Current status and future perspective of nuclear structure calculation for nuclear matrix element of neutrinoless double-beta decay

Nobuo Hinohara

University of Tsukuba

筑波大学 University of Tsukuba

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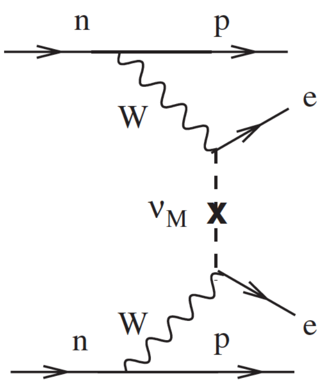
Neutrinoless double-beta decay



Avignone et al., Rev. Mod. Phys. **80**, 481 (2008)

- Majorana neutrino
- neutrino mass hierarchy

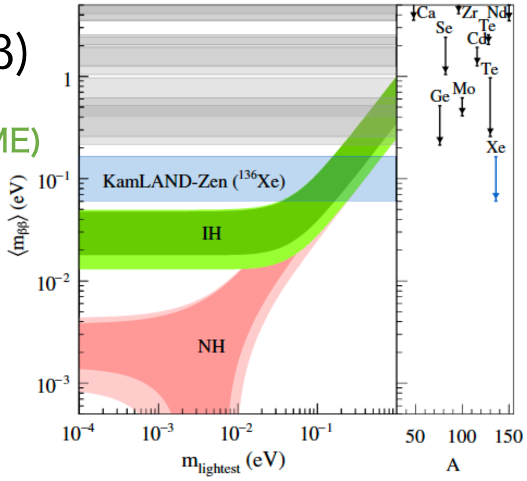
neutrinoless double-beta decay (0νββ)

(light-neutrino exchange)

half-life of 0νββ — nuclear matrix element(NME)

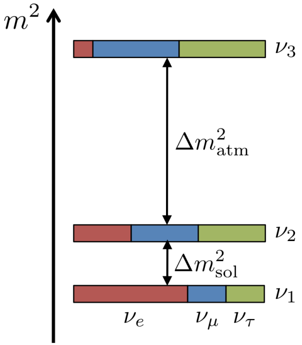
$$(T_{1/2}^{0\nu})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\,|M_{0\nu}|^2 \langle m_{\beta\beta}\rangle^2$$

phase space factor — effective mass of electric neutrino

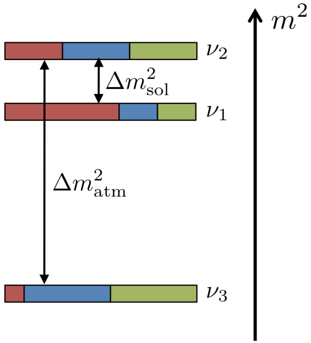


Gando et al., Phys. Rev. Lett. **117**, 082503 (2016)

normal hierarchy (NH)



inverted hierarchy (IH)



JUNO collaboration

Phase space factor

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

phase space factor

- Phase space factor(PSF): emitted electrons under the Coulomb field of final nucleus
- PSF calculated by different groups basically agree

TABLE 2 | PSF for $0\nu\beta^-\beta^-$ decays to final g.s.

Nucleus	$Q^{\beta^-\beta^-}_{g.s.}$	$G^{\beta^-\beta^-}_{0\nu}$ (g.s.) (10^{-15} yr^{-1})					
	(MeV)	[39]	**[11]**	**[3, 35, 36]**	**[5]**	**[47]**	**[46]**
^{48}Ca	4.267	24.65	24.81	26.1	26.0	24.83	24.55
^{76}Ge	2.039	2.372	2.363	2.62	2.55	2.37	2.28
^{82}Se	2.996	10.14	10.16	11.4	11.1	10.18	9.96
^{96}Zr	3.349	20.48	20.58		23.1	20.62	20.45
^{100}Mo	3.034	15.84	15.92	18.7	45.6	15.95	15.74
^{110}Pd	2.017	4.915	4.815			4.83	4.66
^{116}Cd	2.813	16.62	16.70		18.9	16.73	16.57
^{128}Te	0.8665	0.5783	0.5878	0.748	0.671		
^{130}Te	2.528	14.24	14.22	19.4	16.7	14.25	14.1
^{136}Xe	2.458	14.54	14.58	19.4	17.7	14.62	14.49
^{150}Nd	3.371	61.94	63.03	85.9	78.4	63.16	66.0
^{238}U	1.144	32.53	33.61				

Stoica and Mirea, Front. Phys. **7**, 12 (2019)

Nuclear matrix elements (NME)

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

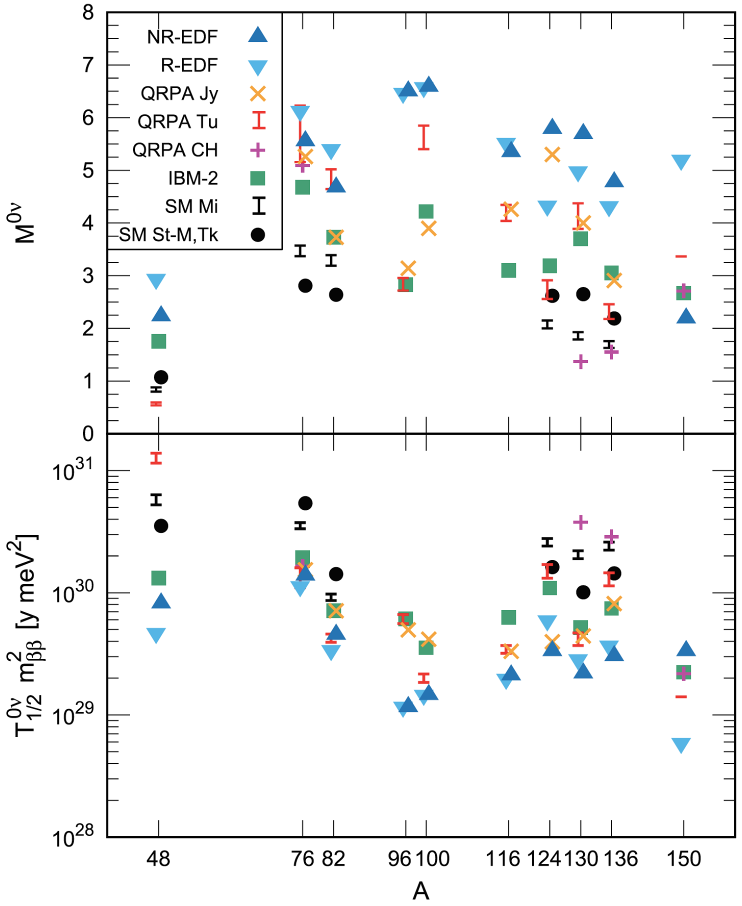
nuclear matrix element

What is the NME?

