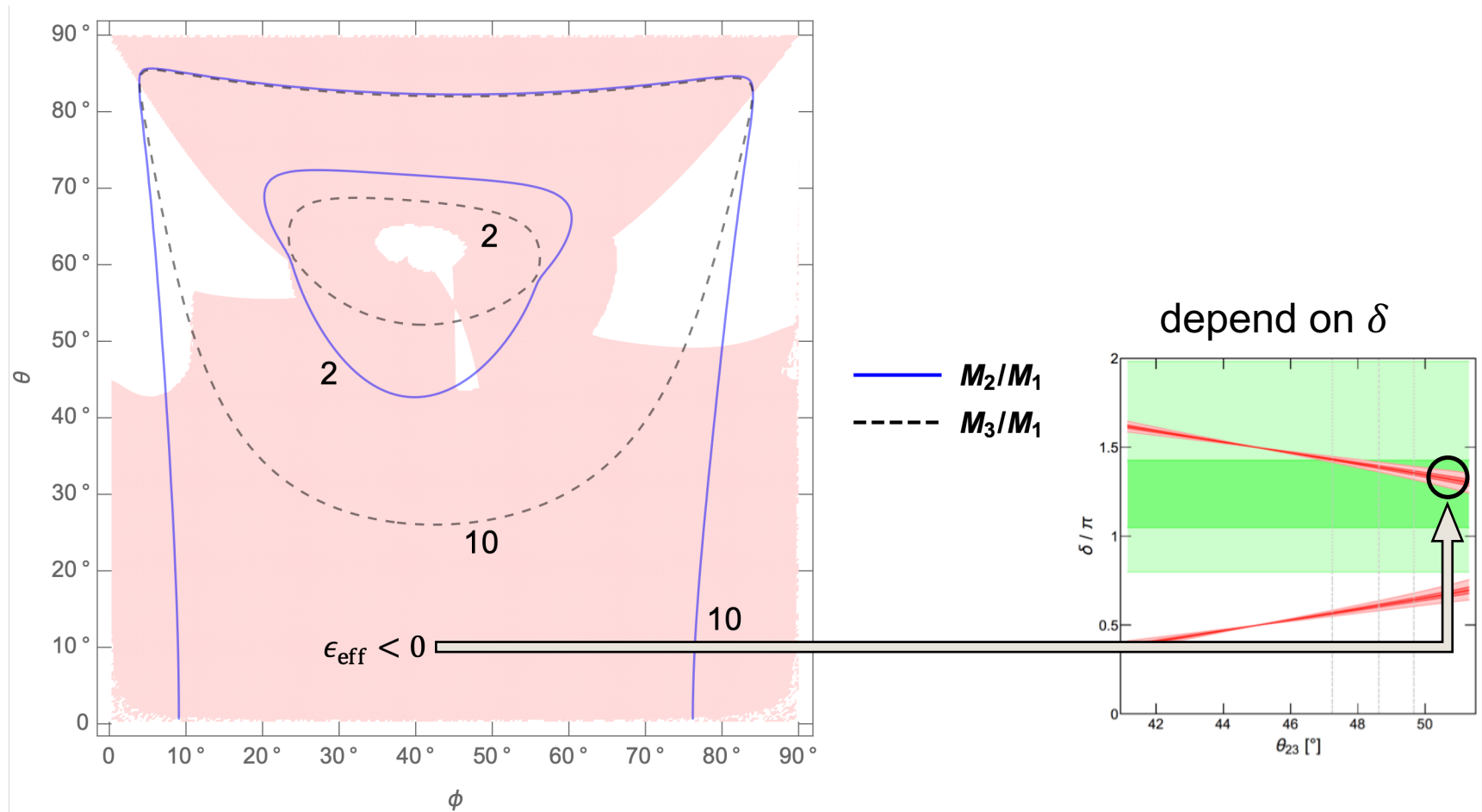
$$Y_B = \frac{n_B}{s} = \frac{n_\varphi}{s} \times \frac{n_N}{n_\varphi} \times \frac{n_L}{n_N} \times \frac{n_B}{n_L}$$

$$2 \sum_i \epsilon_i Br(\varphi \rightarrow N_i N_i) + \sum_{m < n} (\epsilon_m + \epsilon_n) Br(\varphi \rightarrow N_m N_n) \equiv 2 \times \epsilon_{\text{eff}}$$

