Towards general relativistic Boltzmann-radiation-hydrodynamics simulations of core-collapse supernovae

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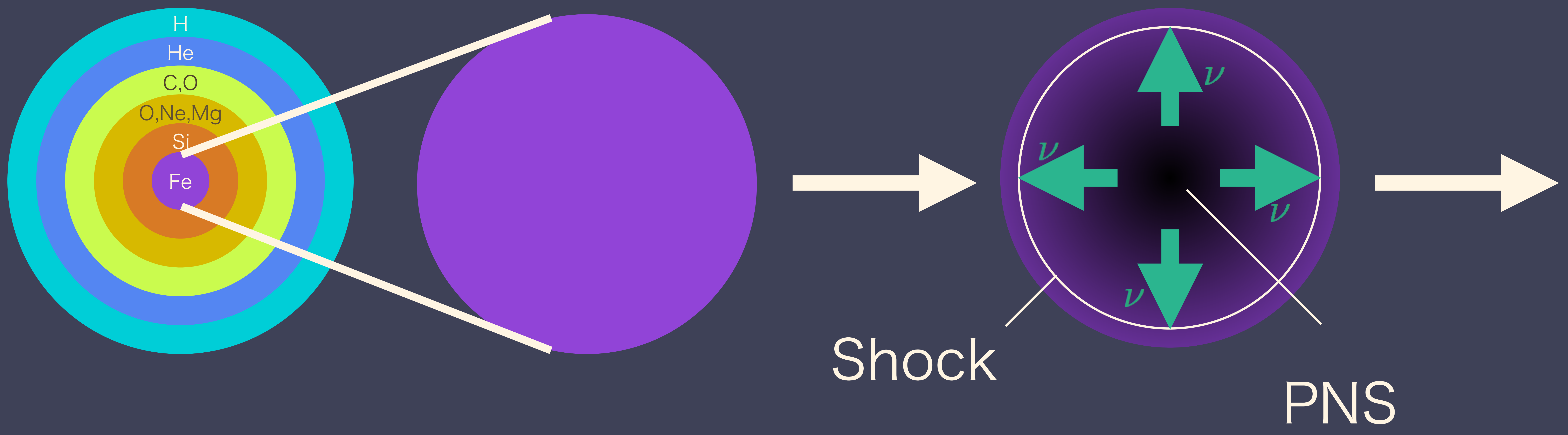
Supernova explosion mechanism

- Core collapse→Core bounce→Stalled shock
- Neutrino emission from the central proto-neutron star (PNS)
- Shock revives by the neutrino heating

→Neutrino heating mechanism



SN1987A
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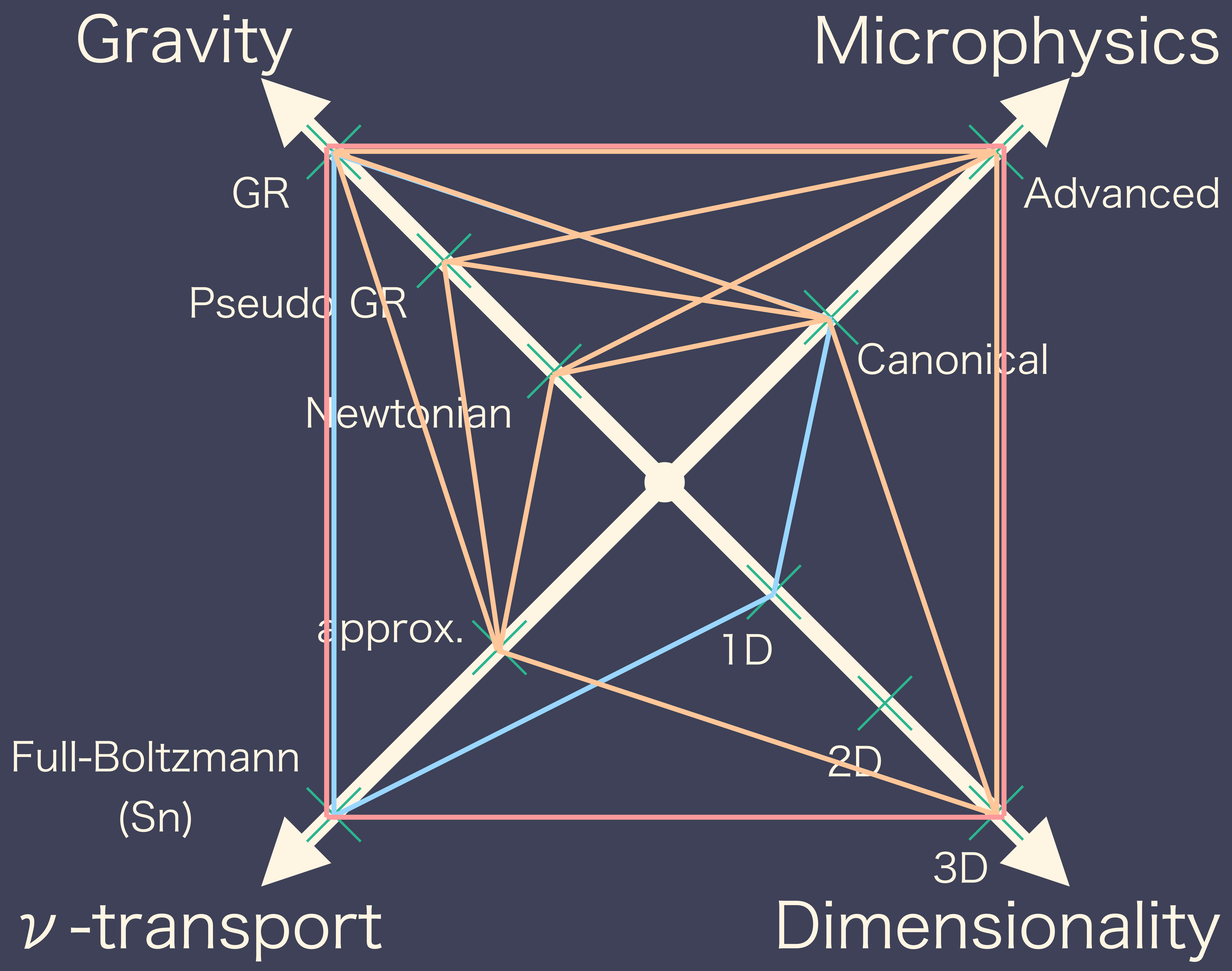
Progresses in CCSNe simulations



1D: Failure

2D/3D: Shock revival but how reliable?

→General Relativistic Boltzmann Simulation!

