The Boltzmann Neutrino Radiation Hydrodynamic Simulation of a Core-collapse Supernova in the Three-dimensional Space

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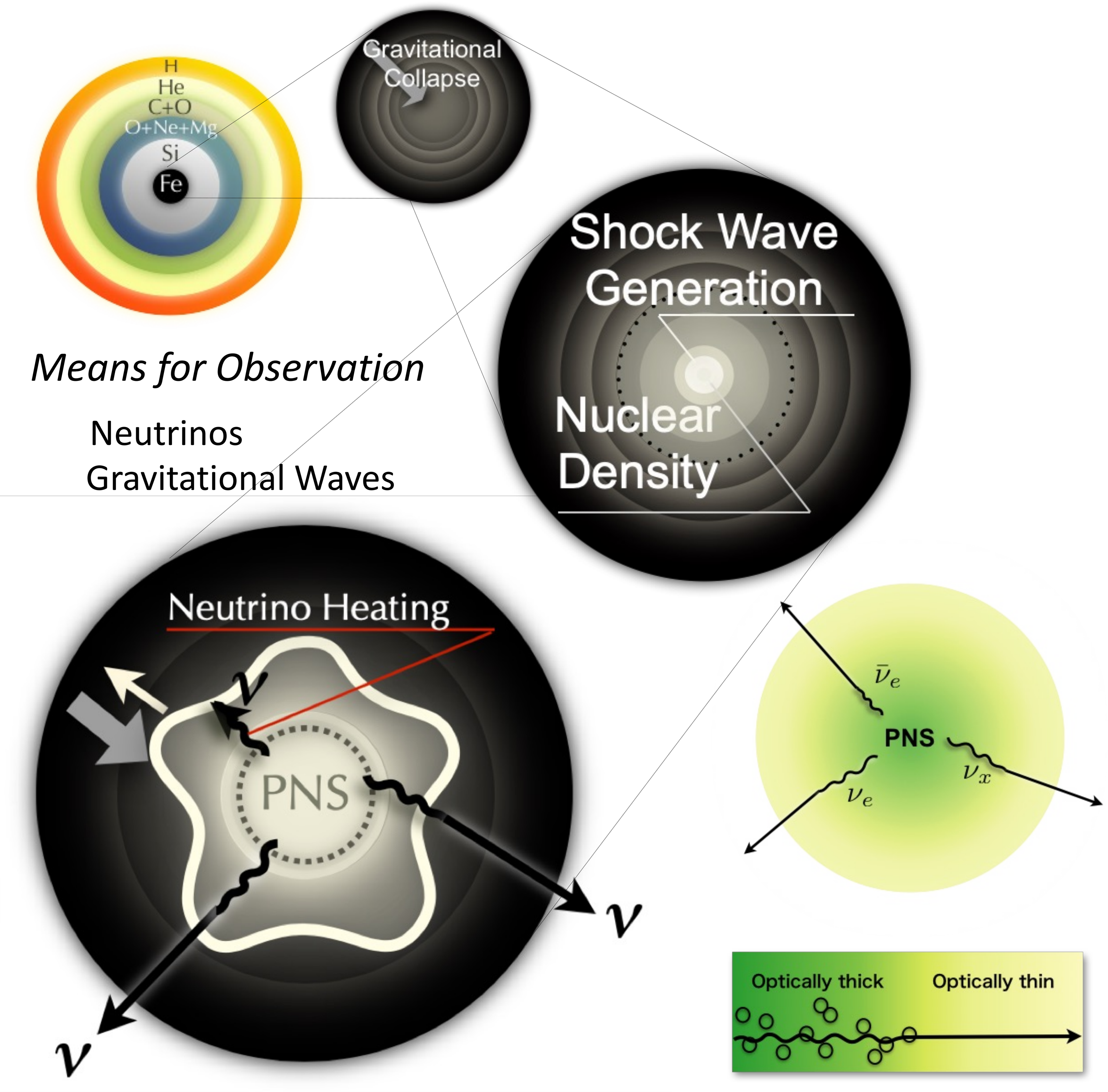
Neutrino Heating Mechanism for Core-collapse Supernovae



Euler Equations

Hydrodynamics

Continuity Equation: $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x^j}\left(\rho v^j\right)=0$$

