A Simulation of Core-collapse Supernovae in Three-dimensional Space with Full Boltzmann Neutrino Transport on the Supercomputer FUGAKU

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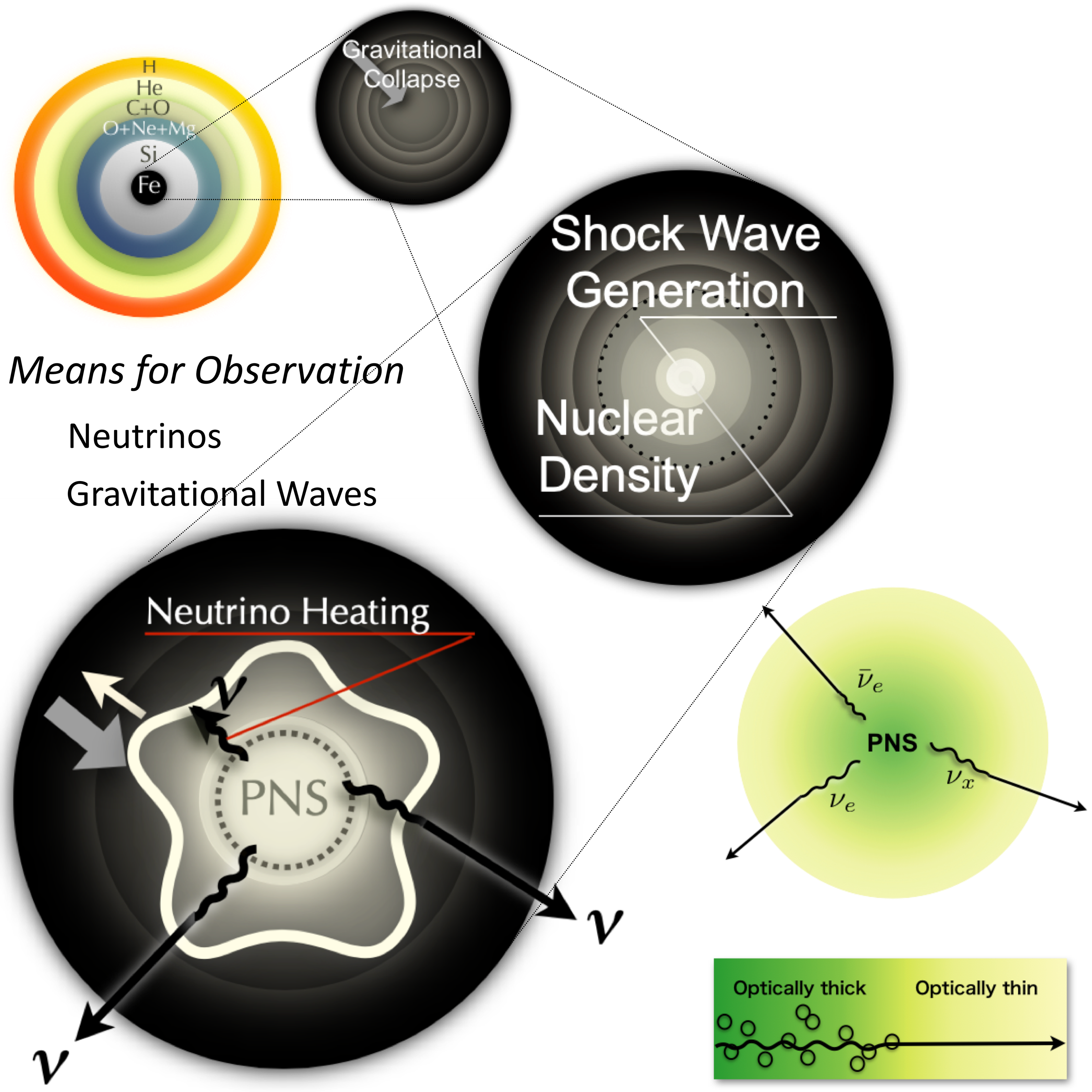
Neutrino Heating Mechanism for Core-collapse Supernovae



Means for Observation

Neutrinos

Gravitational Waves

Euler Equations

Hydrodynamics

Continuity Equation: $\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x^j}\left(\rho v^j\right)=0$

Equations of Motion: $\frac{\partial}{\partial t}\left(\rho v_i\right)+\frac{\partial}{\partial x^j}\left(\rho v_i v^j+P\delta_i^j\right)=-\rho\frac{\partial \psi}{\partial x^j}\boxed{-G^i}$

Energy Equation: $\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+e\right)+\frac{\partial}{\partial x^j}\left[\left(\frac{1}{2}\rho v^2+e+P\right)v^j\right]=-\rho v^j\frac{\partial \psi}{\partial x^j}\boxed{-G^0}$

