

# Non-linear evolution of fast conversions in dense medium

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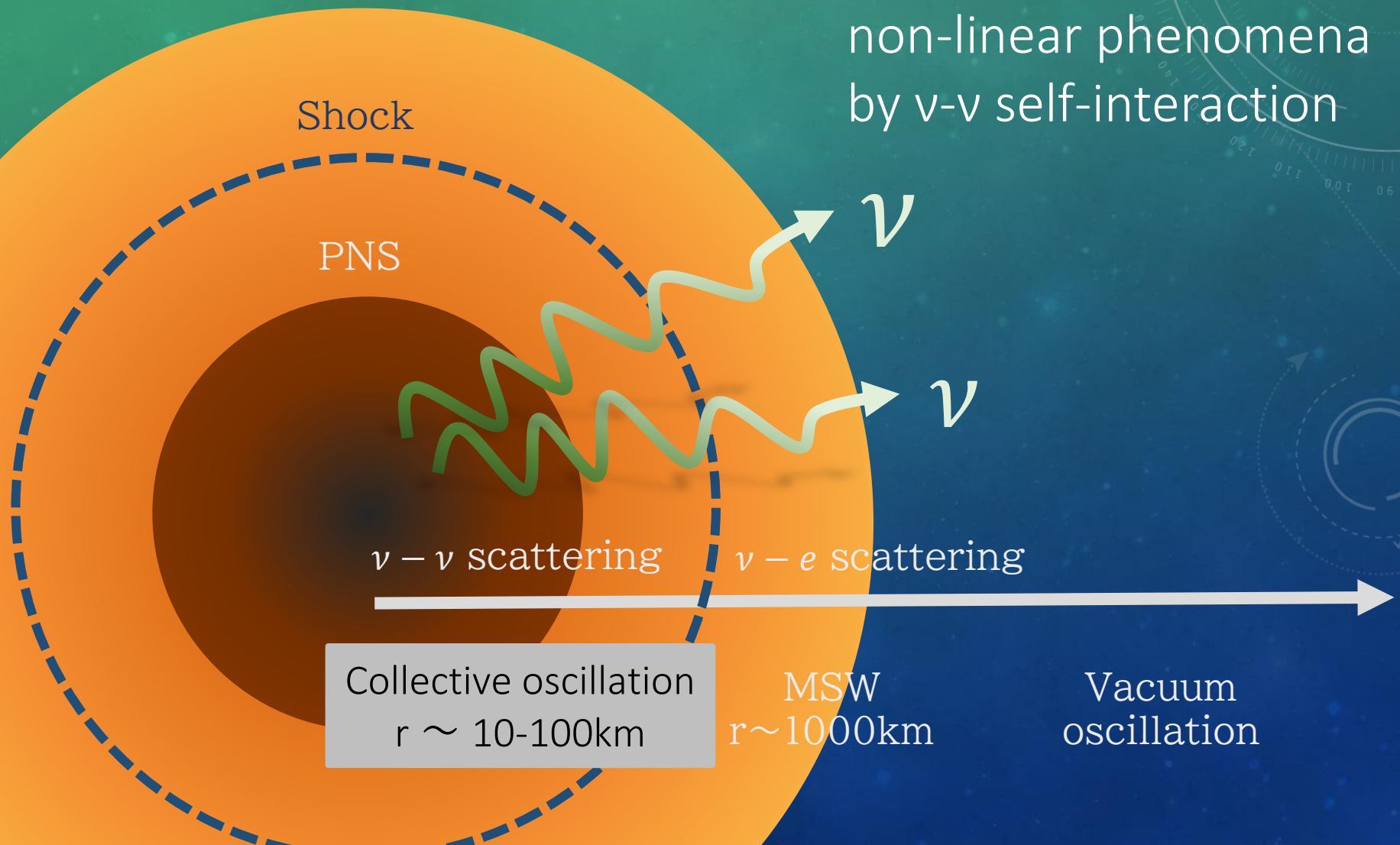


Unraveling the History of the Universe and Matter Evolution with Underground Physics

June, 14<sup>th</sup>, 2022

# Neutrino oscillation in supernovae

Collective oscillations  
non-linear phenomena  
by  $\nu$ - $\nu$  self-interaction



# Quantum Kinetic Equations

$$\nu \quad i \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \rho(\vec{x}, \vec{p}, t) = [\mathcal{H}, \rho(\vec{x}, \vec{p}, t)] + i\mathcal{C}[\rho]$$

$$\bar{\nu} \quad i \left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) \bar{\rho}(\vec{x}, \vec{p}, t) = [\bar{\mathcal{H}}, \bar{\rho}(\vec{x}, \vec{p}, t)] + i\bar{\mathcal{C}}[\bar{\rho}]$$

Oscillation term Collision term

3×3 density matrix

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} & \rho_{e\tau} \\ \rho_{\mu e} & \rho_{\mu\mu} & \rho_{\mu\tau} \\ \rho_{\tau e} & \rho_{\tau\mu} & \rho_{\tau\tau} \end{pmatrix}$$

distribution function of active neutrinos

Hamiltonians

$$\mathcal{H} = \mathcal{H}_{\text{vac}} + \mathcal{H}_{\text{mat}} + \mathcal{H}_{\nu\nu}$$

$$\bar{\mathcal{H}} = \mathcal{H}_{\text{vac}}^* - \mathcal{H}_{\text{mat}}^* - \mathcal{H}_{\nu\nu}^*$$

$$\mathcal{H}_{\text{vac}} = U \frac{M^2}{2E_\nu} U^\dagger$$

$$\mathcal{H}_{\text{mat}} = \sqrt{2}G_F \text{diag}[n_l - \bar{n}_l]$$

$$\mathcal{H}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\vec{p}'}{(2\pi)^3} (1 - \vec{v} \cdot \vec{v}') (\rho' - \bar{\rho}'^*)$$