Equations of Motion: $$\frac{\partial}{\partial t}\left(\rho v_i\right)+\frac{\partial}{\partial x^j}\left(\rho v_i v^j+P\delta_i^j\right)=-\rho\frac{\partial \psi}{\partial x^i}\boxed{-G^i}$$

Energy Equation: $$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+e\right)+\frac{\partial}{\partial x^j}\left[\left(\frac{1}{2}\rho v^2+e+P\right)v^j\right]=-\rho v^j\frac{\partial \psi}{\partial x^j}\boxed{-G^0}$$

Time-Evolution Equation of Electron Number: $$\frac{\partial}{\partial t}\left(\frac{\rho Y_e}{m_A}\right)+\frac{\partial}{\partial x^j}\left(\frac{\rho Y_e}{m_A}v^j\right)=\boxed{-\Gamma}$$

Poisson's equation for gravity: $$\Delta\psi=4\pi G\rho$$

EOS table of Nuclear Matter (LS K=220MeV, Furusawa+Togashi, etc.)

ρ: density, v: velocity, P: pressure, e: internal energy, ψ: the gravitational potential, G: the gravitational constant ($=6.67\times10^{-8}[\mathrm{cm^3g^{-1}s^2}]$), Y_e: electron fraction, m_A: the atomic mass unit, G^0: neutrino radiation energy, G^i: neutrino radiation pressure, Γ: deleptonization rate ($\equiv\Gamma_{\nu_e}-\Gamma_{\bar{\nu}_e}$), Γ_s: neutrino number density

Neutrino Energy Density (μ = 0)
Radiation Pressure (μ = 1, 2, 3)

$$G_s^\mu\equiv\int p_s^\mu\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}}d^3\mathbf{p}\qquad\boxed{G^\mu}\equiv\sum_s G_s^\mu$$

Neutrino Number Density
s: species ($s=\nu_e,\ \bar{\nu}_e,\ \nu_x$)

$$\Gamma_s\equiv\int\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}}d^3\mathbf{p}\qquad\boxed{\Gamma}\equiv\Gamma_{\nu_e}-\Gamma_{\bar{\nu}_e}$$

Quadrupolar Equation of the Gravitational Wave

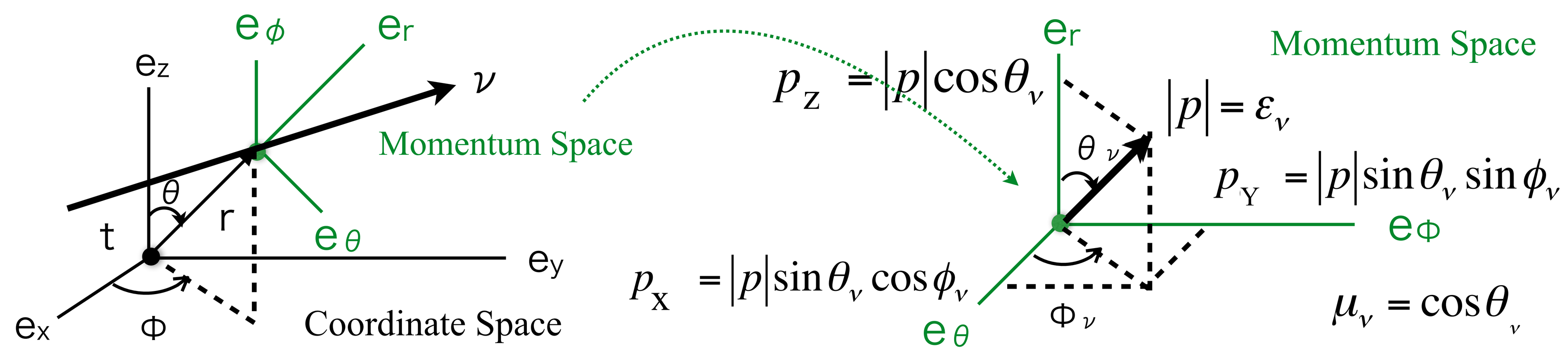
$$h_+=\frac{1}{r}\left[\ddot{I}_{xx}\cos^2\theta+\ddot{I}_{yy}\sin^2\phi+\ddot{I}_{xy}\sin^2 2\phi\cos^2\theta+\ddot{I}_{zz}\sin^2\theta\right.$$
$$\left.-\left(\ddot{I}_{xz}\cos\phi+\ddot{I}_{yz}\sin\phi\right)\sin 2\theta-\ddot{I}_{xx}\sin^2\phi-\ddot{I}_{yy}\cos^2\phi+\ddot{I}_{xy}\sin 2\phi\right]$$