$$M_{0\nu} = \langle f|\hat{M}_{0\nu}|i\rangle$$

- transition amplitude of the nucleus: from initial nucleus (N,Z) to final nucleus (N-2,Z+2)
- transition between 0+ states: spin-zero transition
- NME is not an experimental observable and we need a precise value

- Values of NME depends on theory/group/calculations a factor of 2-3 difference exists

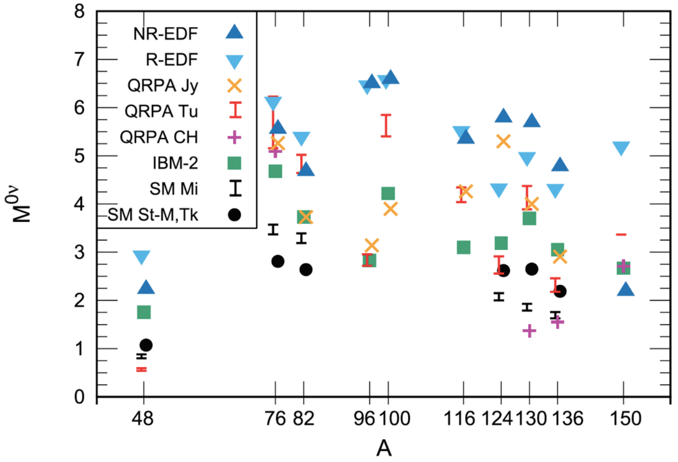


Engel and Menéndez, Rep. Prog. Phys. **80**, 046301 (2017)

Nuclear matrix elements

Why do NME values depend on calculations?

$$M_{0\nu} = \langle f | \hat{M}_{0\nu} | i \rangle$$



Because people use different..

- ☐ decay operator
- ☐ many-body theory for describing initial and final states
- ☐ single-particle model space
- ☐ nucleon-nucleon interaction

Decay operators

$$M_{0\nu} = \langle f | \hat{M}_{0\nu} | i \rangle = \left[M_{0\nu}^{\mathrm{GT}} - \frac{g_V^2}{g_A^2} M_{0\nu}^{\mathrm{F}} + M_{0\nu}^{\mathrm{T}} \right]$$

$$M_{0\nu}^{\mathrm{F}} = \langle f | \sum_{ab} H^{\mathrm{F}}(r_{ab}, \bar{E}) \tau_a^- \tau_b^- | i \rangle$$

$$M_{0\nu}^{\mathrm{GT}} = \langle f | \sum_{ab} H^{\mathrm{GT}}(r_{ab}, \bar{E}) \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \tau_a^- \tau_b^- | i \rangle$$

$$M_{0\nu}^{\mathrm{T}} = \langle f | \sum_{ab} H^{\mathrm{T}}(r_{ab}, \bar{E}) \left[3(\boldsymbol{\sigma}_a \cdot \hat{\boldsymbol{r}}_{ab})(\boldsymbol{\sigma}_b \cdot \hat{\boldsymbol{r}}_{ab}) - \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \right] \tau_a^- \tau_b^- | i \rangle$$

- derived after closure approximation, non-relativistic approximation
- Fermi, Gamow-Teller and tensor parts. Gamow-Teller is dominant
- g_A^4 is included in the phase space factor

- closure approximation: 10-15% error
- tensor part: 10% at most

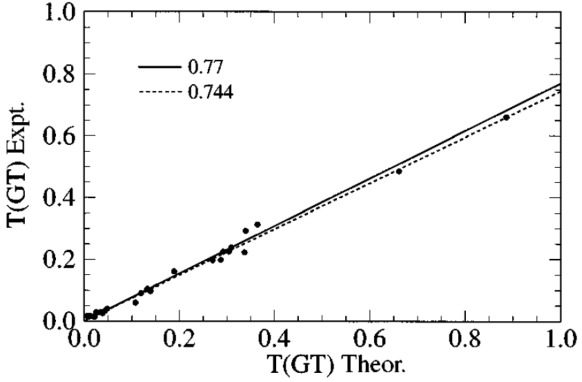
g_A quenching

beta decay

Gamow-Teller operator for β-decay $\hat{\vec{M}}_\beta = g_A \sum_i \vec{\sigma}_i \tau_i^-$

axial-vector coupling constant (bare) $g_A \sim 1.27$

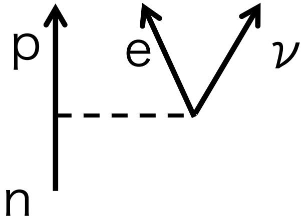
$g_{Aeff} \sim 0.7\text{-}0.8\ g_A$

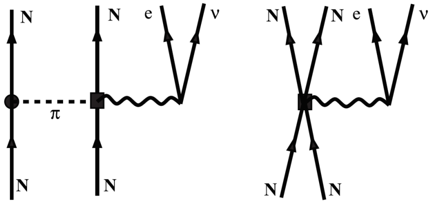


Martinez-Pinedo et al., Phys. Rev. C **53**, R2602 (1996)

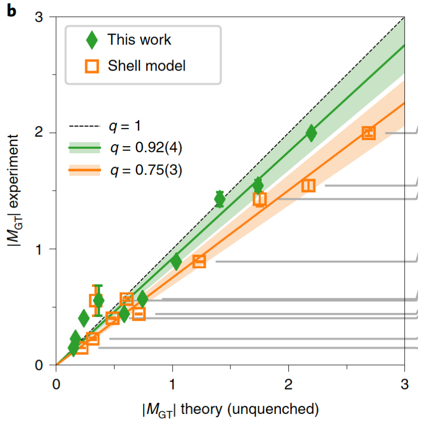
Reason of quenching

- many-body corrections
- many-nucleon weak current





Menendez, et al., PRL**107**, 062501 (2011)