Time-Evolution Equation of Electron Number: $\frac{\partial}{\partial t}\left(\frac{\rho Y_e}{m_A}\right)+\frac{\partial}{\partial x^j}\left(\frac{\rho Y_e}{m_A}v^j\right)=\boxed{-\Gamma}$

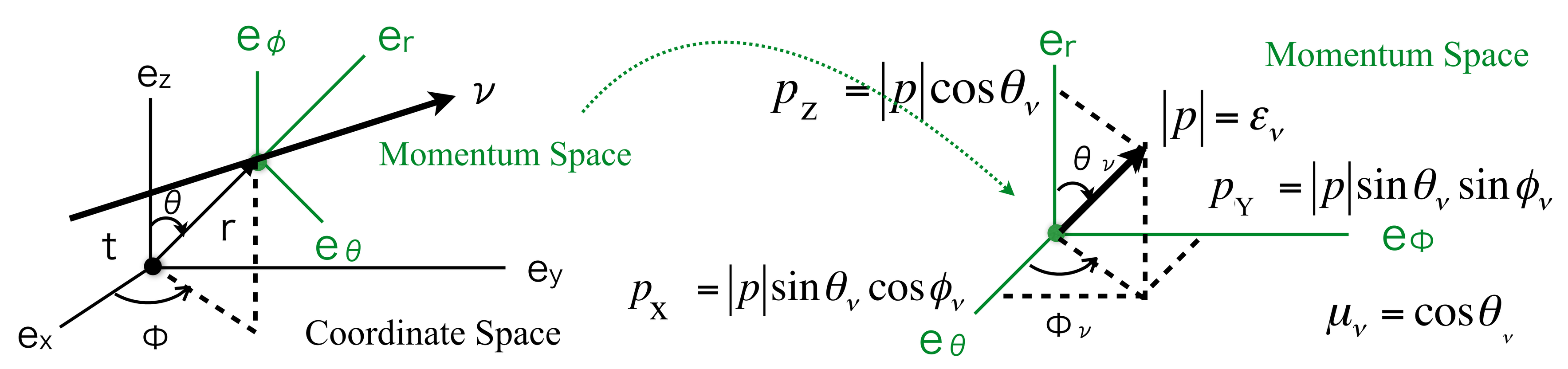
Poisson's equation for gravity: $\Delta\psi = 4\pi G\rho$

EOS table of Nuclear Matter (LS K=220MeV, Furusawa+Togashi, etc.)

ρ: density, v: velocity, P: pressure, e: internal energy, ψ: the gravitational potential, G: the gravitational constant (=6.67×10^{-8}[cm^{3}g^{-1}s^{-2}]), Y_e: electron fraction, m_A: the atomic mass unit, G^0: neutrino radiation energy, G^i: neutrino radiation pressure, Γ: deleptonization rate ($\equiv \Gamma_{\nu_e}-\Gamma_{\bar{\nu}_e}$), Γ_s: neutrino number density

Boltzmann Equation

Neutrino Radiation



Neutrino distribution function

$$f(t,r,\theta,\phi;\varepsilon_\nu,\mu_\nu,\phi_\nu)$$

Boltzmann Equation

$$\frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu}+\frac{dp^i}{d\lambda}\frac{\partial f}{\partial p^i}=\left(\frac{\delta f}{\delta\lambda}\right)_{\text{collision}}$$

Boltzmann Equation in the spherical coordinate

$$\frac{\partial f}{\partial t}+\cos\theta_\nu\frac{\partial f}{\partial r}+\frac{\sin\theta_\nu\cos\theta_\nu}{r}\frac{\partial f}{\partial\theta}+\frac{\sin\theta_\nu\sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial\phi}-\frac{\sin\theta_\nu}{r}\frac{\partial f}{\partial\theta_\nu}-\frac{\cos\theta}{\sin\theta}\frac{\sin\theta_\nu\sin\phi_\nu}{r}\frac{\partial f}{\partial\phi_\nu}=\left(\frac{\delta f}{\delta t}\right)_{\text{collision}}$$

$$\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}}=\left[\frac{\delta f}{\delta\tau}\right]_{\text{emis-abs (s)}}+\left[\frac{\delta f}{\delta\tau}\right]_{\text{scat (s)}}+\left[\frac{\delta f}{\delta\tau}\right]_{\text{pair (s)}}$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{emis-abs}}=-R_{\text{abs}}(\varepsilon,\Omega)f(\varepsilon,\Omega)+R_{\text{emis}}(\varepsilon,\Omega)[1-f(\varepsilon,\Omega)].$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{scat}}=-\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{scat}}(\varepsilon,\Omega;\varepsilon',\Omega')f(\varepsilon,\Omega)\times[1-f(\varepsilon',\Omega')]+\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{scat}}(\varepsilon',\Omega';\varepsilon,\Omega)\times f(\varepsilon',\Omega')[1-f(\varepsilon,\Omega)],$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{pair}}=-\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{pair-anni}}(\varepsilon,\Omega;\varepsilon',\Omega')\times f(\varepsilon,\Omega)\bar{f}(\varepsilon',\Omega')+\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{pair-emis}}(\varepsilon,\Omega;\varepsilon',\Omega')\times[1-f(\varepsilon,\Omega)][1-\bar{f}(\varepsilon',\Omega')],$$