Formulation

Hydrodynamics

$$\partial_t \rho_* + \partial_i(\rho_* v^i) = 0$$

$$\partial_t S_i + \partial_j(S_i v^j + \alpha\sqrt{\gamma}P\delta_i^j) = -S_0\partial_i\alpha + S_j\partial_i\beta^j - \frac{1}{2}\alpha\sqrt{\gamma}S_{jk}\partial_i\gamma^{jk} - \alpha\sqrt{\gamma}\gamma_i{}^\mu C_\mu$$

$$\partial_t(S_0 - \rho_*) + \partial_k((S_0 - \rho_*)v^k + \sqrt{\gamma}P(v^k + \beta^k)) = \alpha\sqrt{\gamma}S^{ij}K_{ij} - S_i D^i\alpha + n^\mu C_\mu$$

$$\partial_t(\rho_* Y_e) + \partial_i(\rho_* Y_e v^i) = \Gamma$$

Boltzmann equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\bigg|_q\left[\left(e^\alpha_{(0)} + \sum_{i=1}^{3}\ell_{(i)}e^\alpha_{(i)}\right)\sqrt{-g}f\right] - \frac{1}{\epsilon^2}\frac{\partial}{\partial\epsilon}(\epsilon^3 f\omega_{(0)}) + \frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}(\sin\theta_\nu f\omega_{(\theta_\nu)}) + \frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}(f\omega_{(\phi_\nu)}) = S_{\rm rad}$$

Einstein equation (BSSN formalism)
+ moving puncture gauge

$$ds^2 = -\alpha^2 dt^2 + e^{4\phi}\bar{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$\partial_\perp\bar{\gamma}_{ij} = -\frac{2}{3}\bar{\gamma}_{ij}\bar{D}_k\beta^k - 2\alpha\bar{A}_{ij}$$

$$\partial_\perp\bar{A}_{ij} = -\frac{2}{3}\bar{A}_{ij}\bar{D}_k\beta^k - 2\alpha\bar{A}_{ik}\bar{A}^k{}_j + \alpha\bar{A}_{ij}K + e^{-4\phi}[-2\alpha\bar{D}_i\bar{D}_j\phi + 4\alpha\bar{D}_i\phi\bar{D}_j\phi + 4\bar{D}_{(i}\alpha\bar{D}_{j)}\phi - \bar{D}_i\bar{D}_j\alpha + \alpha(\bar{R}_{ij} - 8\pi S_{ij})]^{\rm TF}$$

$$\partial_\perp\phi = \frac{1}{6}\bar{D}_k\beta^i - \frac{1}{6}\alpha K$$

$$\partial_\perp K = \frac{\alpha}{3}K^2 + \alpha\bar{A}_{ij}\bar{A}^{ij} - e^{-4\phi}(\bar{D}^2\alpha + 2\bar{D}^i\alpha\bar{D}_i\phi) + 4\pi\alpha(\rho + S)$$

$$\partial_\perp\bar{\Lambda}^i = \bar{\gamma}^{jk}\hat{D}_j\hat{D}k\beta^i + \frac{2}{3}\Delta\Gamma^i\bar{D}_j\beta^j + \frac{1}{3}\bar{D}^i\bar{D}_j\beta^j - 2\bar{A}^{jk}(\delta^i{}_j\partial_k\alpha - 6\alpha\delta^i{}_j\partial_k\phi - \alpha\Delta\Gamma^i_{jk}) - \frac{4}{3}\alpha\bar{\gamma}^{ij}\partial_j K - 16\pi\alpha\bar{\gamma}^{ij}S_j$$

$$\partial_\perp\alpha = -2\alpha K$$

$$\partial_\perp\beta^i = \frac{3}{4}\bar{\Lambda}^i - \eta\beta^i$$

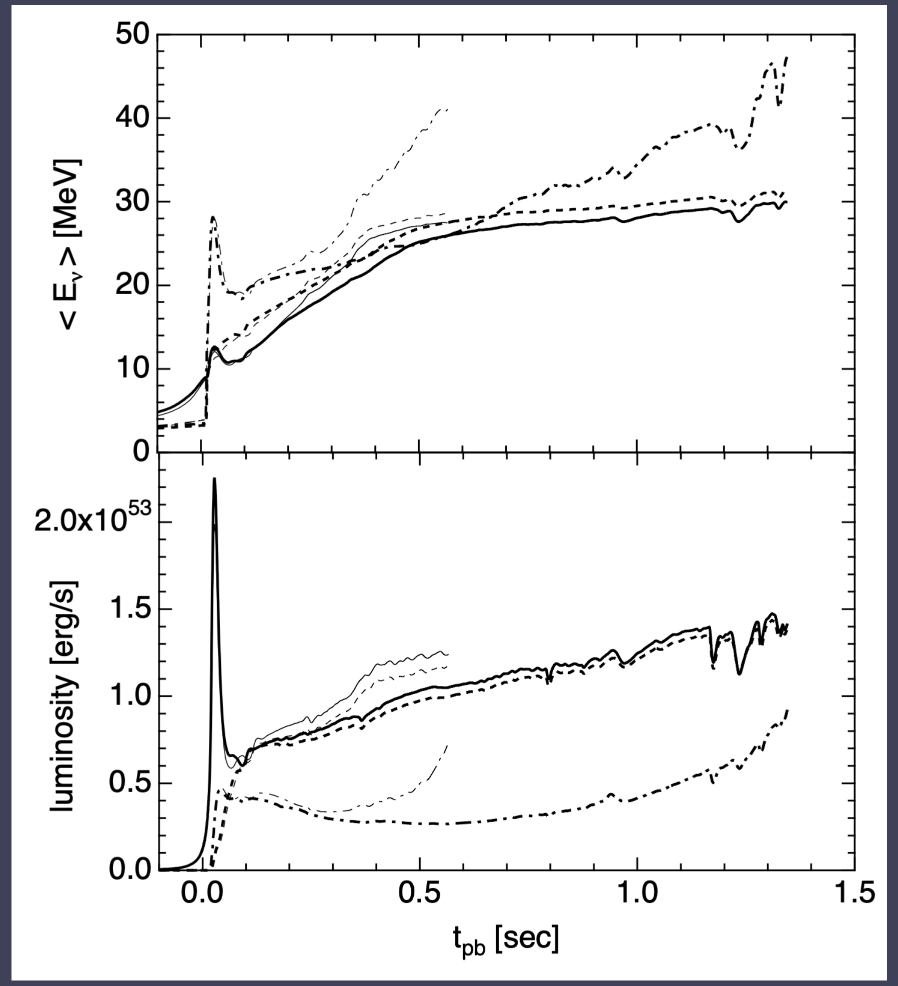
- Solve hydrodynamics, Boltzmann (ν), and Einstein (gravity) equations under moving puncture gauge condition
- Technique 1: factoring out coordinate singularities (r^{-1}, $\sin^{-1}\theta$) to differentiate them by hand
- Technique 2: employ partially implicit Runge-Kutta method