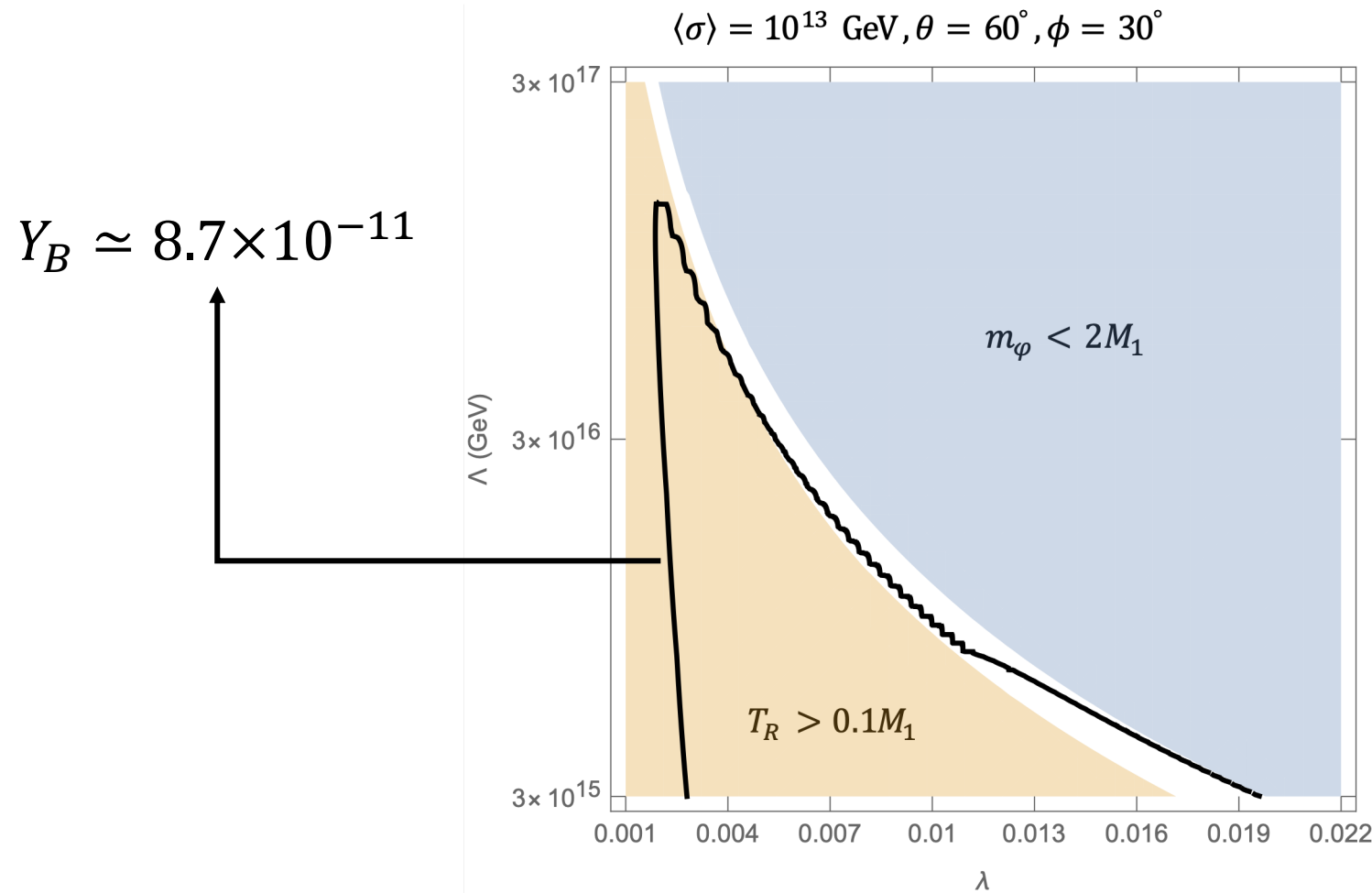
$$\epsilon_{\text{eff}} < 0$$

$$\epsilon_{\text{eff}} = \sum_i \epsilon_i \text{Br}(\phi \rightarrow N_i N_i) + (1/2) \sum_{m < n} (\epsilon_m + \epsilon_n) \text{Br}(\phi \rightarrow N_m N_n)$$

- $\lambda = 0.01$
- $\langle \sigma \rangle = 10^{13} \text{ GeV}$
- $\Lambda = 10^{16} \text{ GeV}$
- NuFIT inputs



Baryon asymmetry : result



cosmic string
 $\langle \sigma \rangle \lesssim 2 \times 10^{13} \text{ GeV}$

It works!

Summary

- We have investigated the baryon asymmetry in the Universe
 - nonthermal leptogenesis with minimal gauged $U(1)_{L_\mu-L_\tau}$ model
- It is shown that Y_B can be reproduced in this framework
- Future prospect
 - thermal leptogenesis, etc

Backup

Introduction

- Light neutrino mass \rightarrow adding RH neutrinos

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Seesaw mechanism

P. Minkowski, Phys. Lett. B67 (1977) 421–428

T. Yanagida, Conf. Proc. C7902131 (1979) 95–99

! mass terms tightly restricted by the $U(1)_{L_\alpha - L_\beta}$ symmetry

$$\begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \simeq \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix} \quad \text{block-diagonal}$$

this simple structure fails to explain the sizable neutrino mixing...

Introduction

- Spontaneous breaking of $U(1)_{L_\alpha - L_\beta}$ symmetry

introduce only one additional scalar field

$$M_\nu^{-1} \simeq \begin{pmatrix} * & * & * \\ * & \boxed{0} & * \\ * & * & \boxed{0} \end{pmatrix}$$

or

$$M_\nu \simeq \begin{pmatrix} * & \boxed{0} & * \\ 0 & \boxed{0} & * \\ * & * & * \end{pmatrix}$$

Two-zero structure leads into strong predictive power!

What are the contents?