$$h_\times=\frac{2}{r}\left[\frac{1}{2}\left(\ddot{I}_{yy}-\ddot{I}_{xx}\right)\sin 2\phi\cos\theta\right.$$
$$\left.+\ddot{I}_{xy}\cos 2\phi\cos\theta+\left(\ddot{I}_{xz}\sin\phi+\ddot{I}_{yz}\cos\phi\right)\sin\theta\right]$$

$$\ddot{I}_{ij}=\frac{G}{c^4}\int d^3x\left[2\rho v_i v_j-\rho\left(x_i\frac{\partial\Phi}{\partial x_j}+x_j\frac{\partial\Phi}{\partial x_i}\right)\right]$$

Boltzmann Equation

Neutrino Radiation



Neutrino distribution function

$$f(t,r,\theta,\phi;\varepsilon_\nu,\mu_\nu,\phi_\nu)$$

Boltzmann Equation

$$\frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu}+\frac{dp^i}{d\lambda}\frac{\partial f}{\partial p^i}=\left(\frac{\delta f}{\delta\lambda}\right)_{\text{collision}}$$

Boltzmann Equation in the spherical coordinate

$$\frac{\partial f}{\partial t}+\cos\theta_\nu\frac{\partial f}{\partial r}+\frac{\sin\theta_\nu\cos\phi_\nu}{r}\frac{\partial f}{\partial\theta}+\frac{\sin\theta_\nu\sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial\phi}$$
$$-\frac{\sin\theta_\nu}{r}\frac{\partial f}{\partial\theta_\nu}-\frac{\cos\theta\sin\theta_\nu\sin\phi_\nu}{\sin\theta}\frac{1}{r}\frac{\partial f}{\partial\phi_\nu}=\left(\frac{\delta f}{\delta t}\right)_{\text{collision}}$$

$$\left(\frac{\delta f}{\delta\tau}\right)_{\text{collision (s)}}=\left[\frac{\delta f}{\delta\tau}\right]_{\text{emis-abs (s)}}+\left[\frac{\delta f}{\delta\tau}\right]_{\text{scat (s)}}+\left[\frac{\delta f}{\delta\tau}\right]_{\text{pair (s)}}$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{emis-abs}}=-R_{\text{abs}}(\varepsilon,\Omega)f(\varepsilon,\Omega)+R_{\text{emis}}(\varepsilon,\Omega)[1-f(\varepsilon,\Omega)].$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{scat}}=-\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{scat}}(\varepsilon,\Omega;\varepsilon',\Omega')f(\varepsilon,\Omega)\times[1-f(\varepsilon',\Omega')]+\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{scat}}(\varepsilon',\Omega';\varepsilon,\Omega)\times f(\varepsilon',\Omega')[1-f(\varepsilon,\Omega)],$$

$$\left[\frac{\delta f}{\delta\tau}\right]_{\text{pair}}=-\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{pair-anni}}(\varepsilon,\Omega;\varepsilon',\Omega')\times f(\varepsilon,\Omega)\overline{f}(\varepsilon',\Omega')+\int\frac{d\varepsilon'\varepsilon'^2}{(2\pi)^3}\int d\Omega' R_{\text{pair-emis}}(\varepsilon,\Omega;\varepsilon',\Omega')\times[1-f(\varepsilon,\Omega)][1-\overline{f}(\varepsilon',\Omega')],$$

Ω' denotes the angle variables after/before the scattering

$\overline{f}(\varepsilon',\Omega')$ denotes the distribution of anti-neutrinos, which is the angle-averaged distribution in the previous time step.