Comparison with other codes

1D-GR-Boltzmann code
(Yamada—Sumiyoshi)

Lagrangian-hydro code
based on Misner-Sharp metric

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dm^2 + R^2 d\Omega^2$$



Sumiyoshi+(2006)

Polar-Slicing
Radial-Gauge coordinate
(Akaho code)

Eulerian-hydro code
based on constraint eq.
(TOV-like equations)

$$ds^2 = -e^{2\Phi}dt^2 + e^{2\Lambda}dm^2 + R^2 d\Omega^2$$

This code

Eulerinan-hydro code
based on BSSN formalism
and moving puncture gauge

$$ds^2 = -\alpha^2 dt^2 + e^{4\phi}\bar{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

- More general form of metric than existing 1D-GR-Boltzmann code, which stops soon after the BH formation
- The BSSN formalism and moving puncture gauge allow to follow the time evolution after the BH formation

From isotropic coordinate to maximal slicing (vacuum case)

isotropic coordinate

$$ds^2 = -\left(\frac{1-\frac{M}{2r}}{1+\frac{M}{2r}}\right)^2 dT^2 + \left(1+\frac{M}{2r}\right)^4 (dr^2 + r^2 d\Omega^2)$$

maximal slicing (trumpet solution)

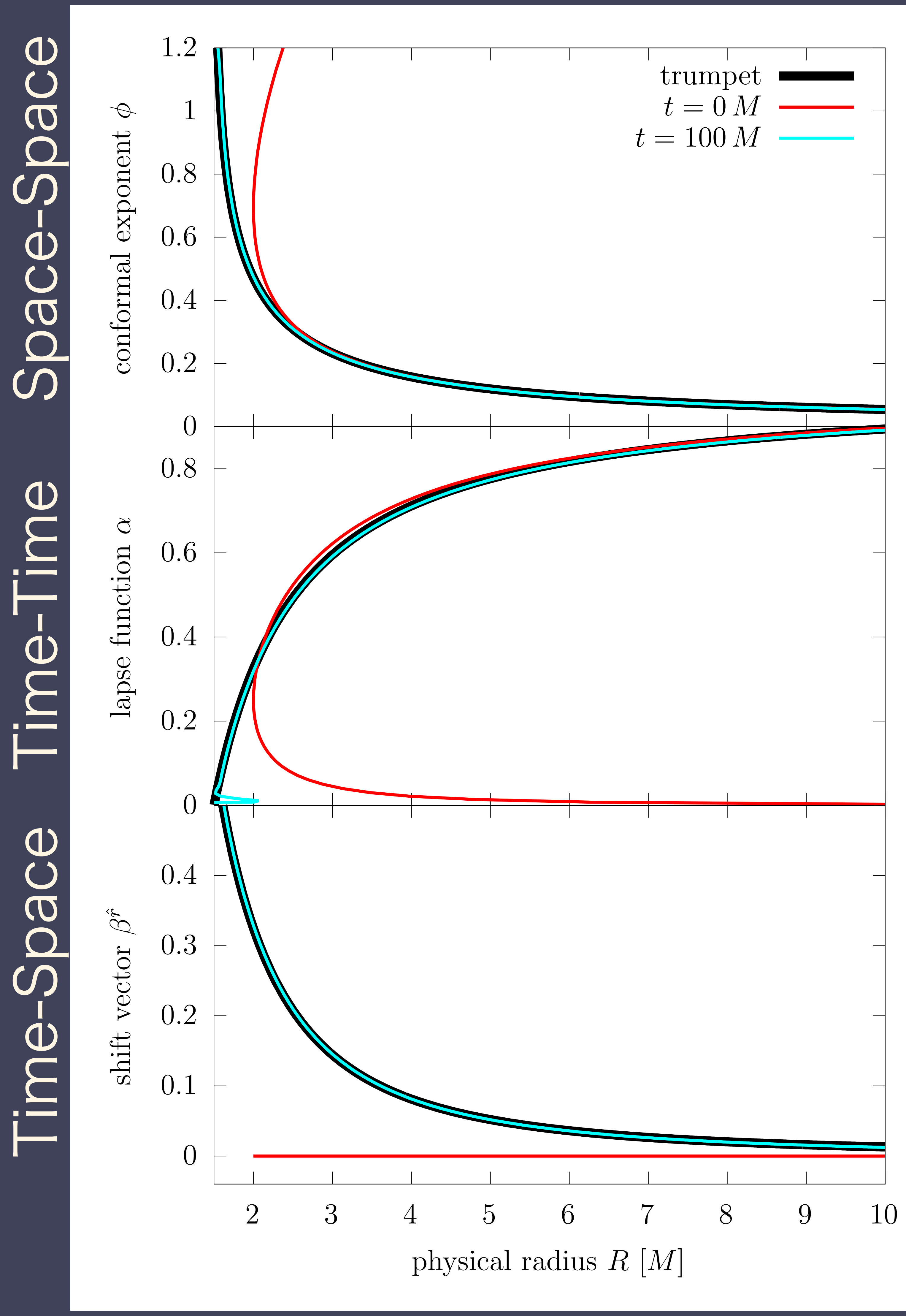
$$\gamma_{RR} = \left(1 - \frac{2M}{R} + \frac{C^2}{R^4}\right)^{-1}$$