# Properties of collective oscillations

✓ two distinct modes: fast mode & slow mode

slow mode : self-interaction  $\lesssim$  vacuum

fast mode : self-interaction  $\ll$  vacuum

$$l_{\text{fast}} \sim (\hbar c)^{-2} G_F^{-1} n_\nu^{-1}$$

✓ small oscillation scales & fine angular structures

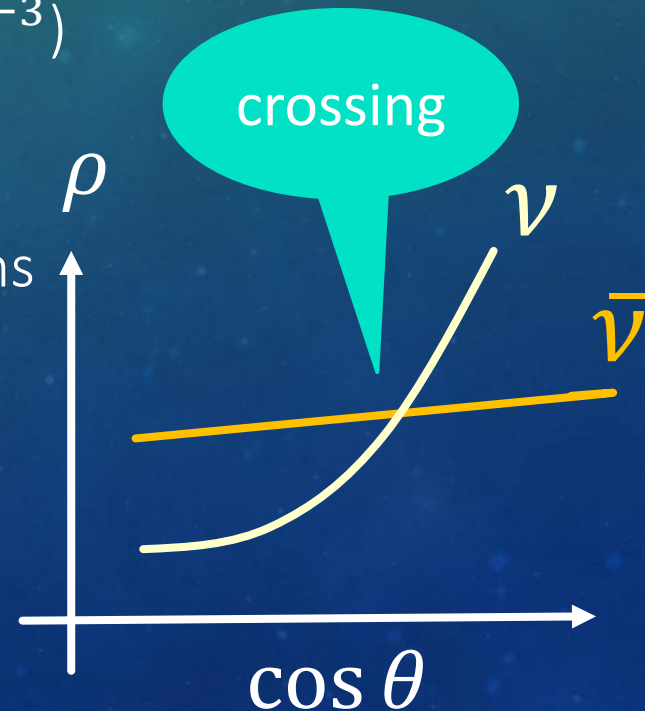
$O(1-100)\text{cm} \ll O(10)\text{km}$ ,  $\cos \theta_\nu \sim O(10^{-3})$

✓ linear regime study

search for the criterion of fast conversions

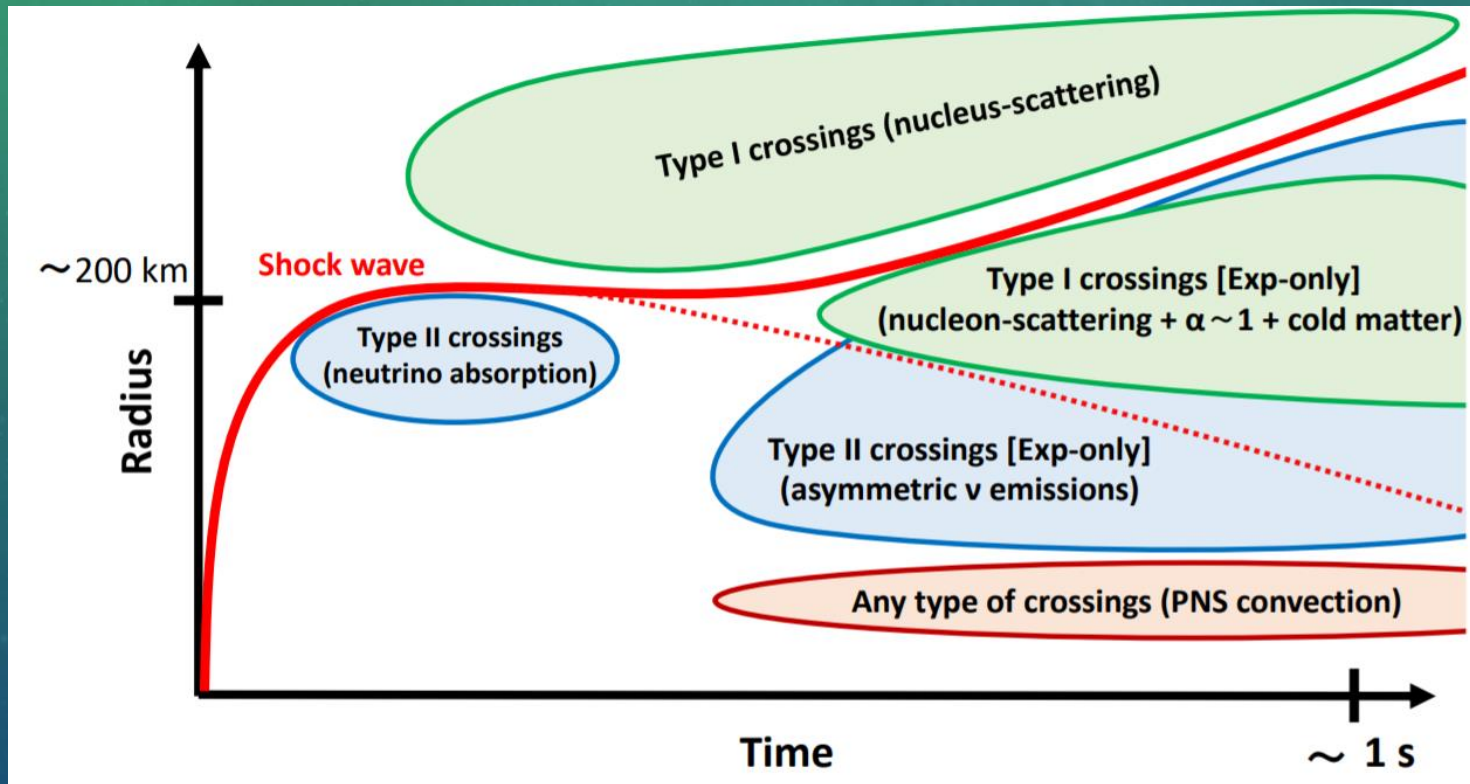
e.g., linear stability analysis, LN crossing

$$L_{\vec{v}} = \sqrt{2} G_F \int \frac{E_\nu^2 dE_\nu}{2\pi^2} (\rho - \bar{\rho})$$