Ω' denotes the angle variables after/before the scattering

$\bar{f}(\varepsilon',\Omega')$ denotes the distribution of anti-neutrinos, which is the angle-averaged distribution in the previous time step.

Emission/Absorption

Electron Capture
$e^- + p \longleftrightarrow \nu_e + n$ [ecp]

Anti-Electron Capture
$e^+ + n \longleftrightarrow \bar{\nu}_e + p$ [aecp]

Electron Capture on nuclei
$e^- + A \longleftrightarrow \nu_e + A'$ [eca]

Scattering

Neutrino-Nucleon scattering
$\nu + N \longleftrightarrow \nu + N$ [nsc]

Neutrino-Nuclei scattering
$\nu + A \longleftrightarrow \nu + A$ [csc]

Neutrino-Electron scattering
$\nu + e \longleftrightarrow \nu + e$ [esc]

Pair Process

Electron-positron process
$e^- + e^+ \longleftrightarrow \nu_i + \bar{\nu}_i$ [pap]

Nucleon-nucleon bremsstrahlung
$N + N \longleftrightarrow N + N + \nu_i + \bar{\nu}_i$ [nbr]

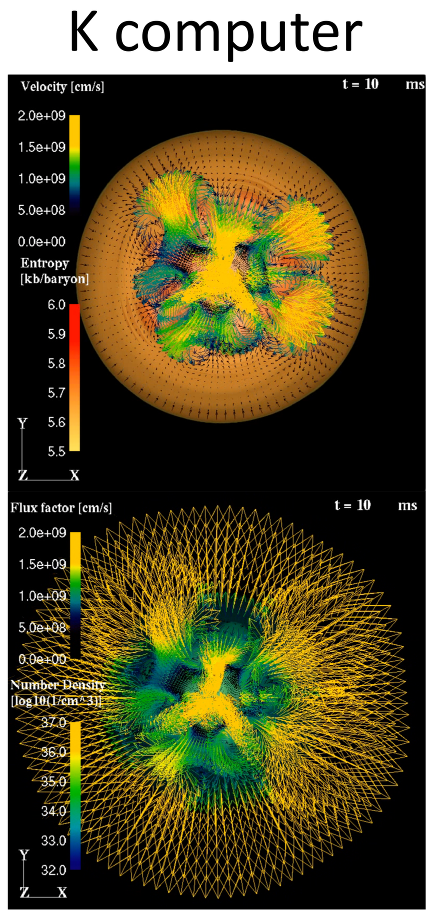
Neutrino Number Density
s: species ($s = \nu_e, \bar{\nu}_e, \nu_x$)

$$\Gamma_s \equiv \int\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{\Gamma}\equiv\Gamma_{\nu_e}-\Gamma_{\bar{\nu}_e}$$

Neutrino Energy Density (μ = 0)
Radiation Pressure (μ = 1, 2, 3)

$$G_s^\mu \equiv \int p_s^\mu\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{G^\mu}\equiv\sum_s G_s^\mu$$

K computer

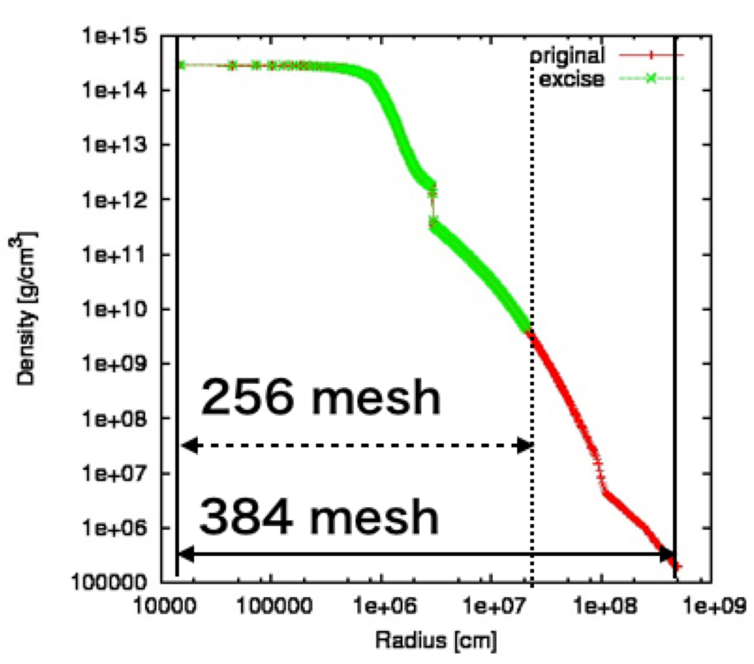


(Iwakami *et al.* 2020)