- The SM

	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	e_R	$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	u_R	d_R	H
$SU(3)_C$	1	1	3	3	3	1
$SU(2)_L$	2	1	2	1	1	2
$U(1)_Y$	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

- Three RH neutrinos
- One extra scalar field

Input parameters

global-fitting group

NuFIT 4.1 (2019)

	Normal Ordering (best fit)		
	bfp $\pm 1\sigma$	3σ range	
with SK atmospheric data	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \rightarrow 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \rightarrow 36.27$
	$\sin^2 \theta_{23}$	$0.563^{+0.018}_{-0.024}$	$0.433 \rightarrow 0.609$
	$\theta_{23}/^\circ$	$48.6^{+1.0}_{-1.4}$	$41.1 \rightarrow 51.3$
	$\sin^2 \theta_{13}$	$0.02237^{+0.00066}_{-0.00065}$	$0.02044 \rightarrow 0.02435$
	$\theta_{13}/^\circ$	$8.60^{+0.13}_{-0.13}$	$8.22 \rightarrow 8.98$
	$\delta_{\text{CP}}/^\circ$	221^{+39}_{-28}	$144 \rightarrow 357$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	$6.79 \rightarrow 8.01$
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.528^{+0.029}_{-0.031}$	$+2.436 \rightarrow +2.618$

Inflaton physics

$$P_\zeta = \frac{V^3}{12\pi^2 M_P^6 V'^2}$$

$$= \frac{\kappa \Lambda^6}{96\pi^2 M_P^6} \left\{ 1 - \left[\frac{\kappa}{\Lambda} \coth \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right) \right]^2 \right\}^4 \sinh^4 \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right) \tanh^2 \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right)$$

$$\simeq \frac{\kappa}{6\pi^2} \left[\frac{N_e (\Lambda^2 - \langle \sigma \rangle^2)}{M_P \Lambda} \right]^2.$$

$$P_\zeta \simeq 2.1 \times 10^{-9}$$

$$\kappa \simeq 3 \times 10^{-6} \times \left(\frac{N_e}{50} \right)^{-2} \left(\frac{\Lambda}{10^{16} \text{ GeV}} \right)^{-2}$$

$$\zeta = -\frac{H_{\text{inf}}}{\dot{\phi}} \delta\phi$$

$$\langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k)$$

$$\mathcal{P}_\zeta(k) = \frac{H_{\text{inf}}^4}{4\pi^2 (\dot{\phi})^2} = \frac{V^3}{12\pi^2 (V')^2 M_G^6}$$

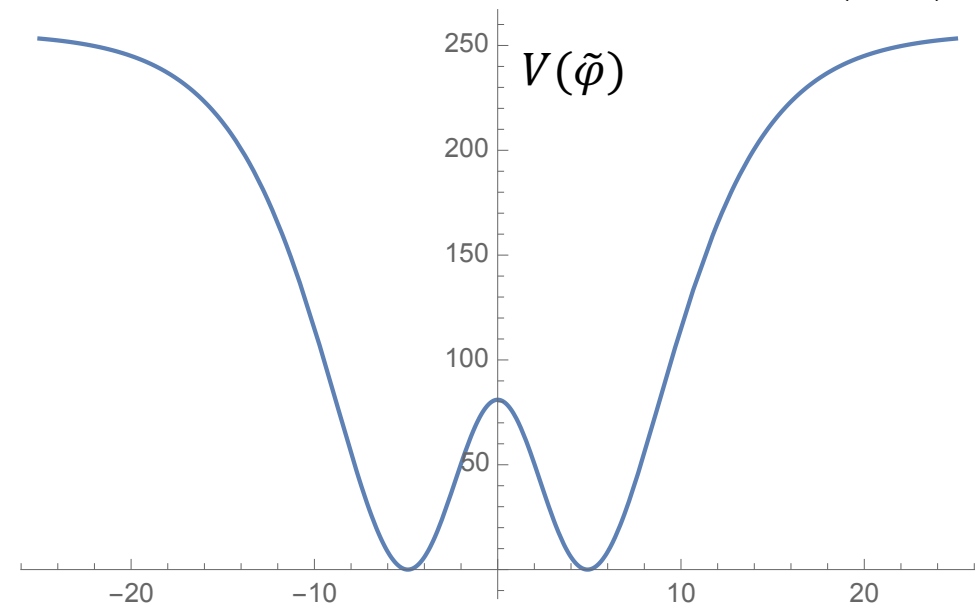
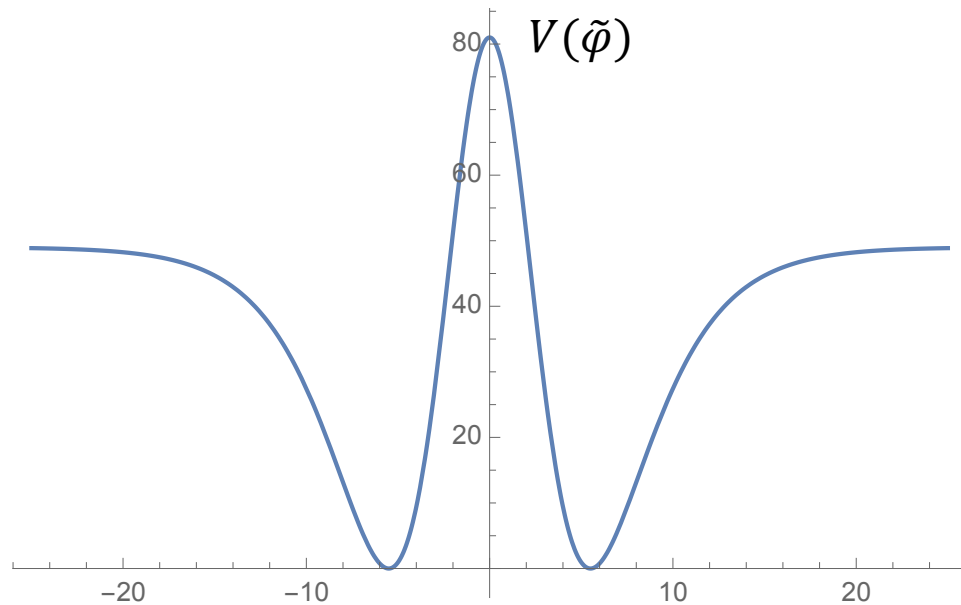
$$m_\sigma^2 = \left. \frac{d^2 V}{d\tilde{\varphi}^2} \right|_{\tilde{\varphi}=\tilde{\varphi}_{\text{min}}} = 4\kappa \langle \sigma \rangle^2 \left(1 - \frac{\langle \sigma \rangle^2}{\Lambda^2} \right)^2$$