# ELN crossing in Supernovae

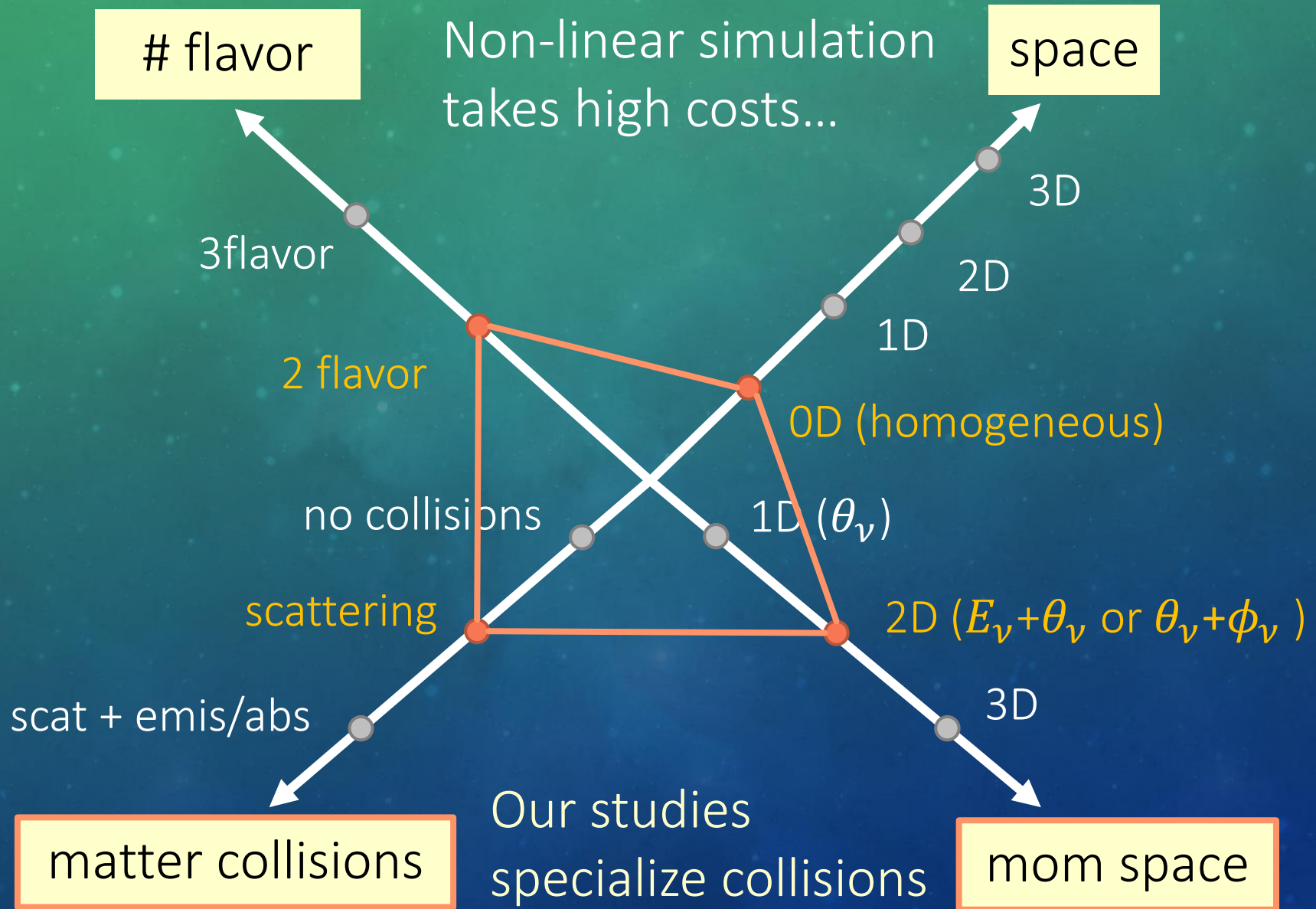
Fast conversions occur everywhere and every time in SN!  
➔ affect the SN dynamics? or observables?



Nagakura 2021

After triggering, how do fast conversions evolve?

# Non-linear simulations of fast conversions



# Fast conversion with matter collisions

Matter collisions affect fast conversion behaviors

- ✓ Collisions change  $\nu$  distribution & make a new crossing  
e.g., halo effects,  $\nu$ -nuclei coherent scattering

Cherry2013, Capozzi2019, Morinaga2020, Zaizen2020

- ✓ Collisions affect the evolution of existing FCs  
Depending on reaction rate and angular distributions,  
FCs are enhanced or suppressed

Martin2021, Shalgar2020, Sigl2021, Shalgar2022

- ✓ Collisional instability = a new instability mode

Lucas2021, Dasgupta 2021

# Energy dependence of fast conversions

✓ Fast conversion itself does not depend on  $\nu$  energy

➔ integrate over  $\nu$  energy

choose representative  $\nu$  energy

$$\mathcal{H}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\vec{p}'}{(2\pi)^3} (1 - \vec{v} \cdot \vec{v}') (\rho' - \bar{\rho}^{*'})$$

✓ However...

Neutrinos have energy spectrum in SN

Reaction rates have  $\nu$  energy dependence

➔ If we consider energy-dependent reaction rates, how are the dynamics of fast conversions changed?



# Our goal & today's targets

## Our ambitious goal

To investigate the effects of fast conversions on  
SN dynamics & observables ?

neutrino spectra, heavy elements etc...