Emission/Absorption

Electron Capture
$e^-+p\longleftrightarrow\nu_e+n$ [ecp]

Anti-Electron Capture
$e^++n\longleftrightarrow\bar{\nu}_e+p$ [aecp]

Electron Capture on nuclei
$e^-+A\longleftrightarrow\nu_e+A'$ [eca]

Scattering

Neutrino-Nucleon scattering
$\nu+N\longleftrightarrow\nu+N$ [nsc]

Neutrino-Nuclei scattering
$\nu+A\longleftrightarrow\nu+A$ [csc]

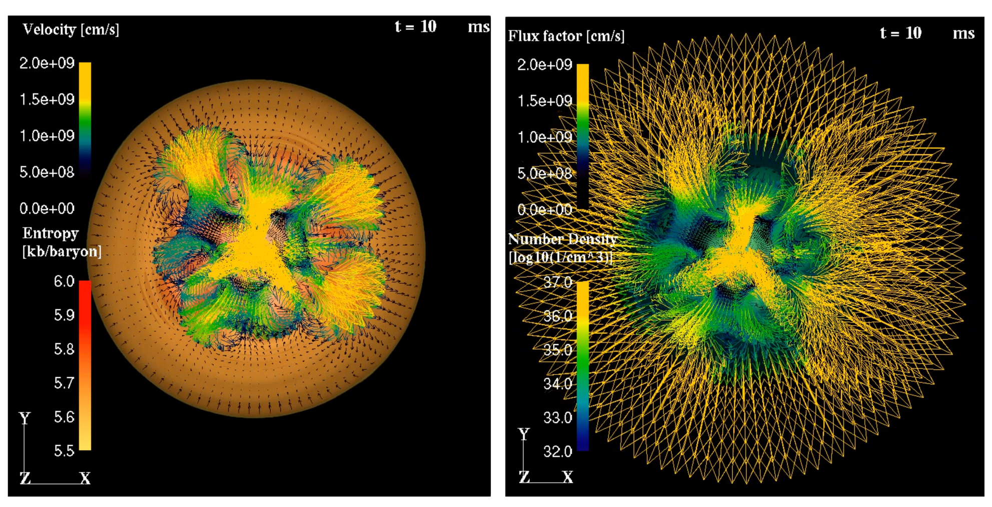
Neutrino-Electron scattering
$\nu+e\longleftrightarrow\nu+e$ [esc]

Pair Process

Electron-positron process
$e^-+e^+\longleftrightarrow\nu_i+\bar{\nu}_i$ [pap]

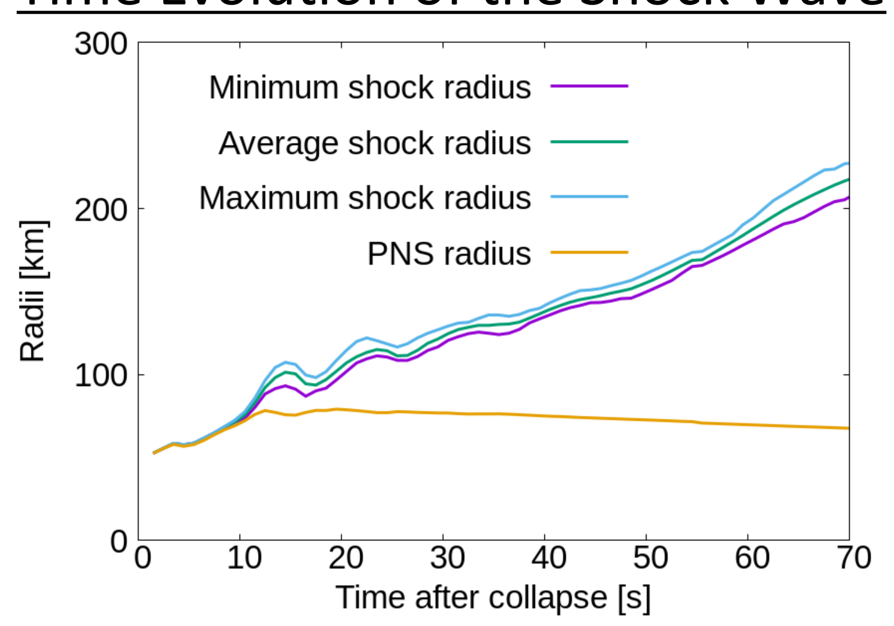
Nucleon-nucleon bremsstrahlung
$N+N\longleftrightarrow N+N+\nu_i+\bar{\nu}_i$ [nbr]