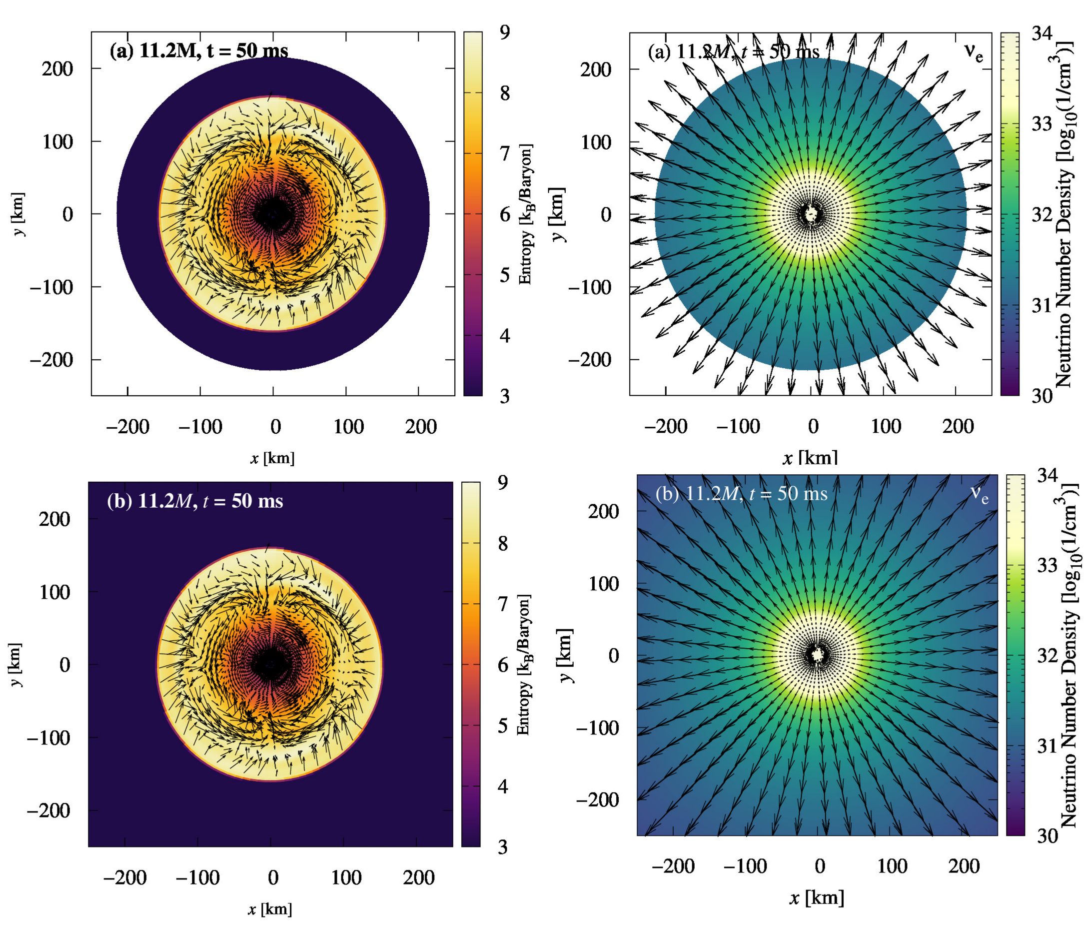
Nr × Nθ × Nϕ × Nve × Nvθ × Nvϕ

= 256 × 48 × 96 × 16 × 6 × 6   0 - 50ms

= 384 × 48 × 96 × 16 × 6 × 6   50 - 300ms



FUGAKU computer



Using FUGAKU computer, I have performed the neutrino radiation hydrodynamic simulations for core-collapse supernovae until t = 50 ms where the shock wave arrives at r ~ 150 km.

Then I extend the computational region from r = 200 km (panel (a)) to 5000 km (panel (b)).

The values between 200 km and 5000 km at t = 50 ms are given by the data of 1D simulations, which is used as the outer boundary condition at r = 200 km for t < 50ms.