## Today's targets

- ✓ Effects of isotropic & isoenergetic collisions
- ✓ Effects of energy-dependent collisions

# Simulation setups

$$i \frac{\partial \rho_a(E_\nu, \cos \theta_\nu, t)}{\partial t} \quad \text{Oscillation term}$$

$$= [\mathcal{H}_{\nu\nu}, \rho_a(E_\nu, \cos \theta_\nu, t)]$$

$$+ i \int_{-1}^1 d \cos \theta'_\nu R(E_\nu) \rho_a(E_\nu, \cos \theta'_\nu, t)$$

$$- i \int_{-1}^1 d \cos \theta'_\nu R(E_\nu) \rho_a(E_\nu, \cos \theta_\nu, t)$$

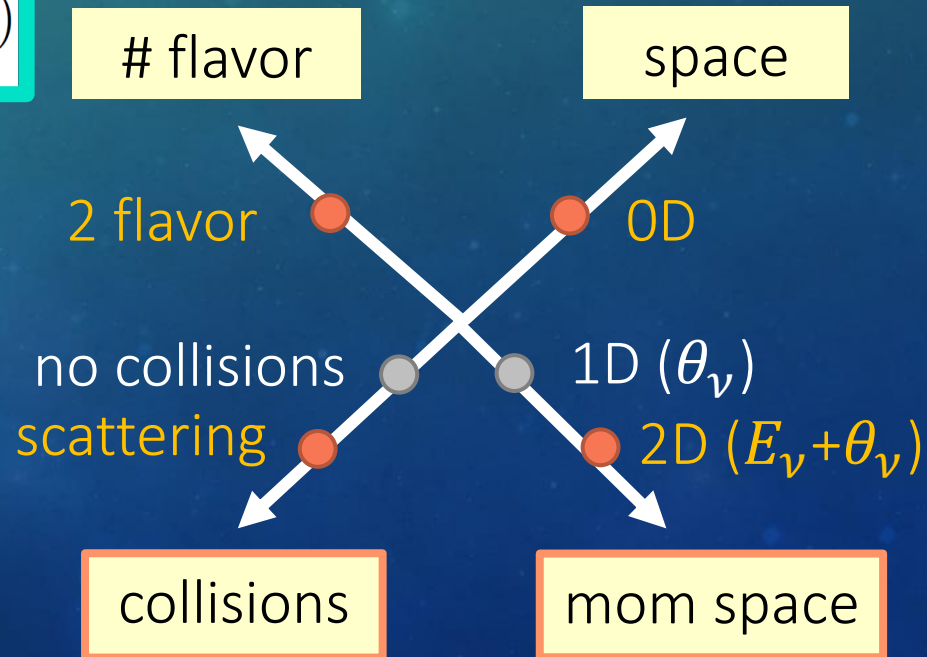
Collision term

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}, \bar{\rho} = \begin{pmatrix} \bar{\rho}_{ee} & \bar{\rho}_{ex} \\ \bar{\rho}_{ex}^* & \bar{\rho}_{xx} \end{pmatrix}$$

$$\rho_a \equiv \int \rho d\phi_\nu$$

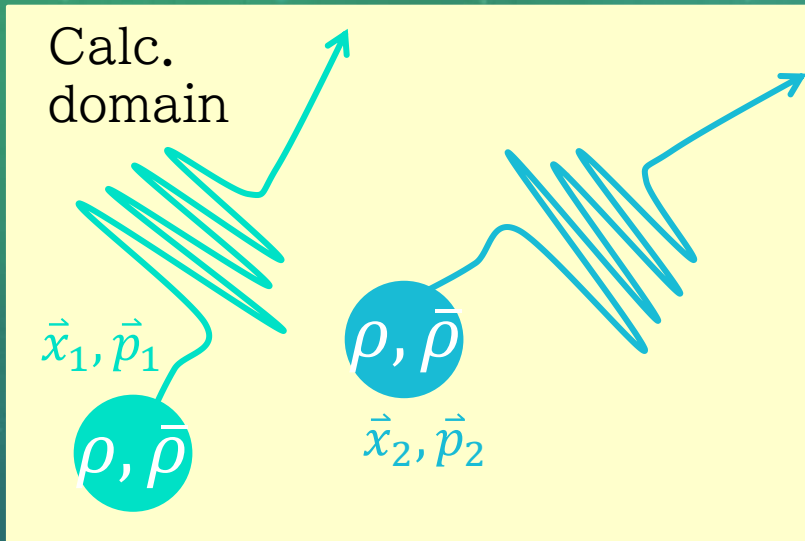
isotropic, isoenergetic scat.  
equivalent reaction rates

1. w/o scattering, 1Dps
2. w scattering, 1Dps
3. w scattering, 2Dps



# QKE-MC $\nu$ transports

CK et al, ApJS, 257, 55, 2021



- ✓ Almost the same as the normal MC method
- ✓ 8 degrees of freedom in each particle

n step

Evolution of sample particles

Solving geodesic equation  
Neutrino reactions

Calculation of  $H_{\nu\nu}$

Summing up MC samples

$$\mathcal{H}_{\nu\nu} = \sqrt{2}G_F \int \frac{d^3\vec{q}}{(2\pi)^3} (1 - \vec{v} \cdot \vec{v}') (\rho' - \bar{\rho}'^*)$$