$$K^i_j = \mathrm{diag}(2C/R^3, -C/R^3, -C/R^3)$$

$$\beta^R = \frac{\alpha C}{R^2}$$

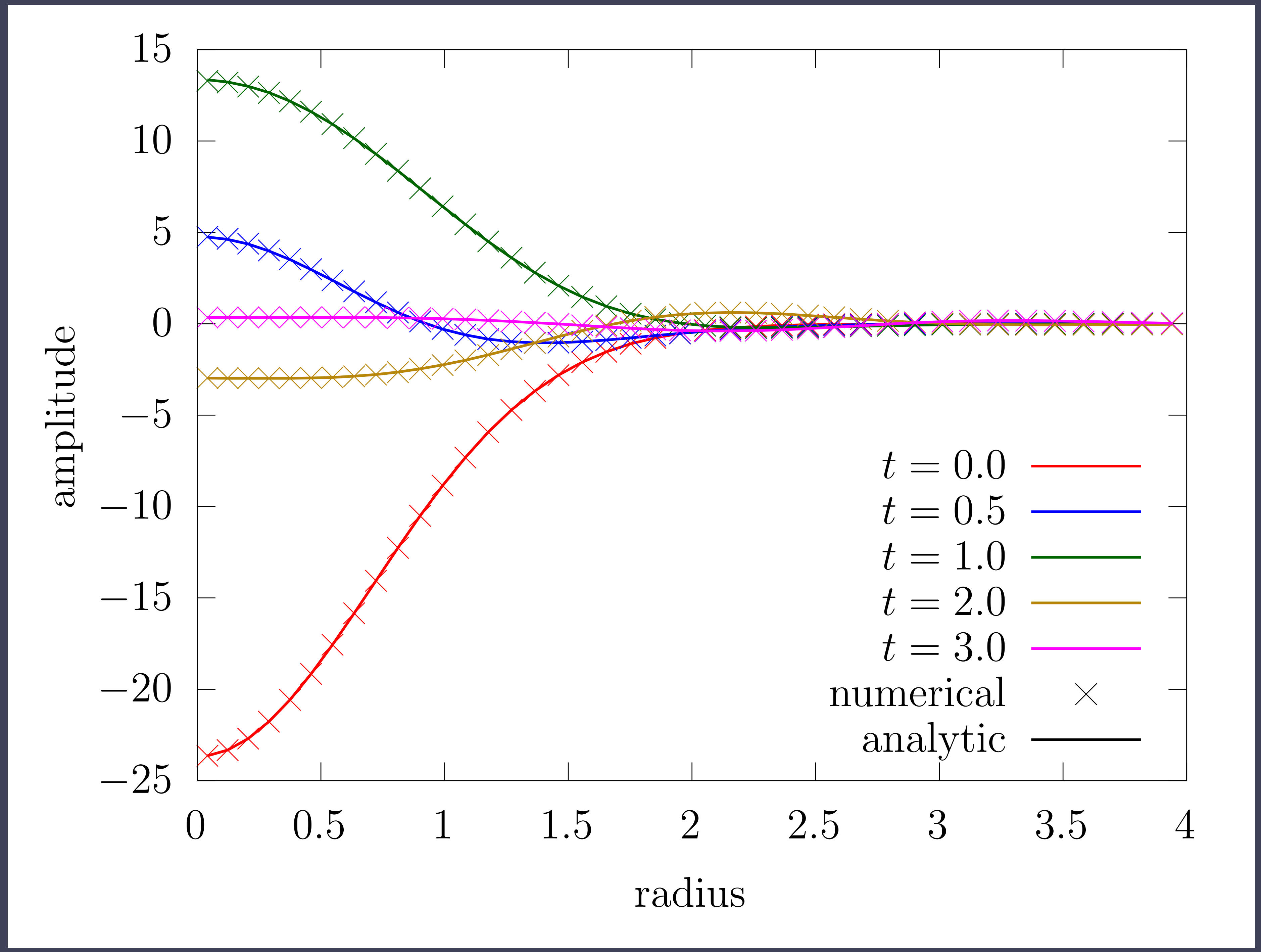
$$\alpha = \sqrt{1 - \frac{2M}{R} + \frac{C^2}{R^4}}$$

- Although Schwarzschild spacetime is a steady state solution, coordinate can evolve depending on gauge conditions
- Initially isotropic coordinate metric evolves into maximal slicing metric



Teukolsky wave

$(\gamma_{rr}-1)\times 10^7$



$\Delta r = 0.1$

$\Delta \theta = 0.1$

$\Delta \phi = 0.1$

$(\theta, \phi) = (1.62, 3.19)$ direction

- BSSN code is implemented in 3D
- Teukolsky (1982) l=2, m=2, $F=10^{-7}(r\pm t)e^{-(r\pm t)^2}$ solution is used for 3D linear wave test in vacuum spacetime
- Numerical solution traces analytic solution

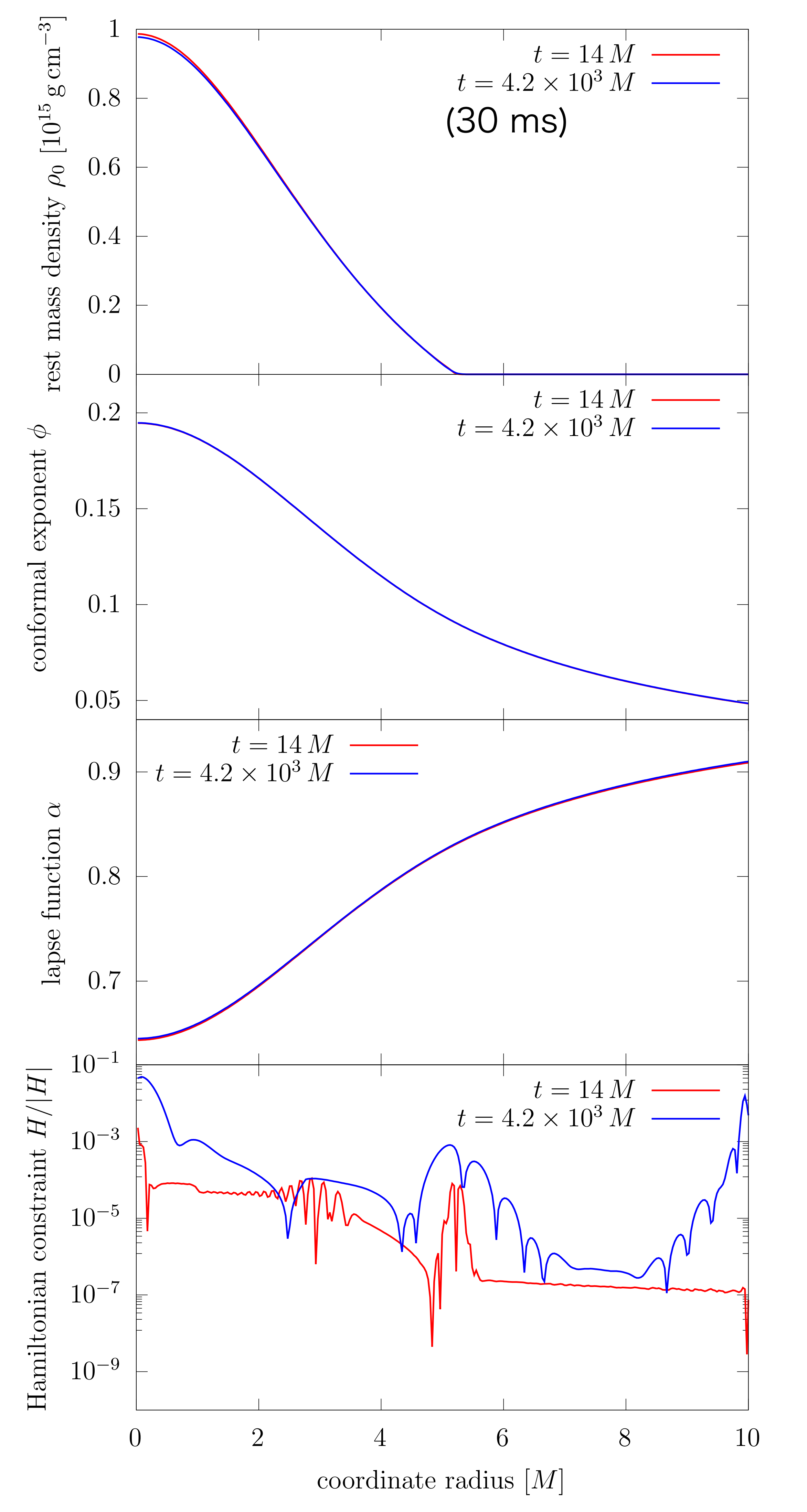
Static NS (γ=2 polytrope)