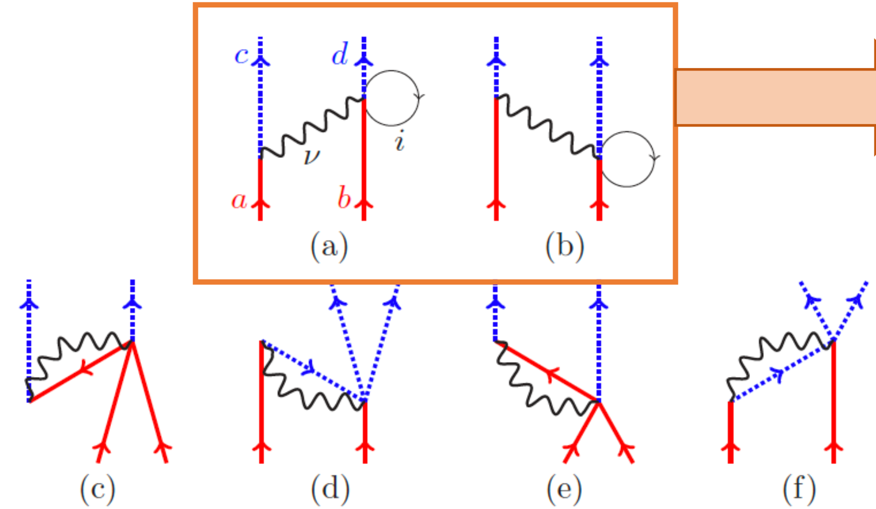


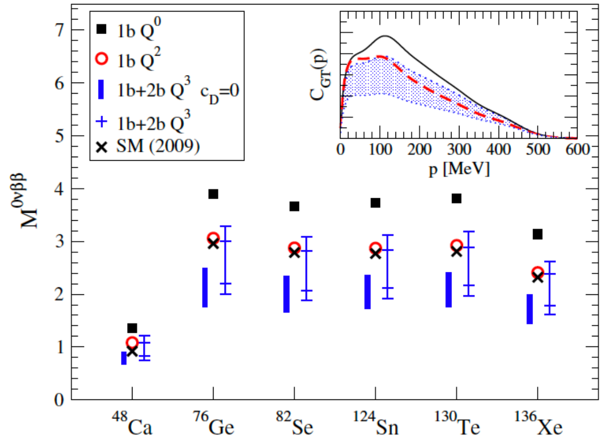
Gysbers et al., Nature Physics **15**, 428 (2019)

g_A quenching

Double-beta decay

two-body current from chiral effective field theory





J. Menéndez et al., Phys. Rev. Lett. **107**, 062501 (2011)

-35% to 10% contribution

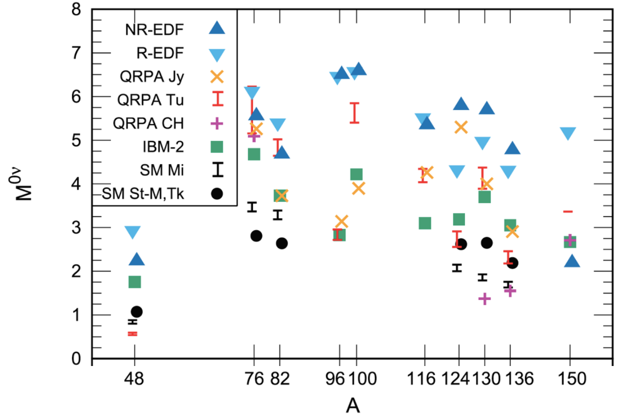
full inclusion of chiral two-body current:
10% quenching

L-J. Wang et al., Phys. Rev. C **98**, 031301(R) (2018)

bare value of g_A is used in the compilation (Menendez and Engel)

Nuclear structure theory

- ☐ decay operator
- ☐ many-body theory (correlations)
- ☐ single-particle model space
- ☐ effective interactions



Shell model
(full correlations)
realistic/phenomenological int.

GCM
(selected collective correlations)
realistic int./EDF

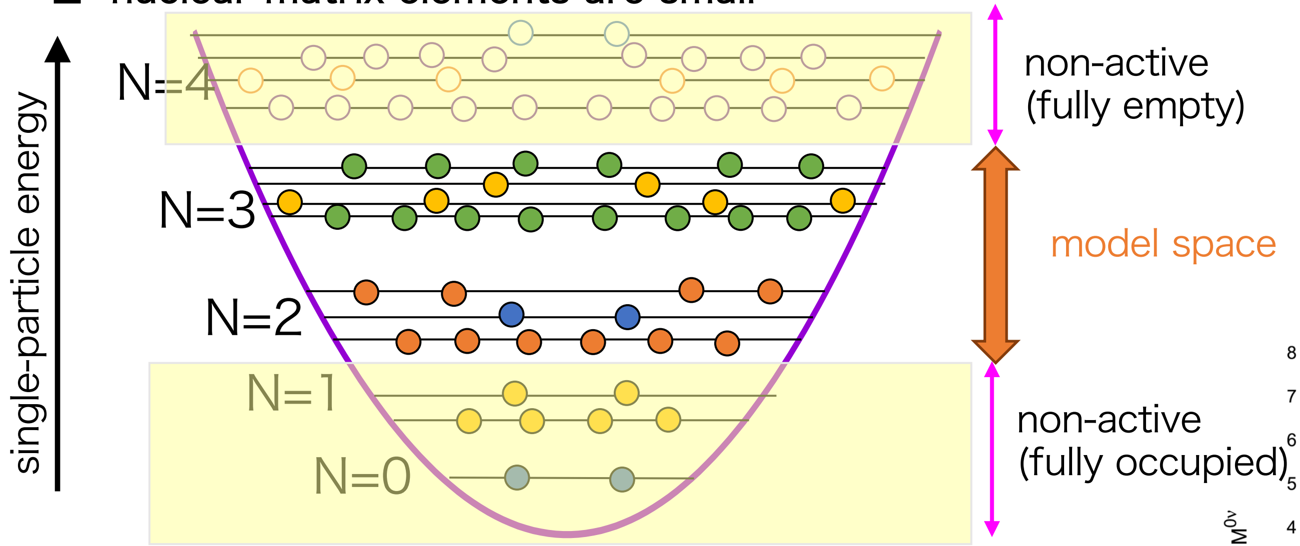
QRPA
(correlations limited to two-quasiparticles)
realistic int./EDF