Evolution of  $\rho, \bar{\rho}$

Solving QKE for each sample particle  
4<sup>th</sup>-order Runge-Kutta method

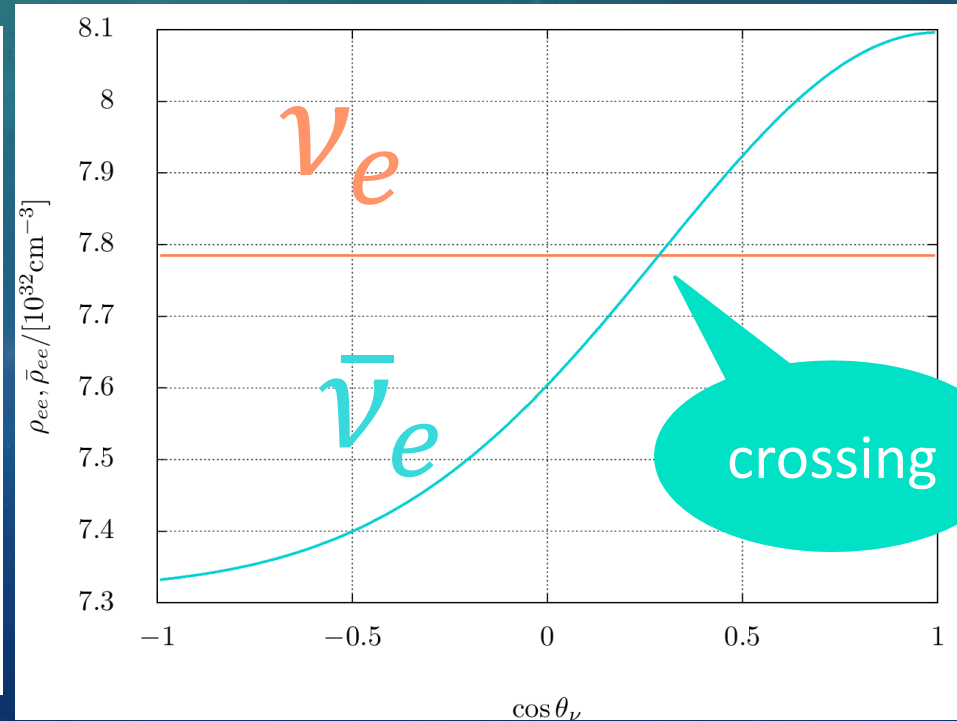
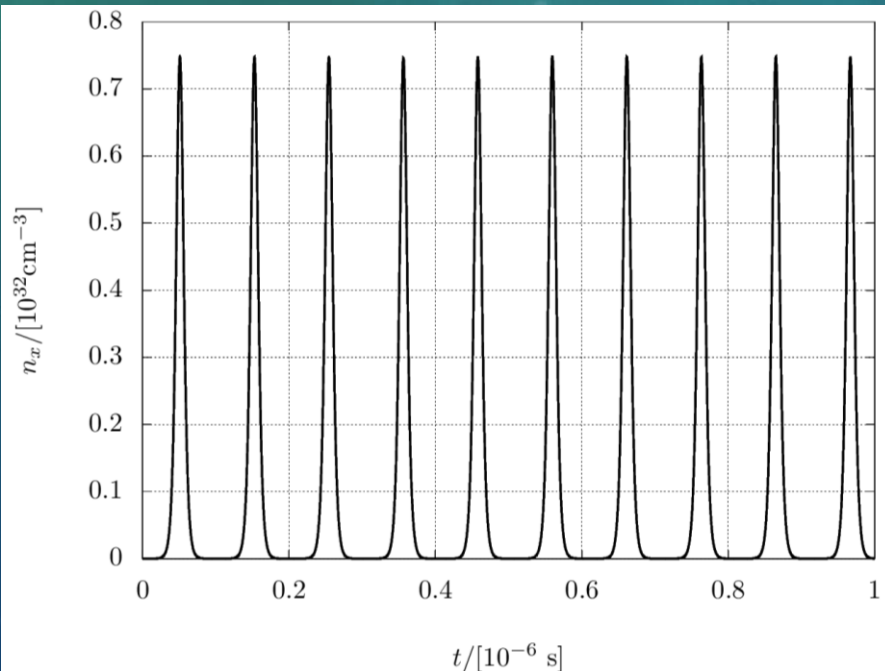
n+1 step

# w/o scattering, 1D phase space

- ✓  $n_{\nu_e} + n_{\nu_x} = \text{constant}$
- ✓ periodic conversion
- ✓ conversion occurs at crossing point

$$\begin{aligned}n_{\nu_e} &= 1.56 \times 10^{33} \text{cm}^{-3} \\ \bar{n}_{\nu_e} &= 1.53 \times 10^{33} \text{cm}^{-3} \\ n_{\nu_x} &= \bar{n}_{\nu_x} = 0\end{aligned}$$