$$H_{\text{inf}} \simeq \frac{\Lambda^2}{M_P} \sqrt{\frac{\kappa}{3}} \simeq 4 \times 10^{10} \text{ GeV} \times \left(\frac{\Lambda}{10^{16} \text{ GeV}} \right) \left(\frac{N_e}{50} \right)^{-1}$$

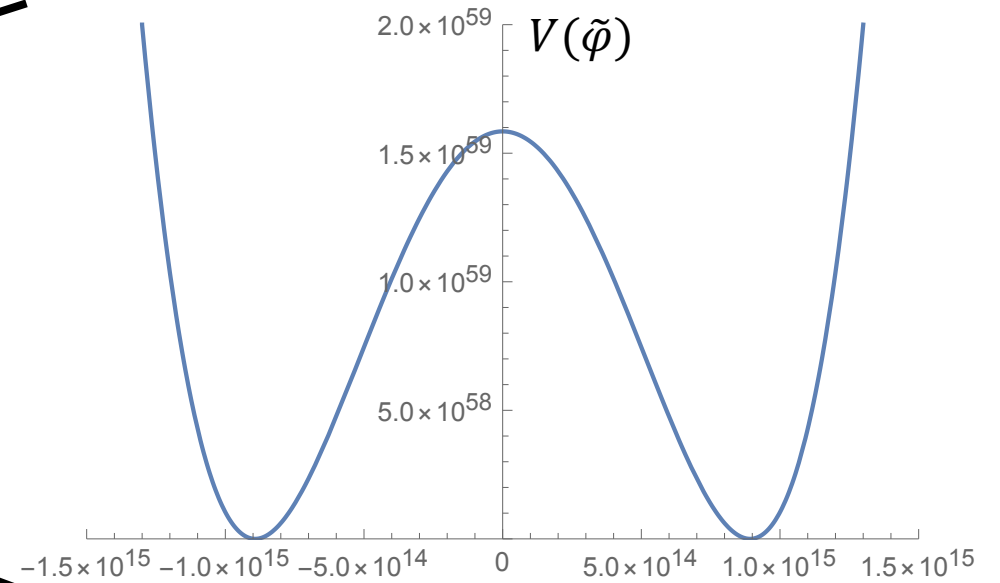
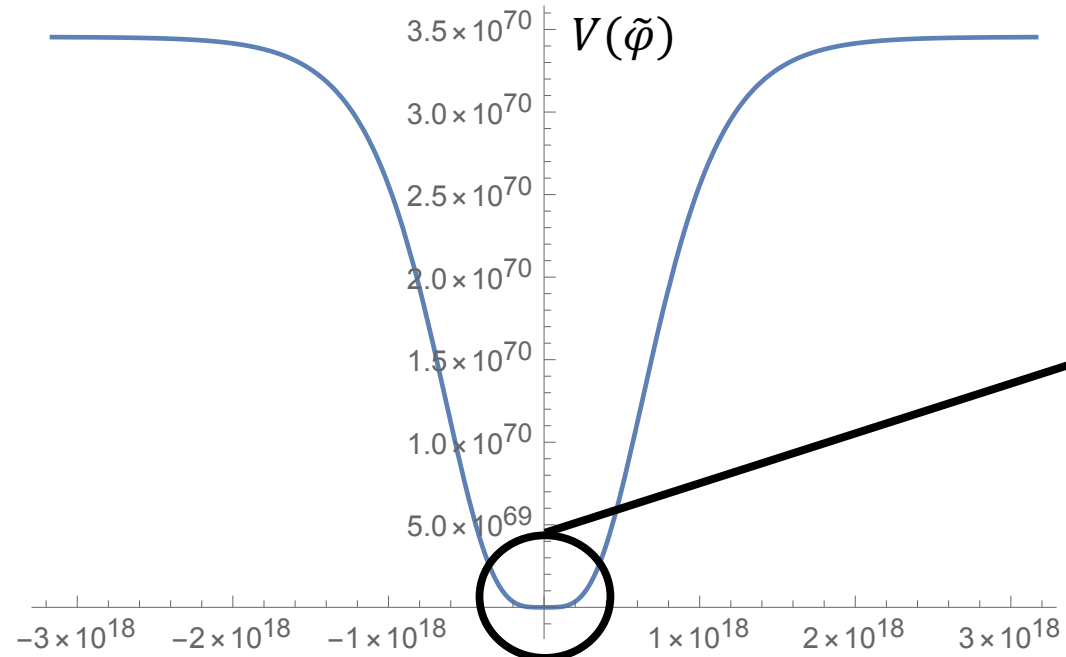
Non-thermal leptogenesis