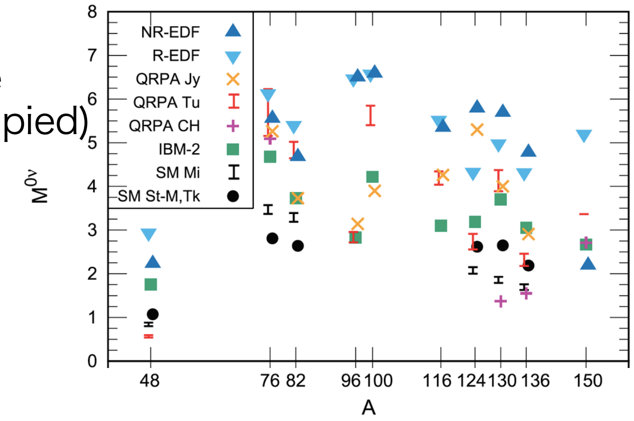
correlations

size of single-particle model space

Shell model

- full many-body correlation included in a limited single-particle model space
- effective interaction determined phenomenologically to reproduce experimental data
- interaction depends on the model space
- **nuclear matrix elements are small**





Shell model many-body basis state

$$|\alpha\rangle = \hat{c}^{\dagger}_{n,n_1 l_1 j_1 m_1} \hat{c}^{\dagger}_{n,n_2 l_2 j_2 m_2} \cdots \hat{c}^{\dagger}_{n,n_N l_N j_N m_N} \hat{c}^{\dagger}_{p,n_1 l_1 j_1 m_1} \hat{c}^{\dagger}_{p,n_2 l_2 j_2 m_2} \cdots \hat{c}^{\dagger}_{p,n_Z l_Z j_Z m_Z} |0\rangle$$

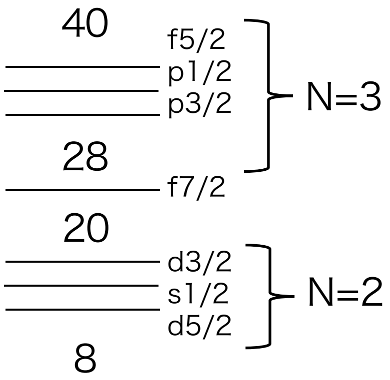
initial/final states: $|\Psi\rangle = \sum_{\alpha} C_{\alpha} |\alpha\rangle$ $\qquad \hat{H}|\Psi\rangle = E|\Psi\rangle$

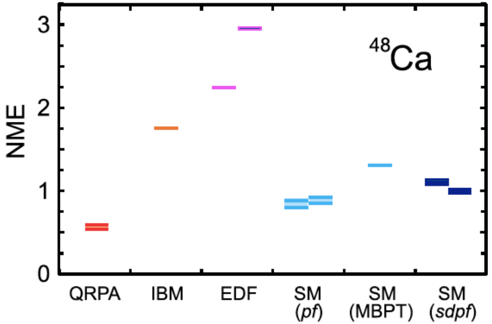
Status of Shell model calculations

Calculations included in the compilation

- ^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{128}Te, ^{136}Xe:
  Menéndez et al Nucl. Phys. A **818**,139 (2009)
- ^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{130}Te, ^{136}Xe:
  Horoi and Neacsu, Phys. Rev. C **93**, 024308 (2016)
- ^{48}Ca: Iwata et al., Phys. Rev. Lett. **116**, 112502 (2016)

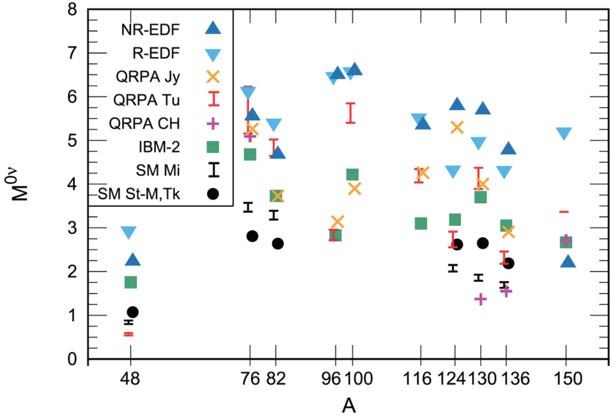
^{48}Ca with full sd+pf model space(Iwata et al.)
dimension: 2×10^9 in ^{48}Ti



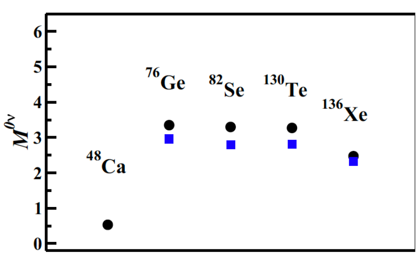


Later results

- ^{76}Ge, ^{82}Se Yoshinaga et al., Prog. Theor. Exp. Phys. **2018**, 023D02
  - phenomenological interaction
  - large g_A (g_{Aeff} ~ 1.13-1.33g_A) to explain 2νββ half-life

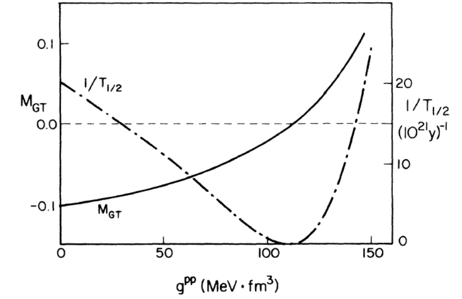


- ^{48}Ca-^{136}Xe: Coraggio et al., Phys. Rev. C **101**, 044315 (2020)
- ^{100}Mo: Coraggio et al., Phys. Rev. C **105**, 034312 (2022)
  - realistic interaction (CD-Bonn, V_{low-k})
  - effective Hamiltonian/decay operator derived for the model space

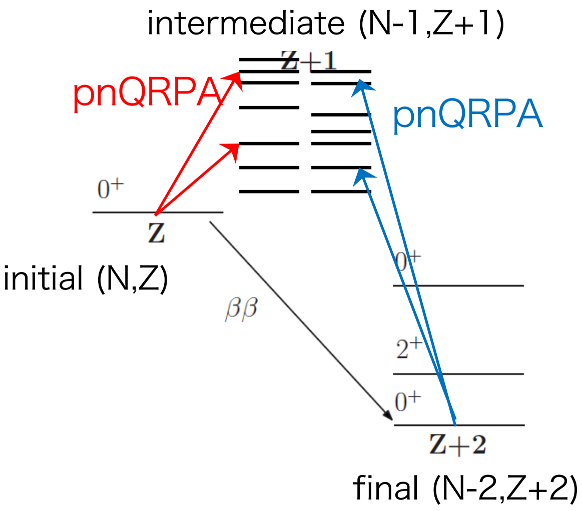


QRPA(quasiparticle random-phase approximation)