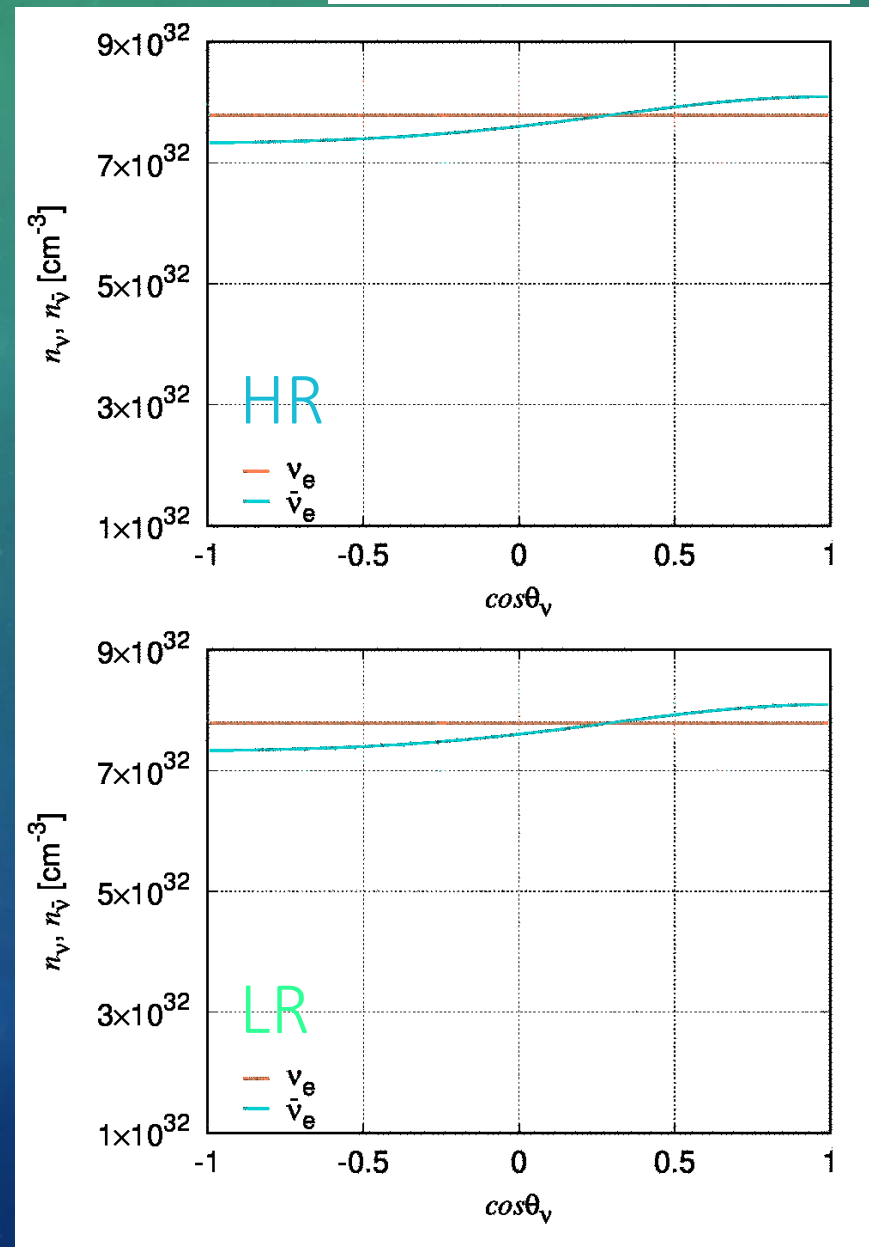
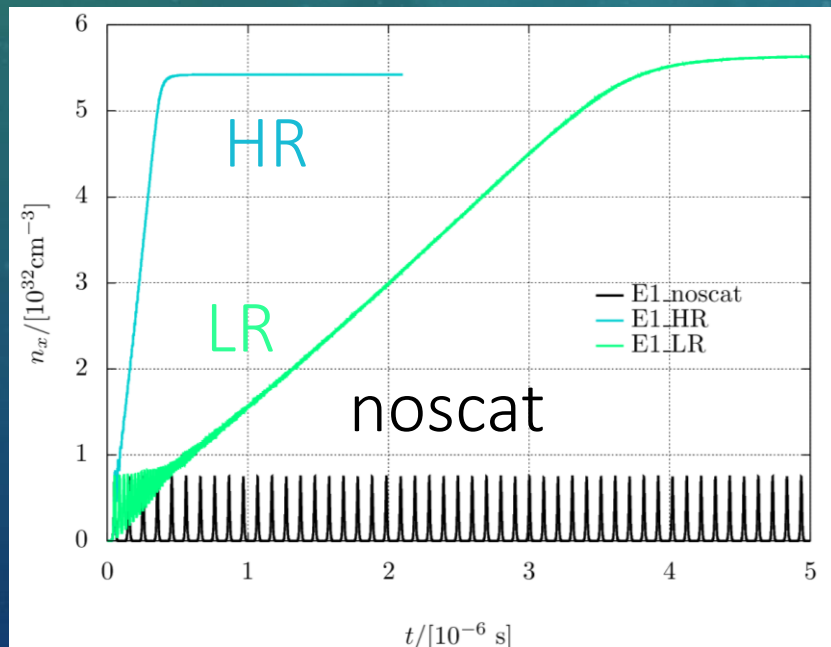
$$\begin{aligned}\text{Im}n_{ex} &= -\text{Re}n_{xe} = 10^{-6}n_{\nu_e} \\ \text{Im}\bar{n}_{ex} &= -\text{Re}\bar{n}_{xe} = 10^{-6}\bar{n}_{\nu_e}\end{aligned}$$



# w scattering, 1D phase space

$1.25 \times 10^{-5} \text{ cm}^{-1}$  (LR)  
 $1.25 \times 10^{-4} \text{ cm}^{-1}$  (HR)

- ✓ collisions enhance conversions
- ✓ low rate  $\rightarrow$  slower growth  
larger number of  $n_x$
- ✓ conversions occur at wide range



# w scattering, 2D phase space

✓ two neutrino energies

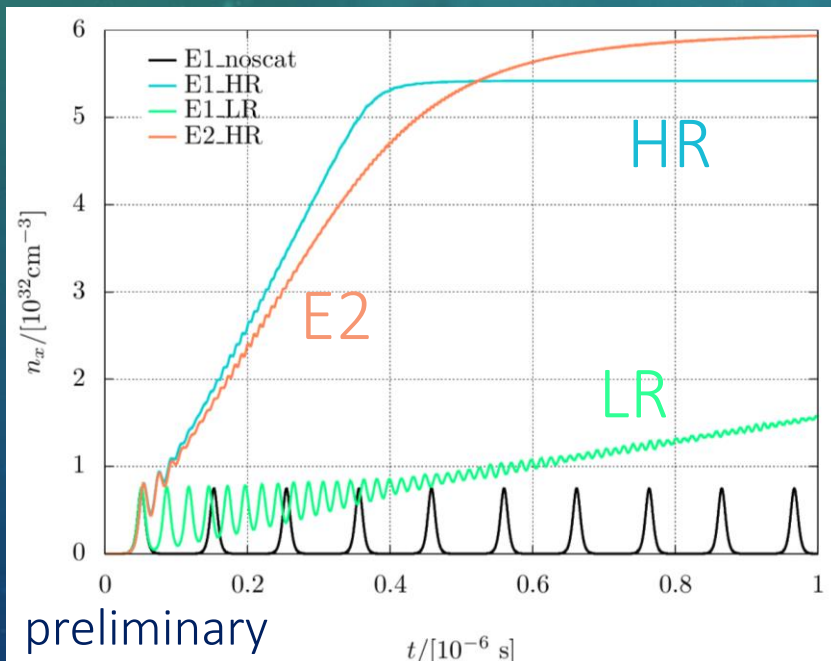
$$E_{low}=10\text{MeV}, E_{high}=30\text{MeV}$$