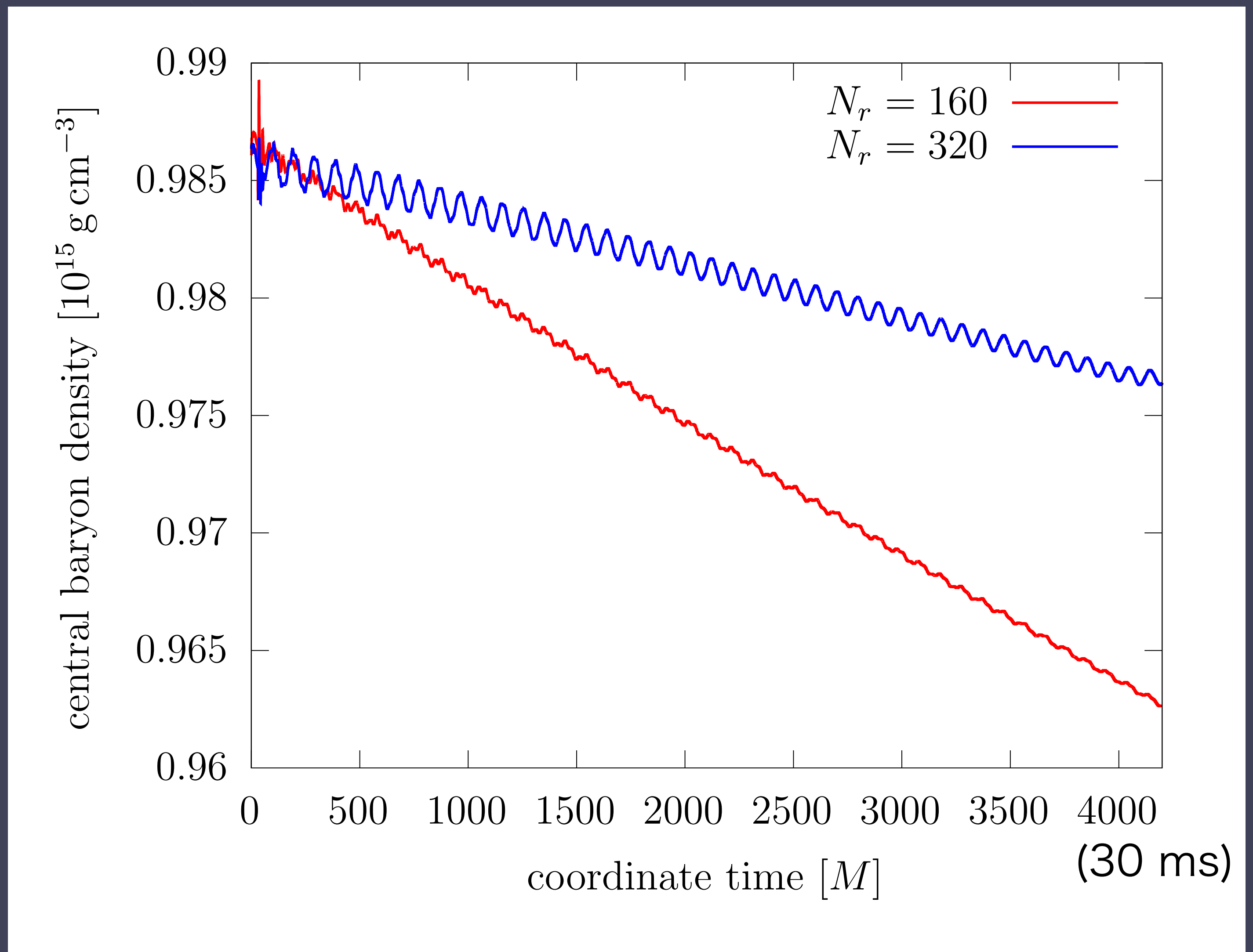
- Hydro+Numerical Relativity
- γ=2 polytropic EOS
- Time evolution with IC of TOV solution
- Less than 1% error at 30 ms = 4200 M

Sp-Sp

Time-Time

$$ds^2 = -\alpha^2 dt^2 + e^{4\phi}\bar{\gamma}_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