- initial and final states are constructed based on mean field theory
- correlation is limited to two-quasiparticle (proton qp and neutron qp) superpositions
- large model space can be employed: same interaction for all nuclei (except for isoscalar pairing)
- isoscalar proton-neutron pairing suppresses the NME
  strength fitted to reproduce the 2νββ NME
- NME values are various

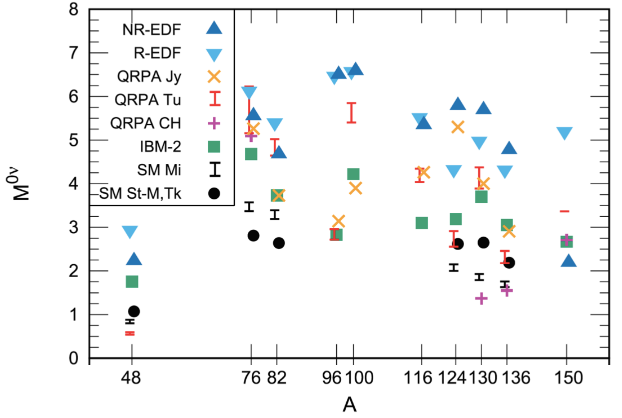


Vogel and Zirnbauer, Phys. Rev. Lett. **57**, 3148 (1986)



$$M^{\mathrm{F}}_{0\nu} = \sum_{abn_in_f} H(r_{ab},\bar{E})\langle f|\tau^-_a|n_f\rangle\langle n_f|n_i\rangle\langle n_i|\tau^-_b|i\rangle$$

$$M^{\mathrm{GT}}_{0\nu} = \sum_{abn_in_f} H(r_{ab},\bar{E})\langle f|\vec{\sigma}_a\tau^-_a|n_f\rangle\langle n_f|n_i\rangle\langle n_i|\vec{\sigma}_b\tau^-_b|i\rangle$$



Status of QRPA calculations

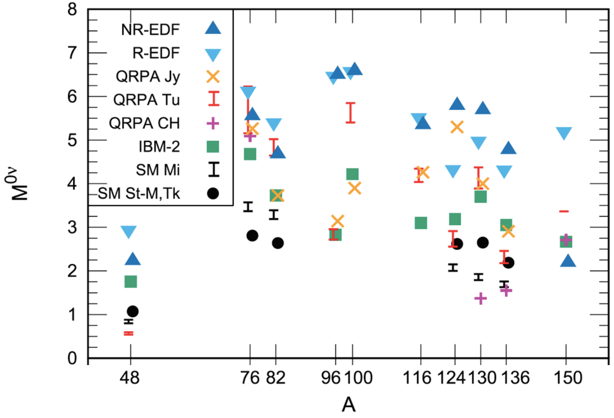
Calculations included in the compilation

	interaction	deformation	proton-neutron pairing
Tübingen(2013,2015)	realistic	spherical (^{150}Nd: deformed)	included
Jyväskylä (2015)	realistic	spherical	included
Chapel Hill (2013)	energy density functional (SkM*)	axially deformed	included

Later results

axially deformed pnQRPA calculation (Tübingen, 2018)

	methods	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe	^{150}Nd
Tübingen(2018)	this work	3.12	2.86	2.90	1.11	3.01
	QRPA-Tü [12]	5.16	4.64	3.89	2.18	–
LNM	QRPA-Jy [13]	5.26	3.73	4.00	2.91	–
	QRPA-NC [14]	5.09	–	1.37	1.55	2.71



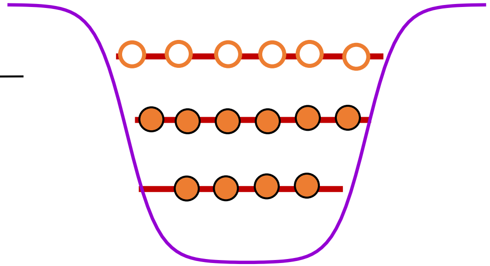
Tübingen : Šimkovic et al., Phys. Rev. C **87**, 045501 (2013)
Fang et al., Phys. Rev. C **92**, 044301 (2015)
Fang et al., Phys. Rev. C **97**, 045503 (2018)
Jyväskylä : Hyvarinen and Suhonen, Phys. Rev. C **9,** 024613 (2015)
Chapel Hill: Mustonen and Engel, Phys. Rev. C **87**, 064302 (2013)

Generator coordinate method

The initial and final states are expressed by superposition of mean fields (Slater determinant)

$$|\Psi_k\rangle = \sum_q f_k(q)|\phi(q)\rangle$$

initial/final states — weight function — mean field



- shell model : large-dimensional superposition of **orthogonal harmonic-oscillator basis**
- GCM : small-dimensional (~100) superposition of **non-orthogonal mean field basis**
- only selected collective correlations are included (quadrupole deformation, pairing···)
- weight function is determined by solving many-body Schrödinger equation (Hill-Wheeler equation)
- drawbacks: (almost) impossible to calculate intermediate (odd-odd) nuclei
  no guarantee on convergence against many-body basis space
- large NME