✓  $n_{low} = n_{high}$  for  $\nu_e$  &  $\bar{\nu}_e$

✓ Energy dependent reaction rate  $R(E_\nu) = R_0 E_\nu^2$

$$\langle R_{ee} \rangle = 1.25 \times 10^{-4} \text{cm}^{-1}$$

$$\langle R_{ee} \rangle = \frac{R(E_{low})n_{low} + R(E_{high})n_{high}}{n_\nu}$$

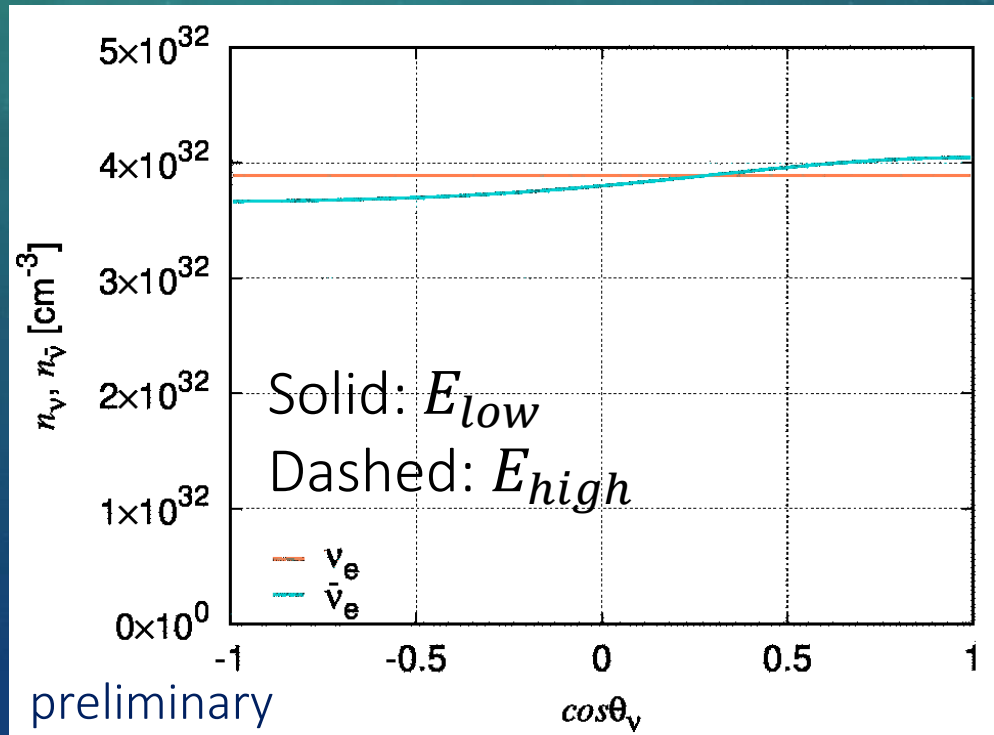


✓ Energy dependence of reaction rate reduces the impact of collisions from the single energy case

# Angular distribution

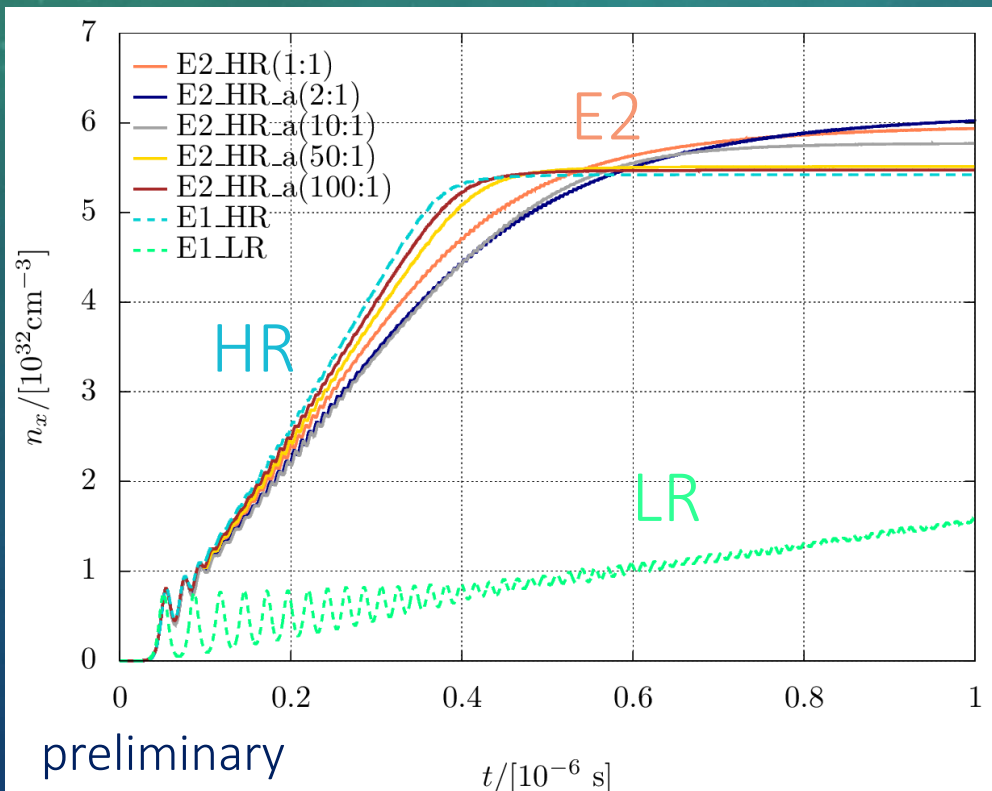
✓ high energy neutrinos experience collisions more frequently