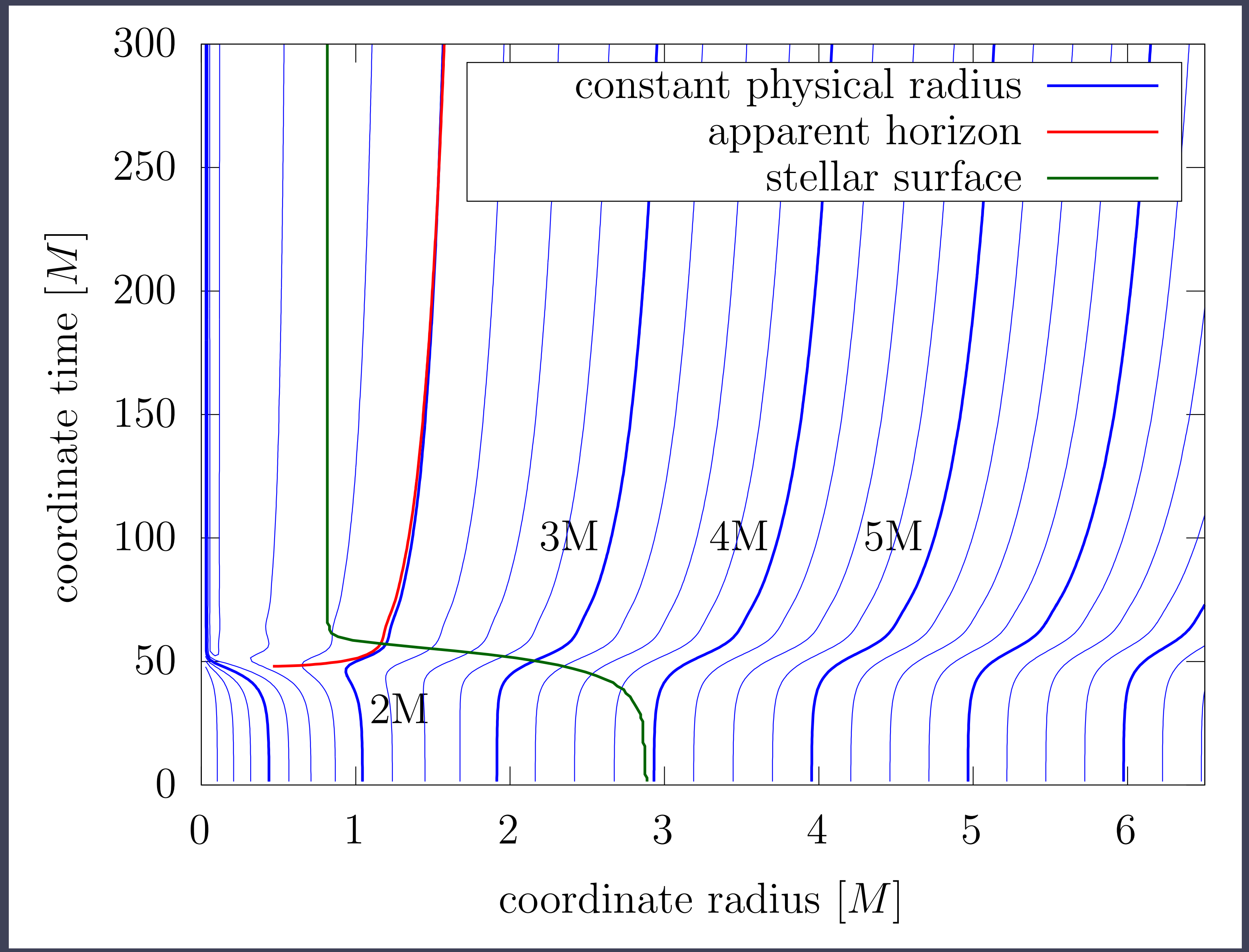




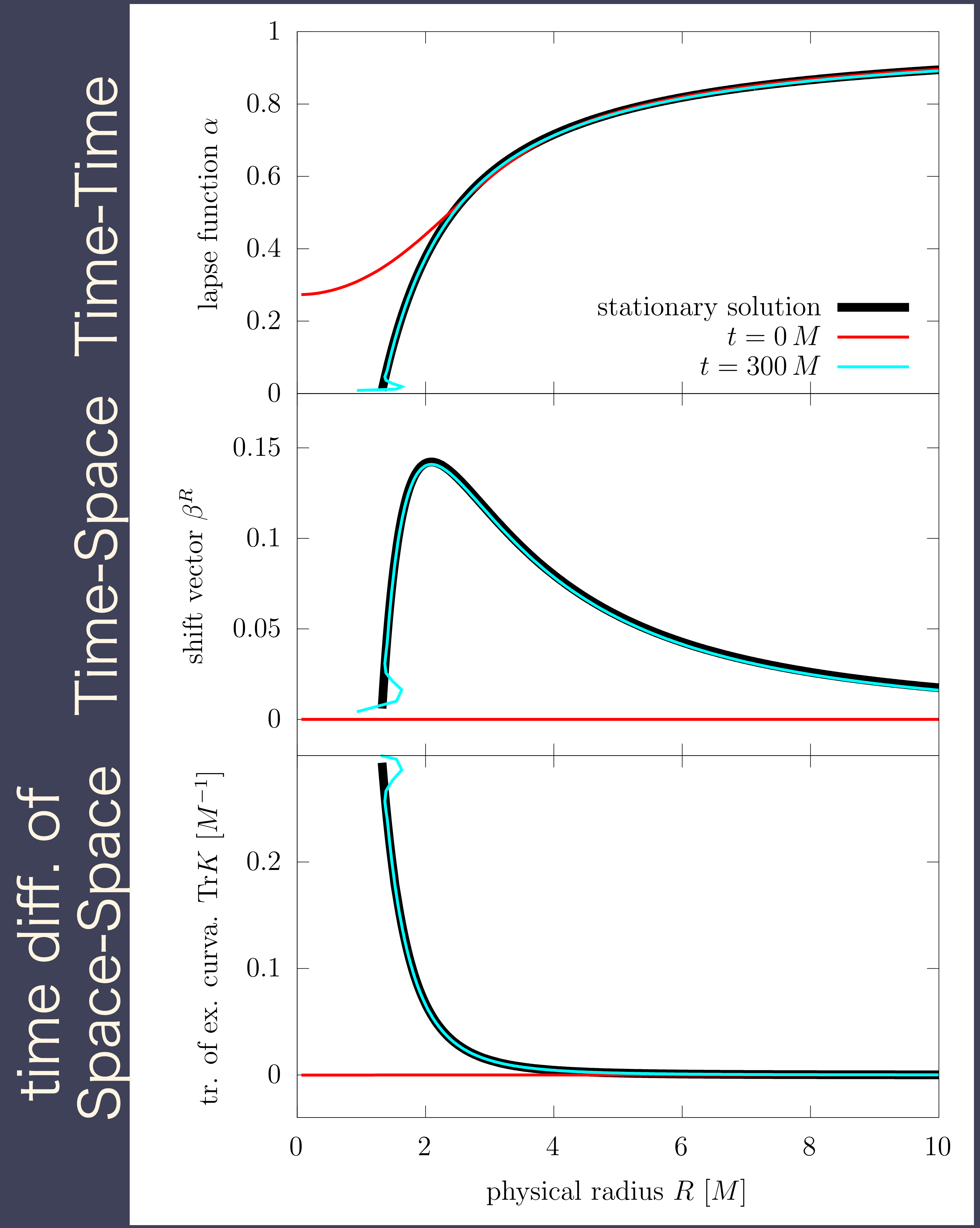
Constraint

0 < r < 22 km= 10 M is divided into equal N_r=320 bin

Unstable NS (γ=2 polytrope)