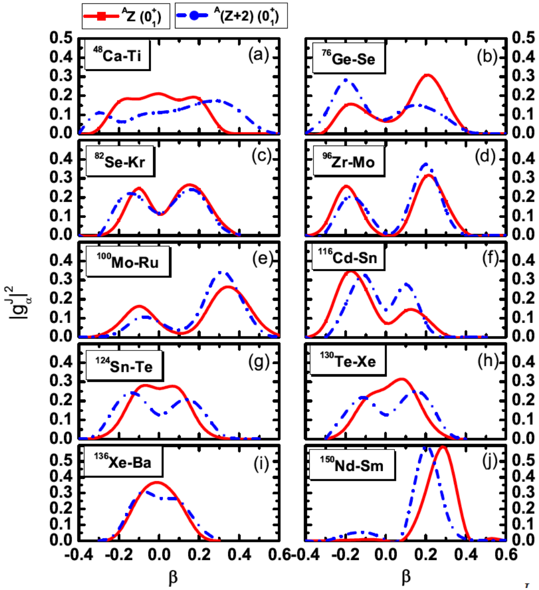
GCM applications to EDF

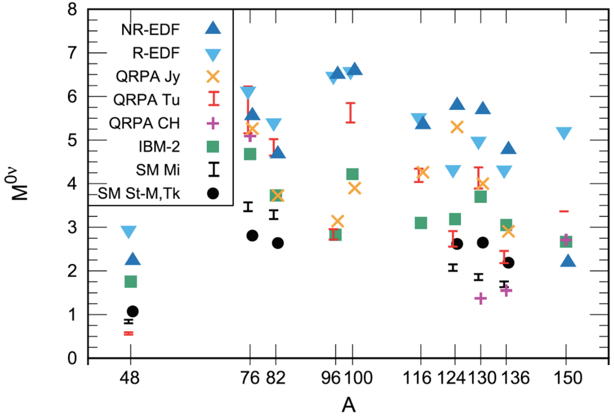
Calculations included in the compilation

NR-EDF: non-relativistic energy density functional (interaction)
R-EDF: relativistic EDF (interaction)

$$|\Psi_k\rangle = \sum_q f_k(q) |\phi(q)\rangle$$

q: quadrupole/octupole deformation
isovector pairing





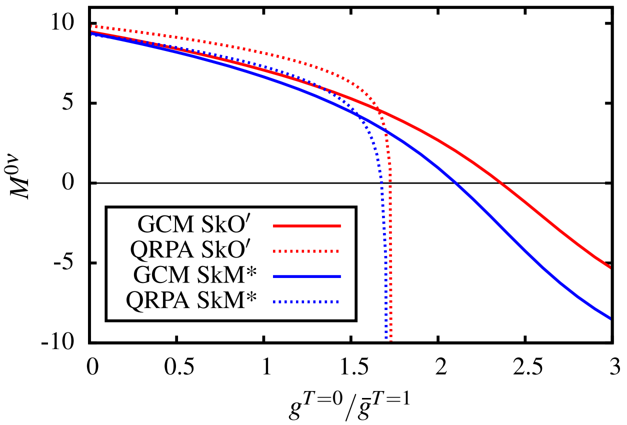
Large NMEs: shorter half-lives

R-EDF: Yao et al. Phys. Rev. C **91**, 024316 (2015), Yao and Engel, Phys. Rev. C **94**, 014306 (2016)
NR-EDF: Vaquero et al., PRL **111**, 142501 (2013)

Which basis should be included in GCM?

NH and Engel, Phys. Rev. C **90**, 031301(R) (2014)

GCM with mean field with isoscalar-pairing amplitude



strength of isoscalar np pairing

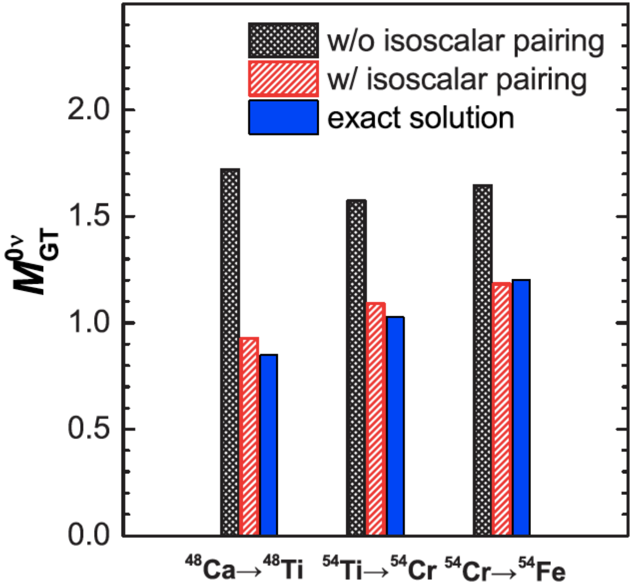
- QRPA: isoscalar proton-neutron pairing interaction known to suppress the NME
- GCM with EDF: large NME: lack of isoscalar np pairing correlation in the initial/final state
- NR-EDF and R-EDF do not include np pairing term in the interaction

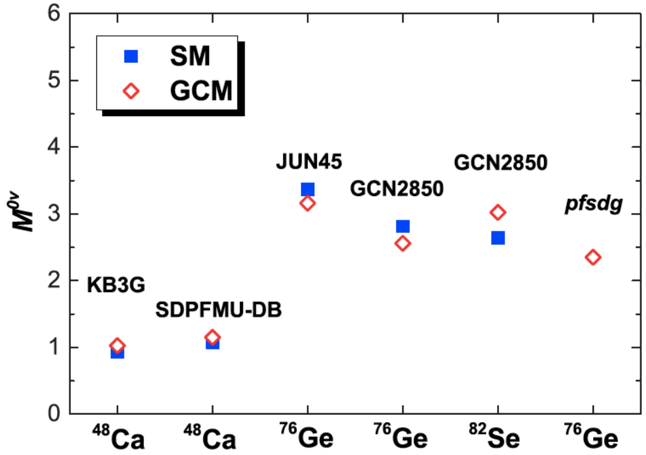
→ Hamiltonian/EDF that includes isoscalar pairing

Later applications of GCM: Shell model Hamiltonian

Jiao et al., Phys. Rev. C **96**, 054310 (2017)

- Solve Shell model Hamiltonian problem with GCM
- Generator coordinates : two quadrupole deformations, isoscalar-pair amplitude