- ➔ detailed balance is achieved between two angles
- ➔ the number of scatterings is effectively reduced



# Number asymmetric case ( $n_{low} \neq n_{high}$ )

- ✓ we change the number ratio between two  $\nu$  energies
- ✓ In all the models, the impact of collisions is reduced
- ✓ But some models have very close results



$$\langle R_{ee} \rangle = 1.25 \times 10^{-4} \text{ cm}^{-1}$$

$$\langle R_{ee} \rangle = \frac{R(E_{low})n_{low} + R(E_{high})n_{high}}{n_{\nu}}$$

	$R_0/[10^{-4} \text{ cm}^{-1}]$	$n_{low}/n_{\nu}$	$n_{high}/n_{\nu}$	$\beta$
E2_HR(1:1)	1.00	1/2	1/2	1
E2_HR_a(2:1)	1.36	2/3	1/3	2
E2_HR_a(10:1)	2.89	10/11	1/11	10
E2_HR_a(50:1)	4.32	50/51	1/51	50
E2_HR_a(100:1)	4.63	100/101	1/101	100



# Validity of monochromatic assumption

✓ monochromatic assumption in two  $\nu$  energies

$$= R_{low}n_{low} \gg R_{high}n_{high}$$

$$= R_{low}n_{low} \ll R_{high}n_{high}$$

✓ monochromatic assumption in general

= collisions in a specific  $\nu$  energy dominate others

✓ a versatile index for measuring multi-energy effects

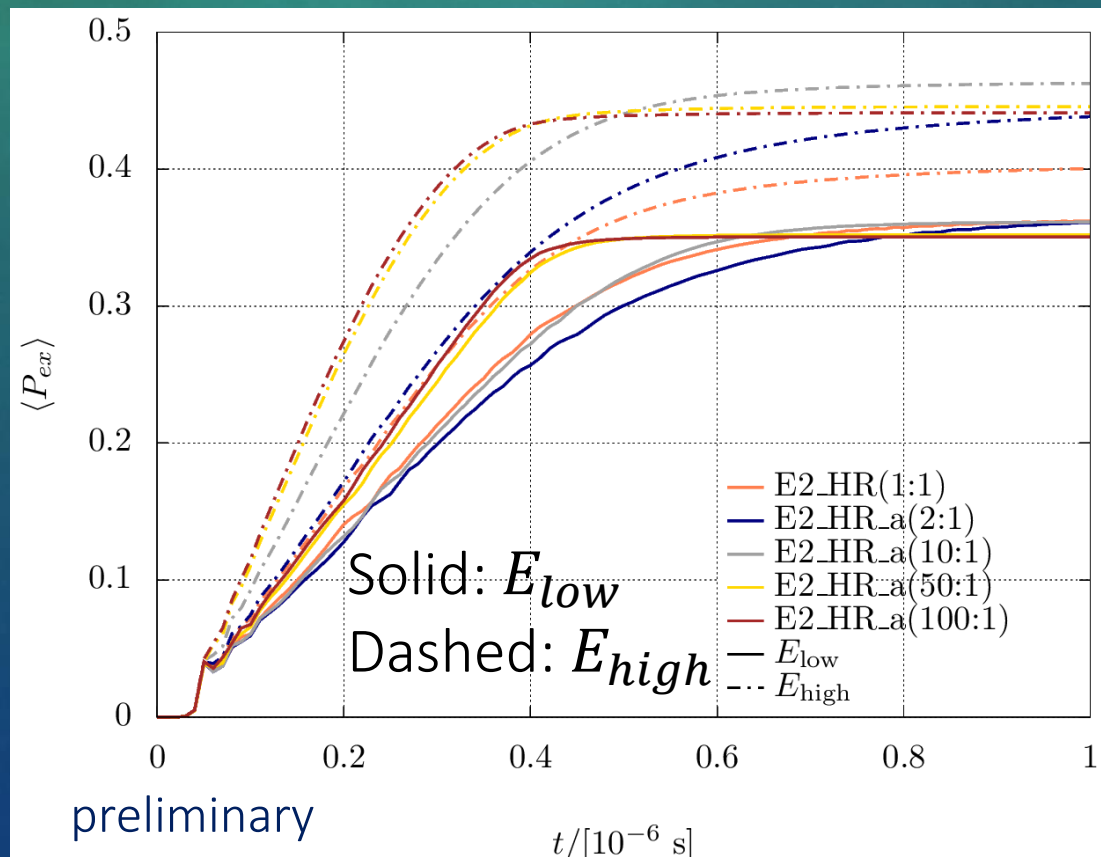
$\chi \rightarrow 0 \Leftrightarrow$  monochromatic

$$\chi = \left| \frac{\langle E_\nu \rangle - \langle RE_\nu \rangle}{\langle E_\nu \rangle + \langle RE_\nu \rangle} \right|$$

$$\langle E_\nu \rangle = \frac{\int d^3\vec{p} E_\nu \rho}{\int d^3\vec{p} \rho}$$
$$\langle RE_\nu \rangle = \frac{\int d^3\vec{p} RE_\nu \rho}{\int d^3\vec{p} R \rho}$$

# Energy dependence of fast conversions

- ✓ fast conversion itself does not depend on  $\nu$  energy
- ➔ energy-dependent collisions introduce dependence!



# Summary & Future works

## Summary

- ✓ collective oscillation will affect SN dynamics & observables
- ✓ Homogeneous fast conversions have periodic feature
- ✓ Collisions break this feature and larger number of  $n_x$  is produced
- ✓ If we introduce energy-dependence into reaction rates, the results deviate from the monochromatic results.
- ✓ If collisions are localized in a certain energy, the monochromatic assumption is valid

## Future works

- ✓ Emission & absorption, realistic scatterings (non-isoenergetic, non-isotropic)
- ✓ Realistic neutrino energy spectrum & angular distributions
- ✓ How FCs affects SN dynamics & observables?