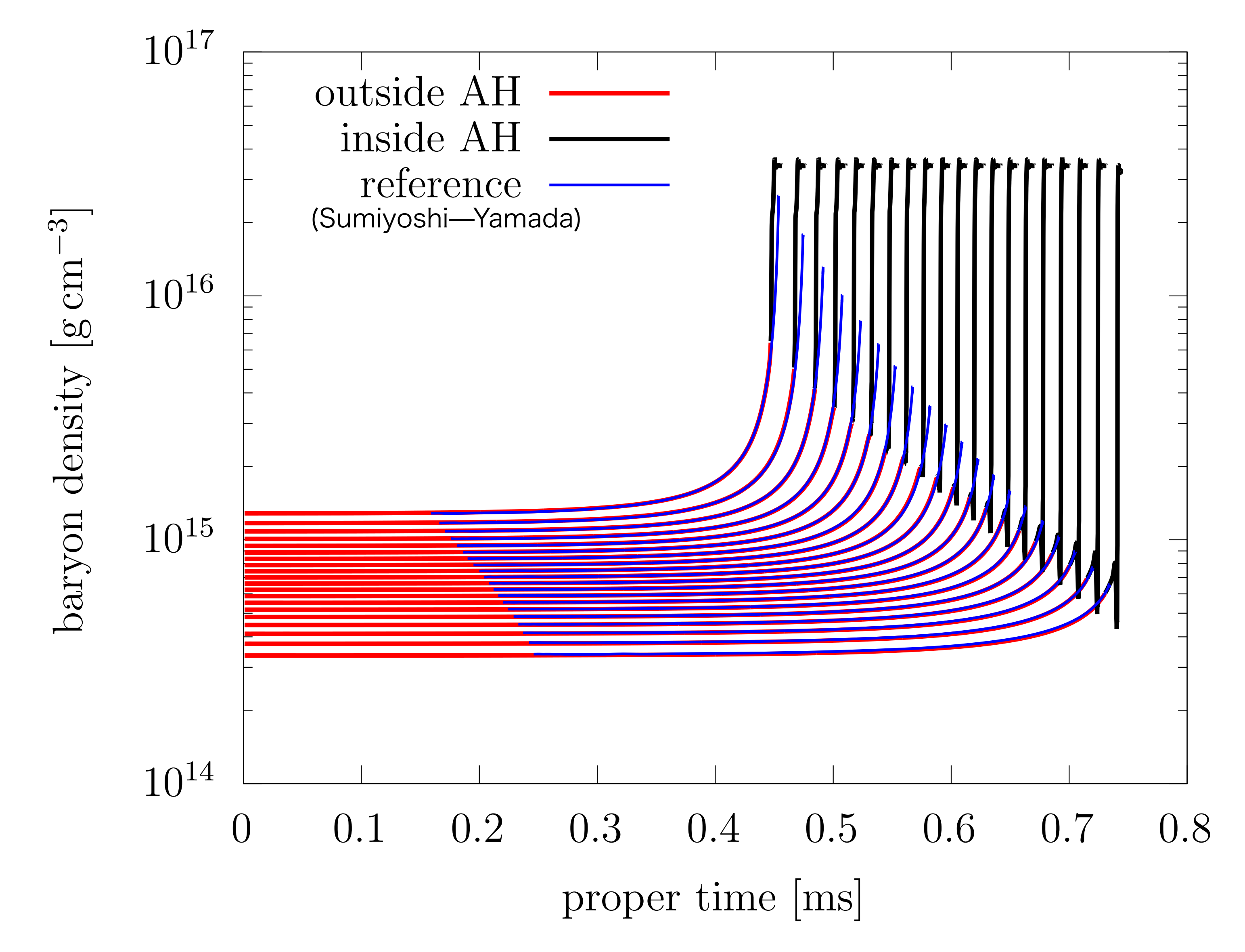


⊿r = 70 m = 0.03 M



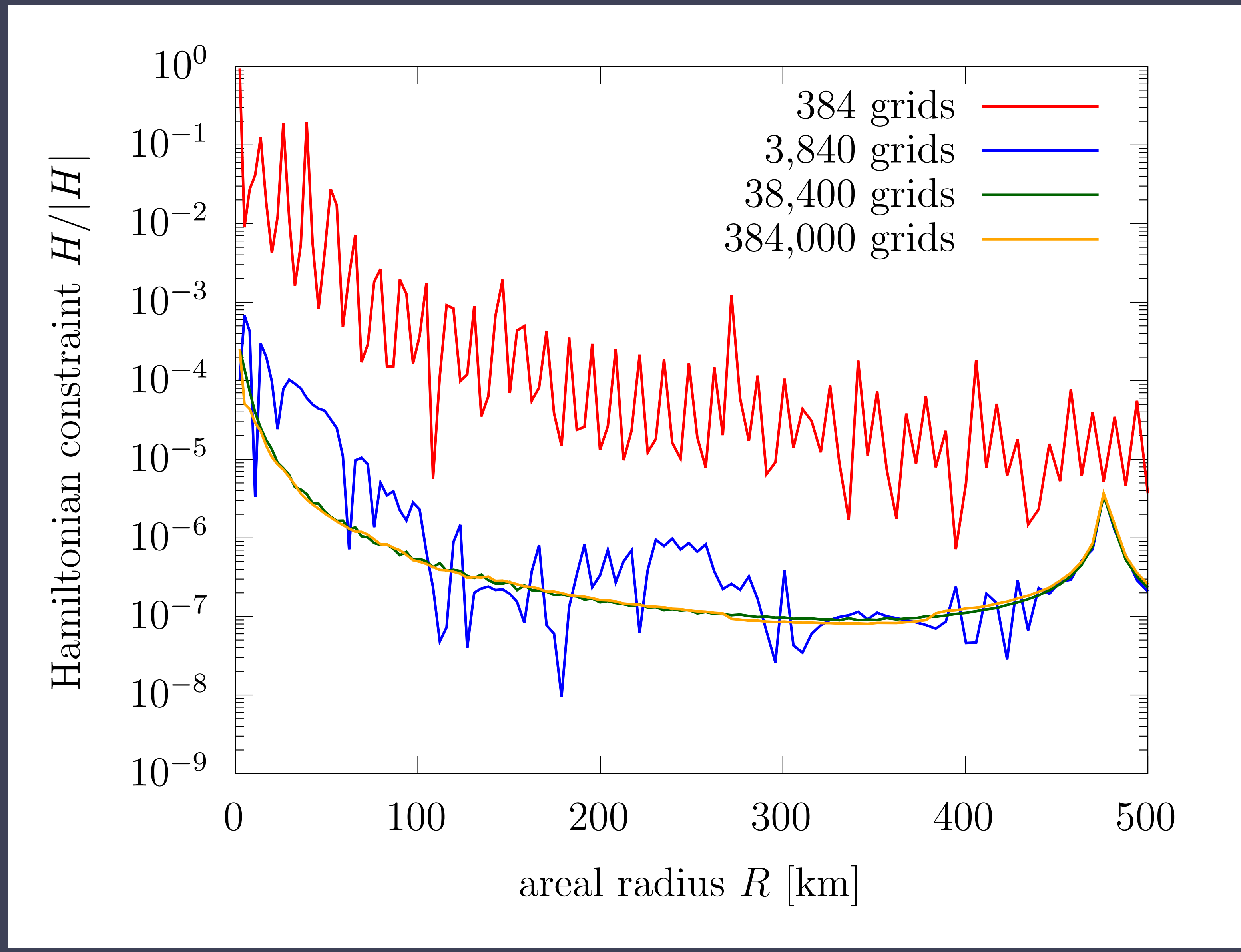
- Gravitational collapse of dynamically unstable TOV solution (γ=2 polytrope EOS)
- Apparent horizon is formed at the Schwarzschild radius, and metric approaches stationary solution