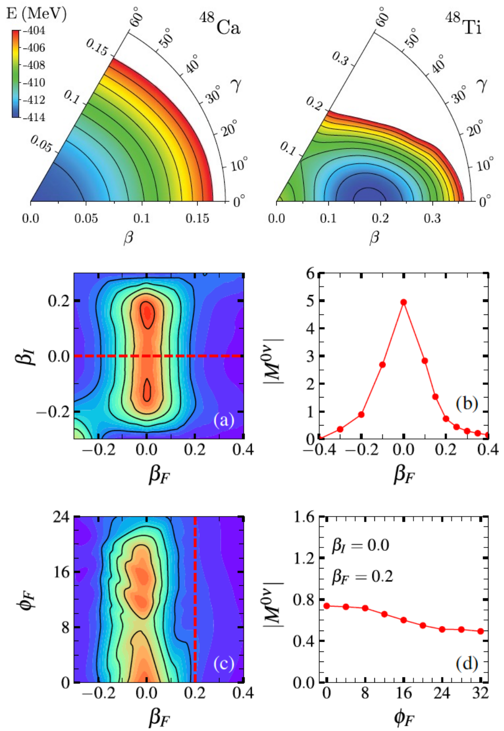


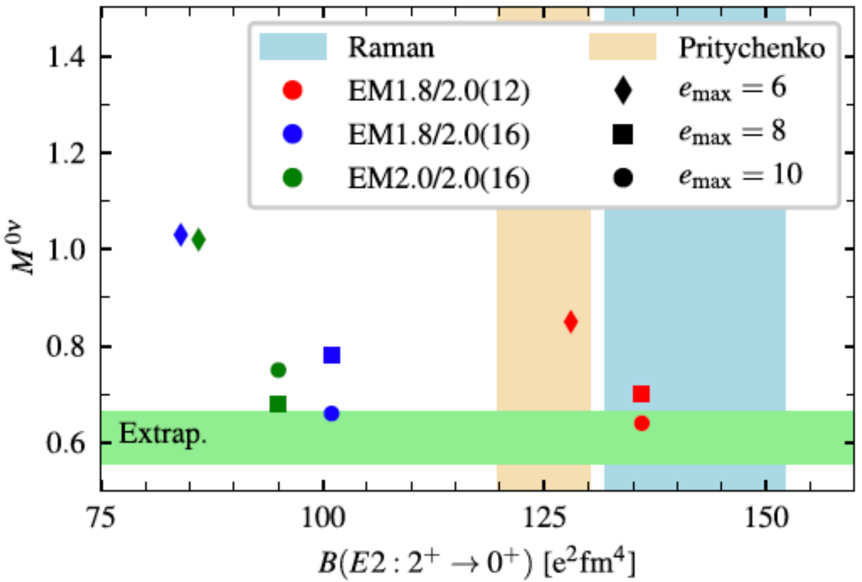
NME values agree between (direct) Shell model calc and GCM
GCM can go beyond shell model limit (pfsdg)

Ab-initio applications with GCM wave functions

Yao et al. Phys. Rev. C **98**, 054311 (2018)
Yao et al., Phys. Rev. Lett. **124**, 232501 (2020)

In-medium Similarity Renormalization Group (IMSRG)
applications so far: spherical nuclei
Hamiltonian: chiral EFT
IMSRG + GCM for deformed nuclei, quadrupole deformation (β) and isoscalar pairing (φ) correlations

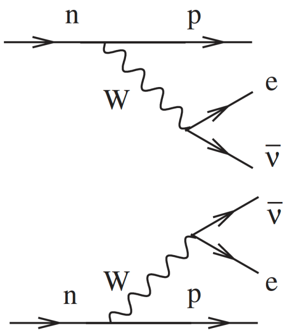




^{48}Ca $M^{0\nu}$ = 0.61

Related processes: 2νββ

precise expression of 2νββ half-life

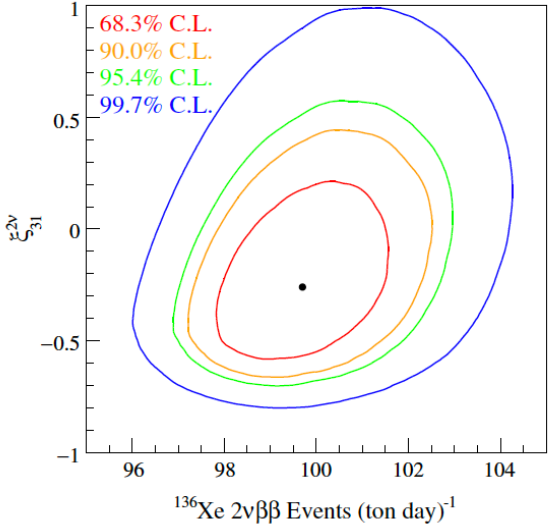


$$[T^{2\nu}_{1/2}]^{-1} = g_A^4 |M^{2\nu}_{\rm GT}|^2 \left[G^{2\nu}_0 + \xi^{2\nu}_{31} G^{2\nu}_2\right]$$

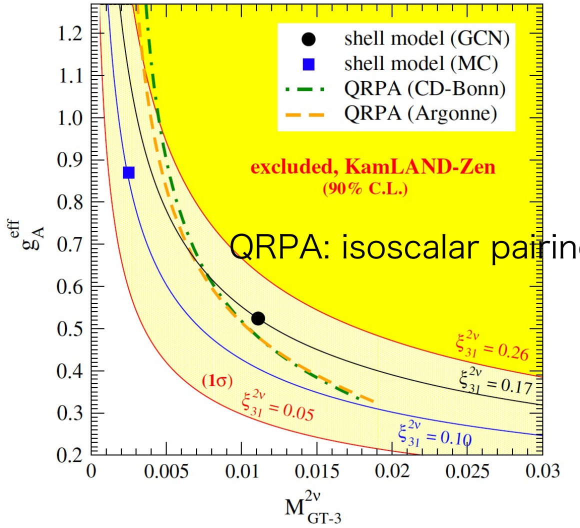
$$\xi^{2\nu}_{31} = M^{2\nu}_{\rm GT-3}/M^{2\nu}_{\rm GT}$$

G_0 and G_2 has different lepton energy dependence

ξ_{31} determined by experiment



$\xi_{31}^{2\nu}$ < 0.26 (KamLAND-Zen)



QRPA: isoscalar pairing strength dependence

Simkovic et al., Phys. Rev. C **97**, 034315 (2018)
Gando et al., Phys. Rev. Lett. **122**, 192501 (2019)

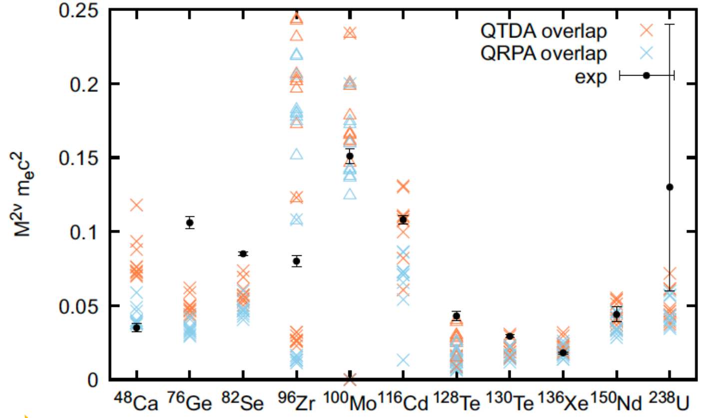
2νββ decay prediction

NH and Engel, Phys. Rev. C **105**, 044314 (2022)