Prompt Convection

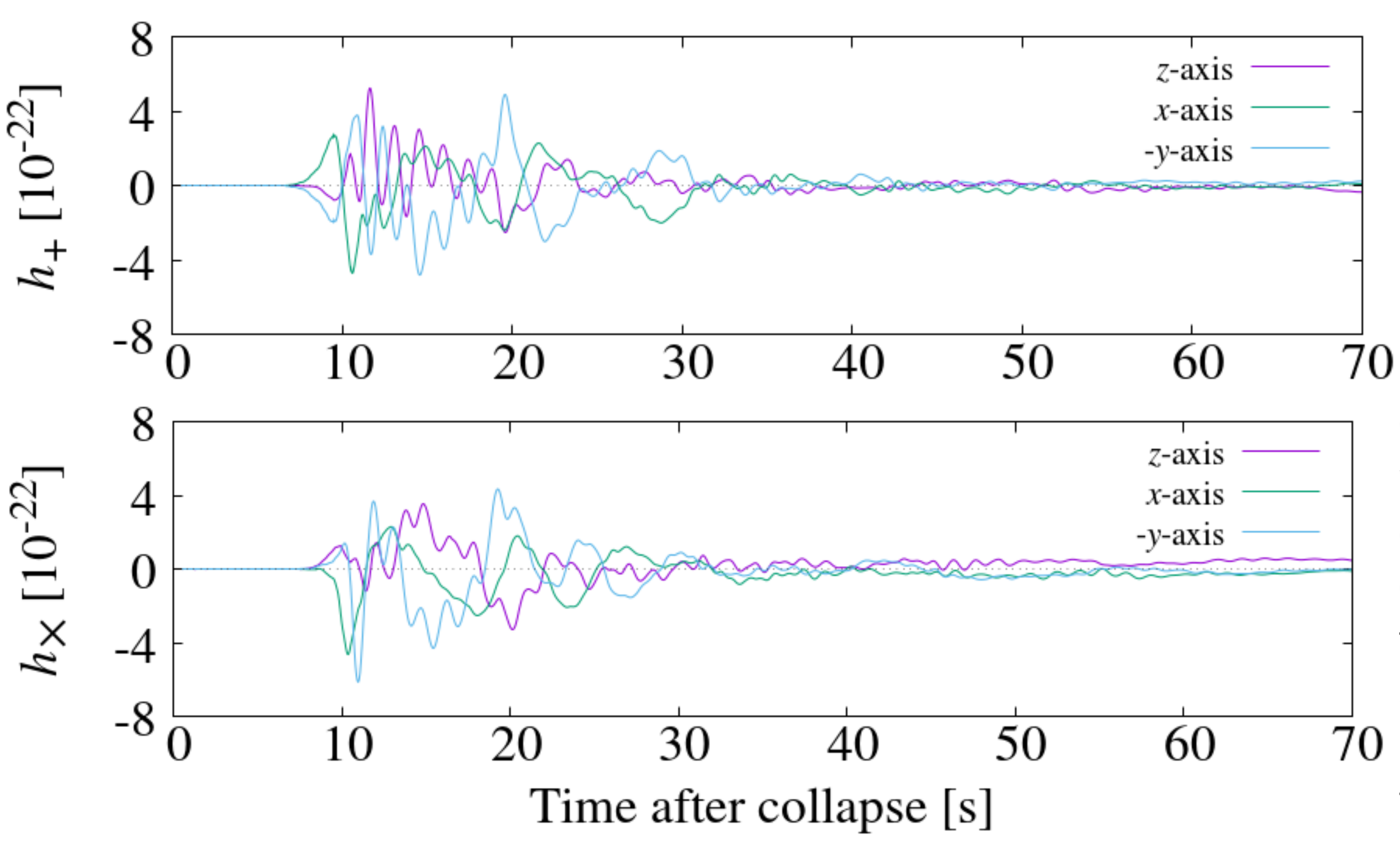


(Iwakami *et al.* 2020)

Time Evolution of the Shock Wave



Time History of the Gravitational Wave from Matter



GW originated from matter is radiated in the various directions when the prompt convection is strongly developed.

GW from neutrino will be also calculated and the correlation with the neutrino signal is investigated in the future work.