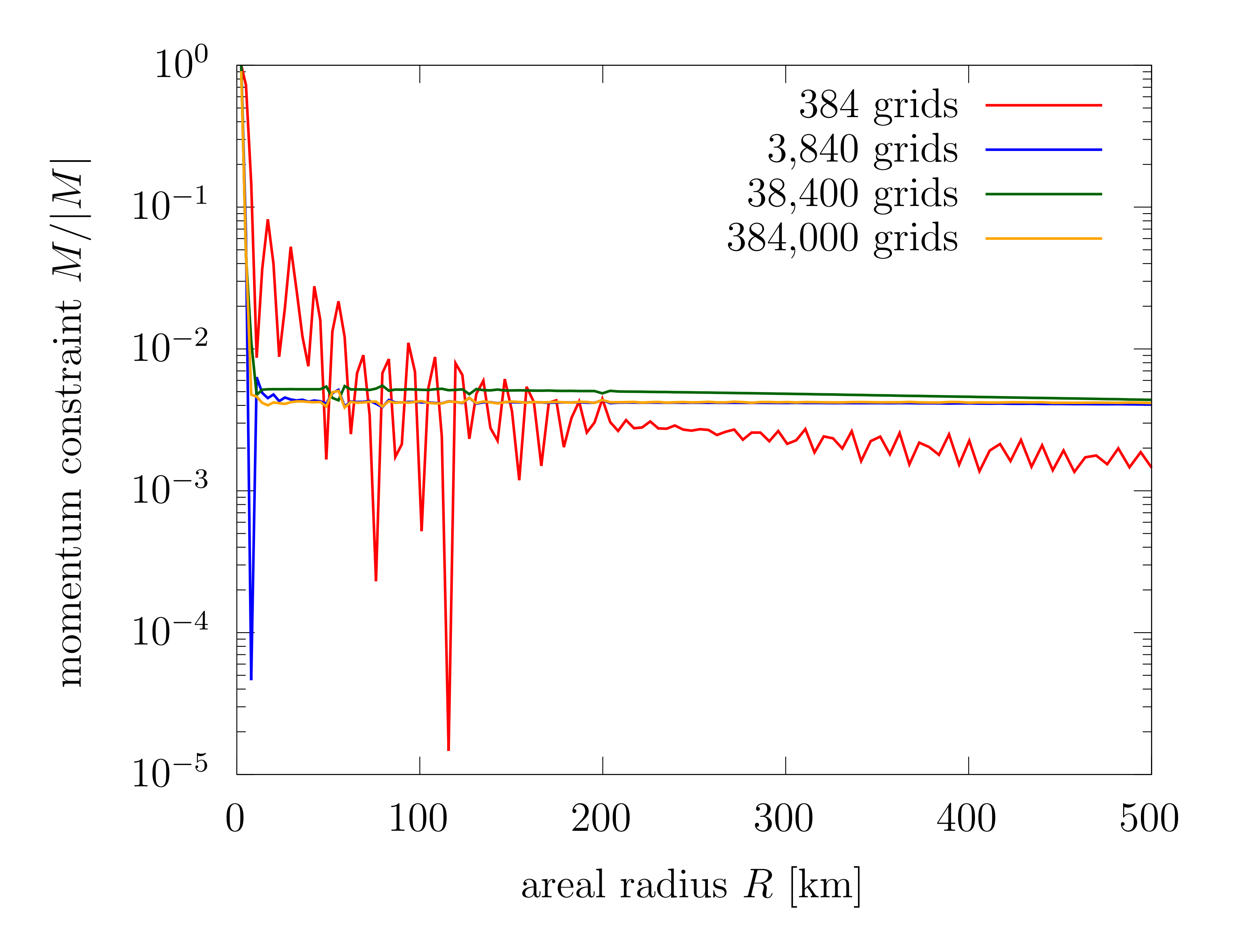
Unstable NS (Shen TM1e EOS)



- Comparison of two independent codes (BSSN-Eulerian-Hydro code vs Sumiyoshi—Yamada code)
- Dynamically unstable NS (Shen TM1e EOS based on RMF and density-dependent symmetry slope)
- Both solution coincide outside the horizon (red-BSSN-Hydro code vs blue-Sumiyoshi—Yamada code)

Initial condition solver





- Initial condition for the numerical relativity should satisfy the Hamiltonian and momentum constraints
- Need to solve constraint equations for some metric quantities
- Initial moment of 40 $M_\odot$ progenitor with Shen TM1e EOS
- Constraints are satisfied very accurately with increasing resolution

BSSN-Boltzmann-Hydro simulation

is under debugging … stay tuned!

Summary

- General Relativistic Boltzmann-Radiation-Hydrodynamics code has been developed.
- GR part is developed based on BSSN formalism in spherical coordinate and moving puncture gauge.
- Vacuum spacetime and dynamics of NS are accurately evolved.
- BSSN-Boltzmann-Hydrodynamics is under debugging… stay tuned!