- Taking σ as inflaton

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial\tilde{\varphi})^2 - \kappa\Lambda^4 \left[\tanh^2\left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda}\right) - \left(\frac{\langle\sigma\rangle}{\Lambda}\right)^2 \right]^2 \quad \frac{\varphi}{\sqrt{2}\Lambda} \equiv \tanh\left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda}\right)$$



Inflaton potential



Cosmic string

$$\mathcal{L} = D_\mu \Phi D^\mu \Phi^\dagger - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda (\Phi^\dagger \Phi - \sigma^2/2)^2$$

$$\mu \equiv \frac{dH}{dz} = \int r dr d\theta \mathcal{H} = - \int r dr d\theta \mathcal{L}$$

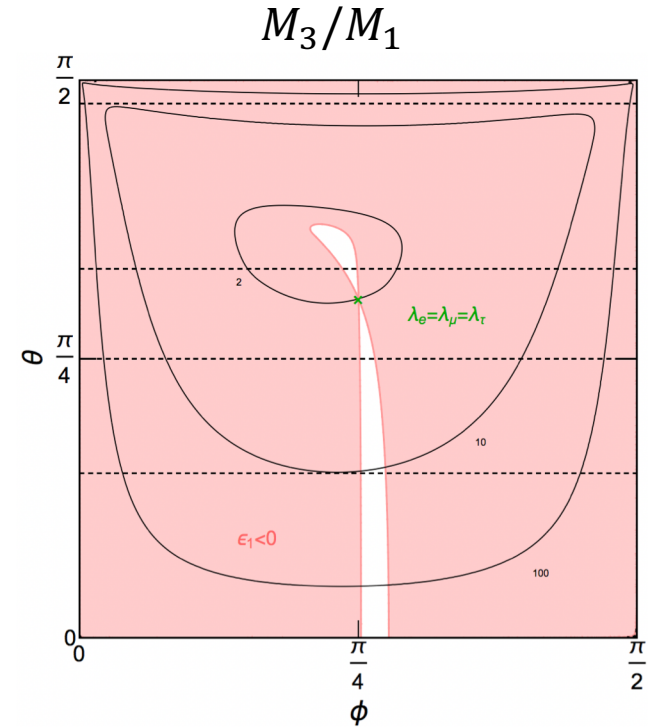
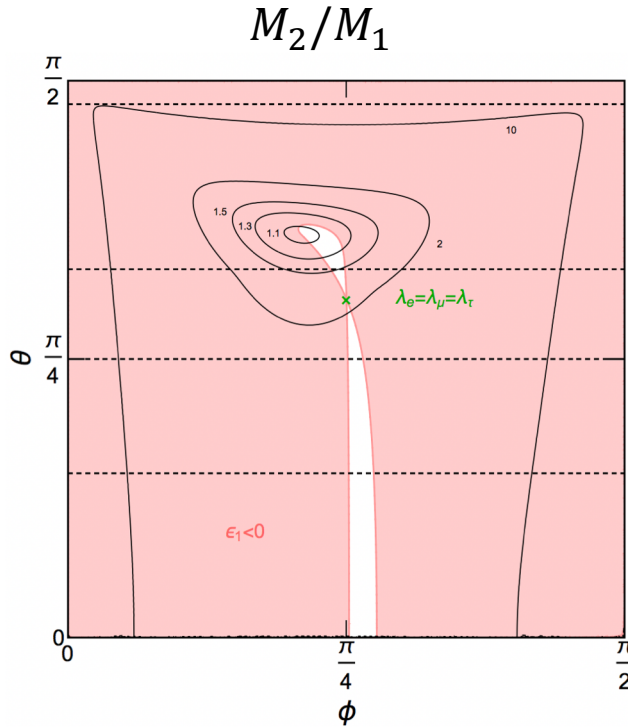
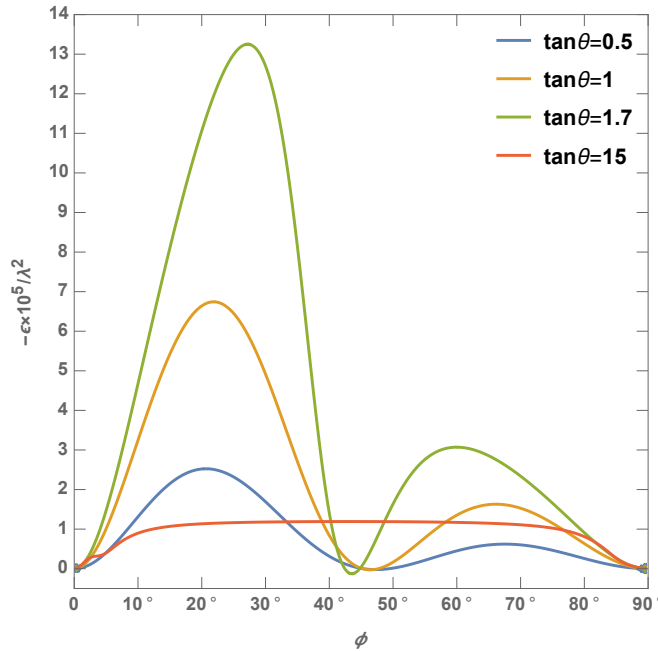
$$= \int_0^\infty \int_0^{2\pi} r dr d\theta \left[\left| \frac{\partial \Phi}{\partial r} \right|^2 + \left| \frac{1}{r} \frac{\partial \Phi}{\partial \theta} - ie A_\theta \Phi \right|^2 + V(\Phi) + \frac{B^2}{2} \right]$$