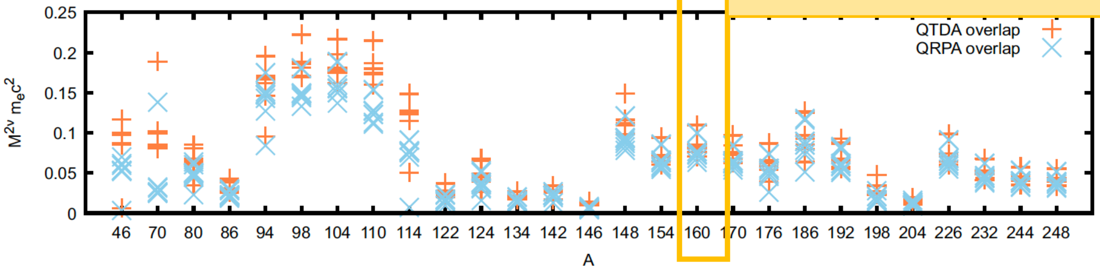
- In QRPA, usually 2νββ NME(half-life) is used to fit the unknown isoscalar pairing strength
  → prediction of 2νββ NME is impossible
- QRPA calculations using EDF with isoscalar pairing fitted to β decay half-lives globally
  → prediction of 2νββ NME possible

QRPA (Finite-amplitude Method) calculations with 10 EDFs, two overlap prescriptions

Small uncertainties in heavier isotopes (A>130)



prediction of unmeasured 2νββ NME



^{160}Gd NME

predicted previously 0.0455 MeV^{-1}
(Hirsch et al., Phys. Rev. C **66**,015502 (2002))
EDF: 0.12 – 0.21 MeV^{-1}

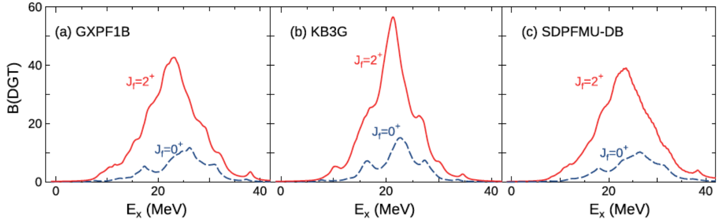
PIKACHU experiment for ^{160}Gd
(Poster P25: Takashi Iida)

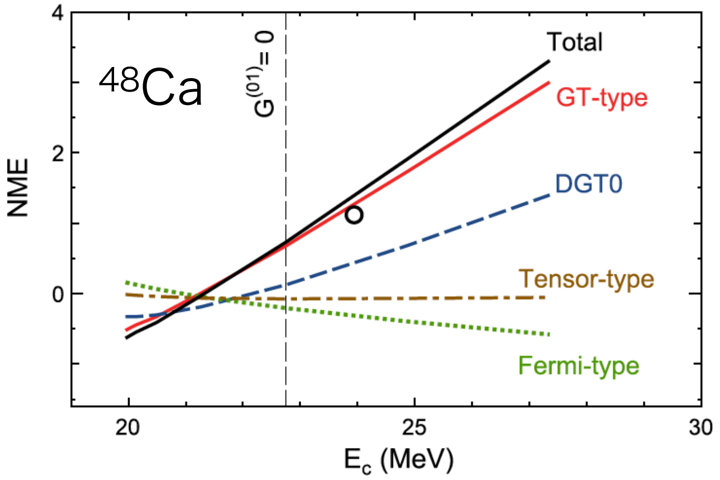
Extension to 0νββ and double electron capture is in progress

Double Gamow-Teller transition

Constraining 0νββ NME from double Gamow-Teller transitions

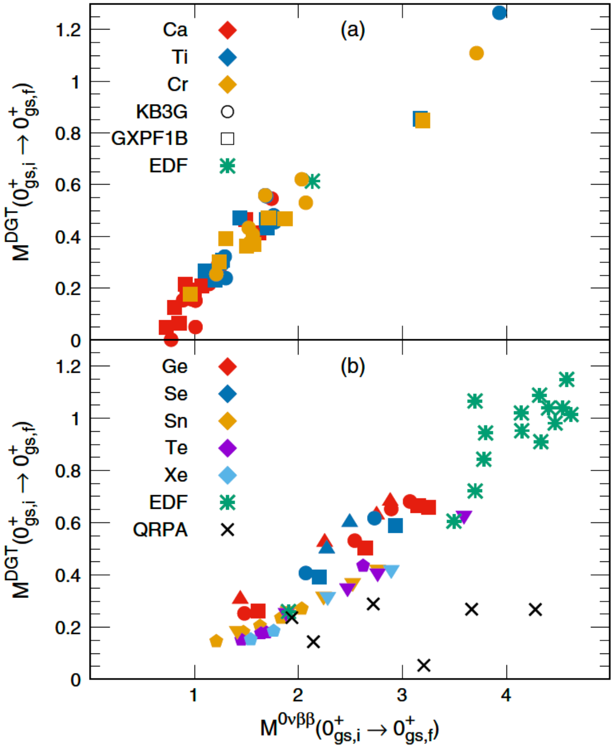
double Gamow-Teller giant resonance ($^{48}Ca \rightarrow ^{48}Ti$)





DGT GR (2+)

ground-state transition



Shimizu et al., Phys. Rev. Lett. **120**, 142502 (2018)

Summary

- Neutrinoless double-beta decay: Majorana neutrino, neutrino mass hierarchy

- Nuclear matrix element (NME) calculation
- Uncertainties depending on the nuclear structure theories (shell model, QRPA, GCM)

  - decay operator, g_A quenching, two-body currents

  - improvements of the nuclear structure theories
    - realistic interaction in shell model
    - inclusion of deformation in QRPA
    - inclusion of isoscalar pairing in GCM
    - combination of GCM with shell model/ab-initio method

- related process to constrain $0\nu\beta\beta$ NME
  - precise measurement of $2\nu\beta\beta$ decay
  - double Gamow-Teller transitions