$$\mu \simeq \pi \sigma^2$$

Leptogenesis

$$(\lambda_e, \lambda_\mu, \lambda_\tau) \equiv \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$$



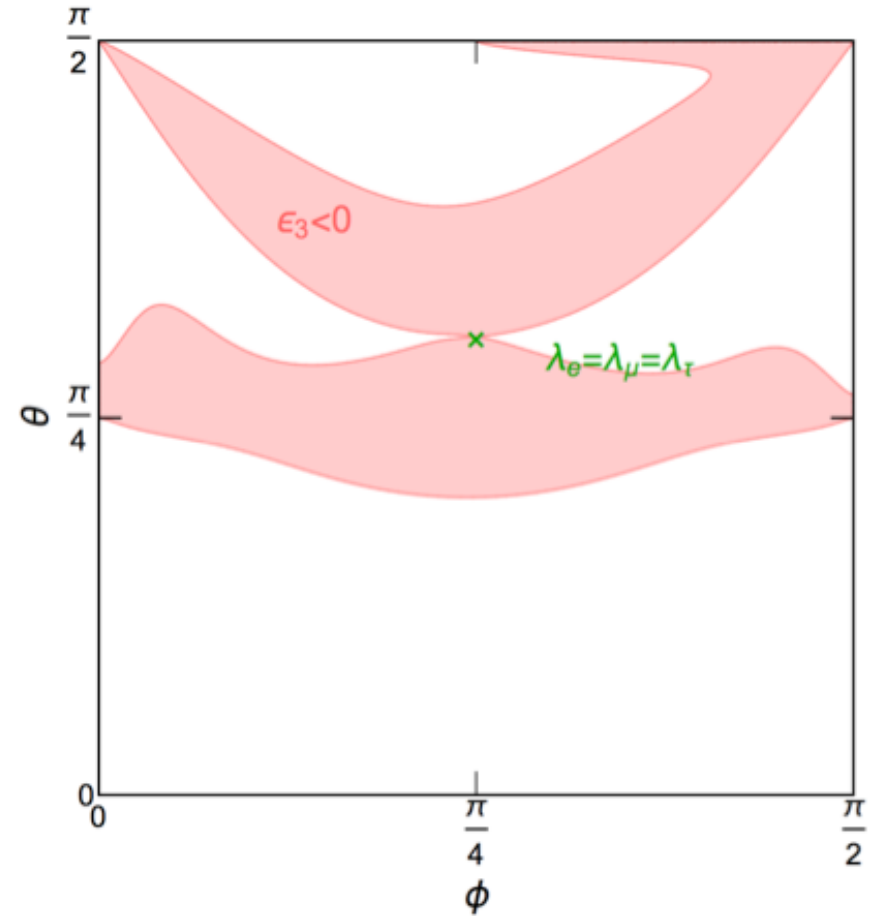
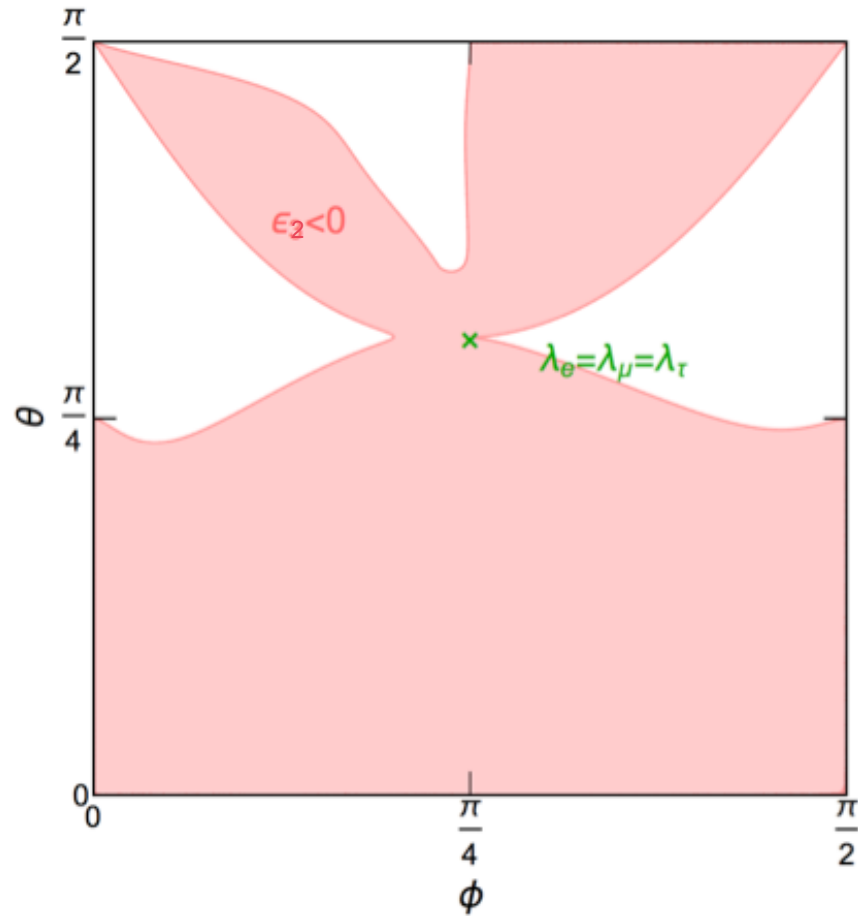
$$Y_{\Delta B} \equiv \frac{n_B}{s} \simeq 8.7 \times 10^{-11} > 0$$

$$\frac{n_B}{n_L} = -\frac{28}{79}$$



$$\epsilon_1 < 0$$

Leptogenesis

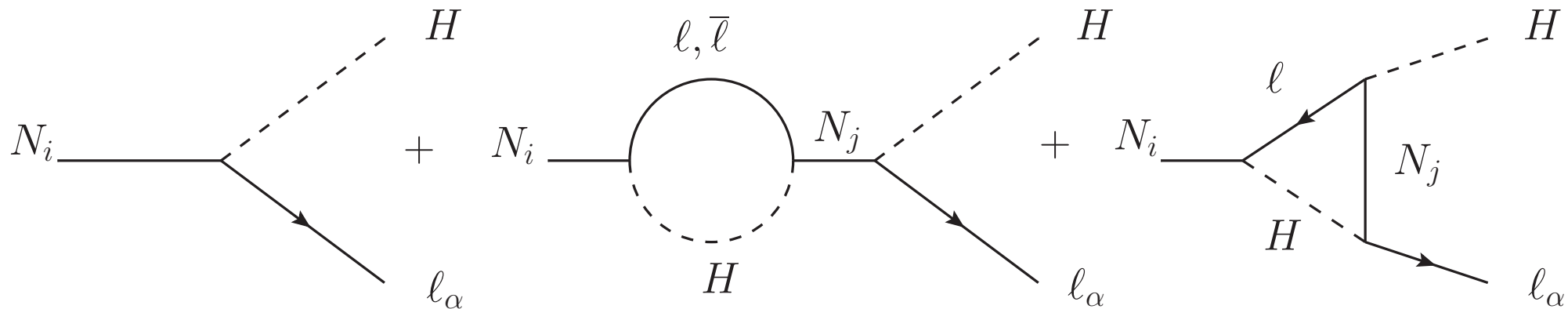


Leptogenesis

M. Fukugita, T. Yanagida, Phys. Lett. B174, 45 (1986)

- $Y_B \equiv n_B/s \sim 8.7 \times 10^{-11}$
- Decays of RH neutrinos generate lepton asymmetry

$$\epsilon = \frac{\Gamma(N \rightarrow lH) - \Gamma(N \rightarrow \bar{l}H^\dagger)}{\Gamma(N \rightarrow lH) + \Gamma(N \rightarrow \bar{l}H^\dagger)}$$



Leptogenesis

- Baryon asymmetry in the universe
- Decays of RH neutrinos generate lepton asymmetry

$$\frac{1}{(8\pi)} \frac{1}{[\lambda^\dagger \lambda]_{11}} \sum_j \text{Im} \left\{ [(\lambda^\dagger \lambda)_{1j}]^2 \right\} g(x_j)$$

λ : Yukawa coupling

$$g(x) = \sqrt{x} \left[\frac{1}{1-x} + 1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] \quad x_j = \frac{M_j^2}{M_1^2}$$

Leptogenesis

- Baryon asymmetry in the universe

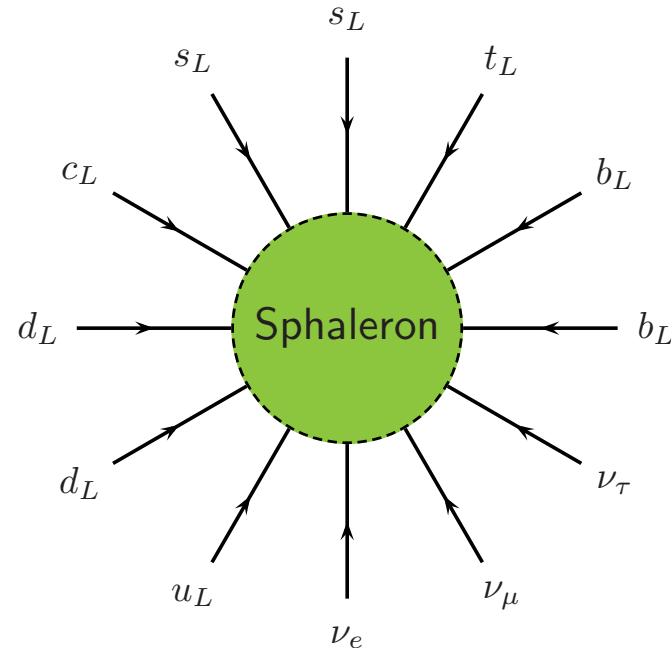
$$Y_{\Delta B} \equiv \frac{n_B}{s} \simeq 8.7 \times 10^{-11}$$

- Decays of RH neutrinos generate lepton asymmetry

- Sphaleron processes

V. A. Kuzmin et al., Phys. Lett. 155B (1985) 36

$$B = \frac{28}{79} \times (B - L)$$



Right-handed neutrino sector

- RH neutrino mass matrix $M_R = -M_D^T M_\nu^{-1} M_D$
- Parameters $(\lambda_e, \lambda_\mu, \lambda_\tau) \equiv \lambda(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$

$$\Delta\mathcal{L} = - \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^3 \hat{\lambda}_{\alpha i} (L_\alpha \cdot H) \hat{N}_i^c - \frac{1}{2} \sum_{i=1}^3 M_{Ri}^{\text{diag}} \hat{N}_i^c \hat{N}_i^c + \text{h.c.}$$

$$M_R = \Omega^* M_R^{\text{diag}} \Omega^\dagger, \quad \hat{N}_i^c = \sum_{\alpha} \Omega_{i\alpha}^\dagger N_\alpha^c, \quad \hat{\lambda}_{\alpha i} = \sum_{\beta} \lambda_{\alpha\beta} \Omega_{\beta i}$$

Non-thermal leptogenesis

- As a concrete example, we first consider this simple scenario
- Pairs of RH neutrinos generated from inflaton decays
- N_R out-of-equilibrium decay : $T_R \simeq \left(\frac{90}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{\Gamma_\varphi M_P} < M_1$
- Taking σ as inflaton

$$\begin{aligned} \Delta\mathcal{L} = & - y_e e_R^c L_e H^\dagger - y_\mu \mu_R^c L_\mu H^\dagger - y_\tau \tau_R^c L_\tau H^\dagger \\ & - \lambda_e N_e^c (L_e \cdot H) - \lambda_\mu N_\mu^c (L_\mu \cdot H) - \lambda_\tau N_\tau^c (L_\tau \cdot H) \\ & - \frac{1}{2} M_{ee} N_e^c N_e^c - M_{\mu\tau} N_\mu^c N_\tau^c - \lambda_{e\mu} \sigma N_e^c N_\mu^c - \lambda_{e\tau} \sigma^* N_e^c N_\tau^c + \text{h.c.} \end{aligned}$$

Non-thermal leptogenesis

- Pairs of RH neutrinos generated from inflaton decays

- N_R out-of-equilibrium decay : $T_R \simeq \left(\frac{90}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{\Gamma_\varphi M_P} < M_1$

- Taking σ as inflaton

$$\mathcal{L}_\sigma = \frac{|\partial_\mu \sigma|^2}{(1 - |\sigma|^2/\Lambda^2)^2} - \kappa(|\sigma|^2 - \langle \sigma \rangle^2)^2$$

Non-thermal leptogenesis

- Pairs of RH neutrinos generated from inflaton decays

- N_R out-of-equilibrium decay : $T_R \simeq \left(\frac{90}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{\Gamma_\varphi M_P} < M_1$

- Taking σ as inflaton

$$\mathcal{L}_\sigma = \frac{1}{2}(\partial\tilde{\varphi})^2 - \kappa\Lambda^4 \left[\tanh^2 \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right) - \left(\frac{\langle\sigma\rangle}{\Lambda} \right)^2 \right]^2 \quad \varphi \equiv \sqrt{2}\text{Re}(\sigma)$$
$$\frac{\varphi}{\sqrt{2}\Lambda} \equiv \tanh \left(\frac{\tilde{\varphi}}{\sqrt{2}\Lambda} \right)$$

Non-thermal leptogenesis

- Pairs of RH neutrinos generated from inflaton decays

- N_R out-of-equilibrium decay : $T_R \simeq \left(\frac{90}{\pi^2 g_*} \right)^{\frac{1}{4}} \sqrt{\Gamma_\varphi M_P} < M_1$

- Taking σ as inflaton

$$m_\varphi \simeq 2\sqrt{\kappa} \langle \sigma \rangle \simeq 3 \times 10^{10} \text{ GeV} \times \left(\frac{\langle \sigma \rangle}{10^{13} \text{ GeV}} \right) \left(\frac{\Lambda}{10^{16} \text{ GeV}} \right)^{-1} \left(\frac{N_e}{50} \right)^{-1}$$

from CMB normalization

Non-thermal leptogenesis

- Inflaton decay

$$\Gamma_\varphi = \sum_{i,j} \frac{m_\varphi}{32\pi} \left[1 - \frac{2(M_i^2 + M_j^2)}{m_\varphi^2} + \frac{(M_i^2 - M_j^2)^2}{m_\varphi^4} \right]^{\frac{1}{2}} \\ \times \left[\text{Re}(\hat{h}_{ij})^2 \left\{ 1 - \frac{(M_i + M_j)^2}{m_\varphi^2} \right\} + \text{Im}(\hat{h}_{ij})^2 \left\{ 1 - \frac{(M_i - M_j)^2}{m_\varphi^2} \right\} \right]$$