Neutrino Nucleon Scattering for the Boltzmann neutrino transfer

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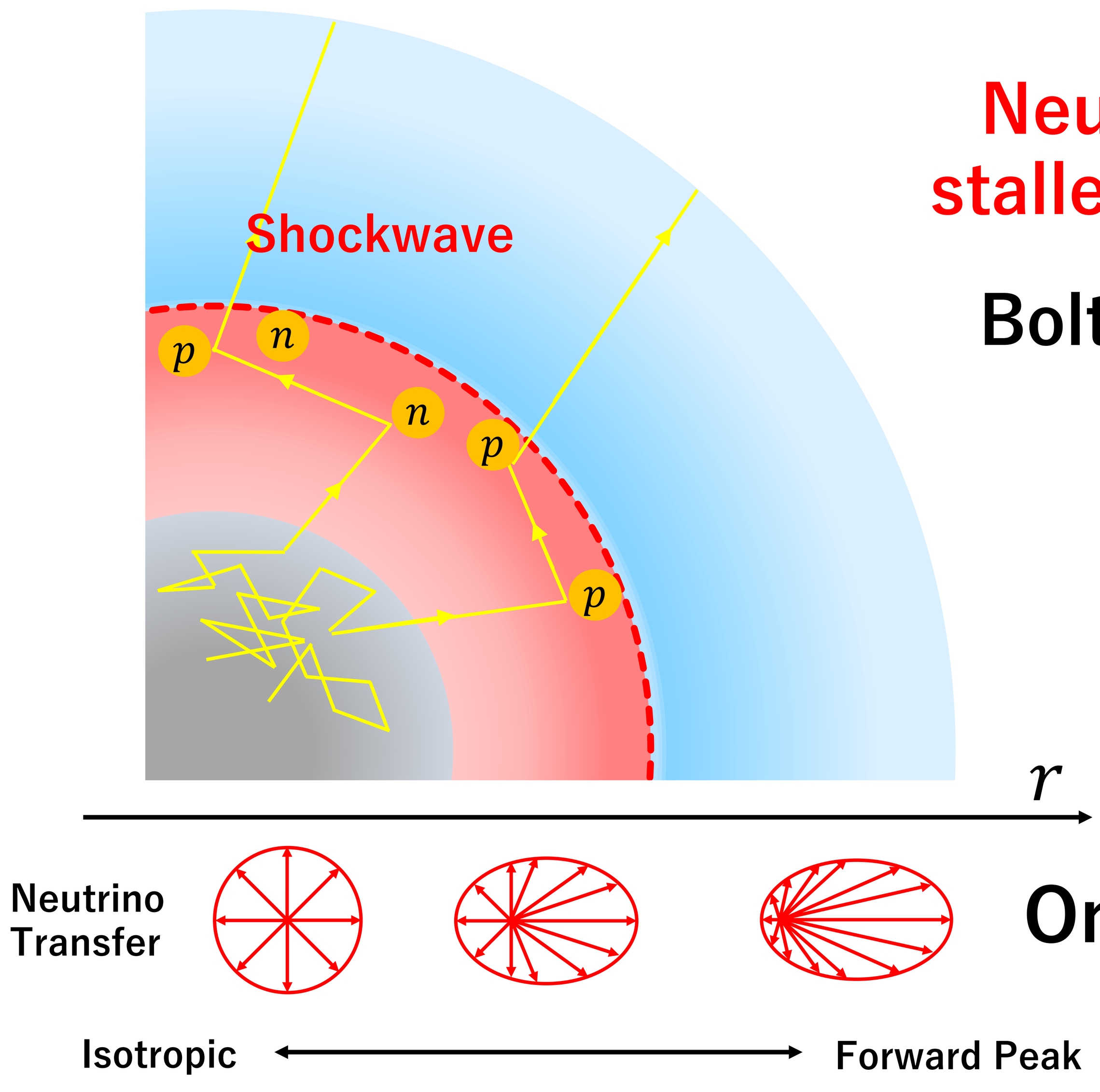


Neutrinos revive stalled shockwaves

Boltzmann Eq.

$$\frac{1}{c}\frac{\partial f}{\partial t}+\frac{\partial f}{\partial s}=\frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$$

Ignore the space dependence

OneZone Calculation

$$\frac{1}{c}\frac{\partial f}{\partial t}=\frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$$

Scattering Process in OneZone Calculation

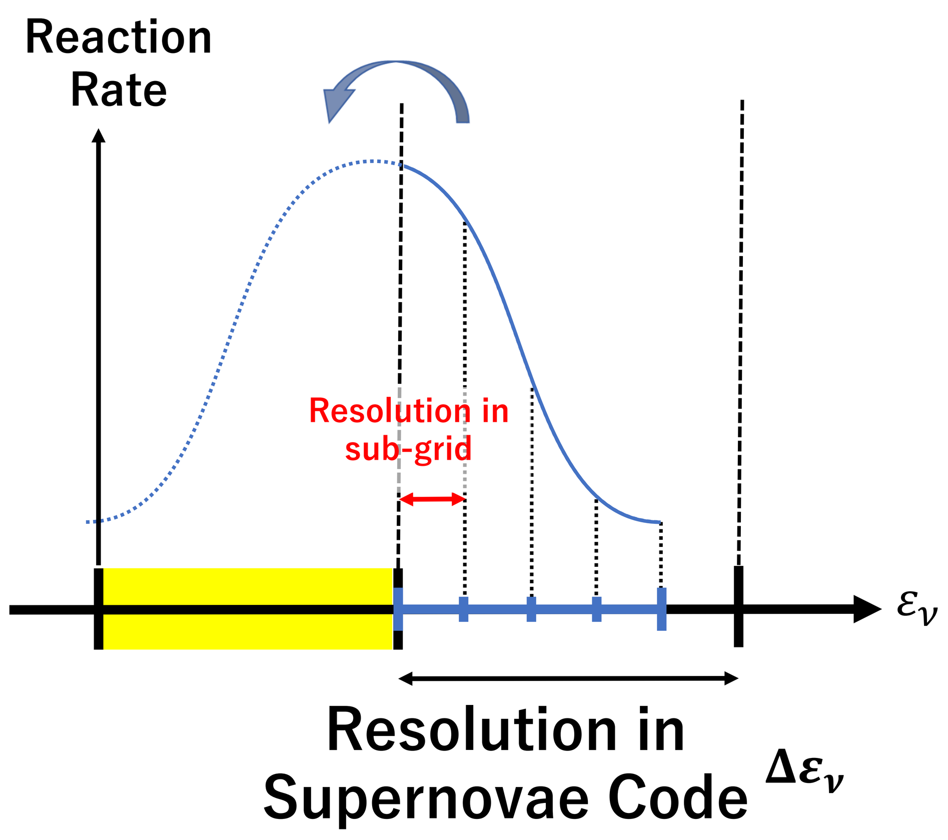
$$\frac{1}{c}\frac{\partial f}{\partial t}=\frac{1}{c}\int\frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' R_{scat}(\epsilon,\Omega;\epsilon',\Omega')\,f(\epsilon,\Omega)\,[1-f(\epsilon',\Omega')]$$
$$-\frac{1}{c}\int\frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' R_{scat}(\epsilon',\Omega';\epsilon,\Omega)\,f(\epsilon',\Omega')\,[1-f(\epsilon,\Omega)]$$

$$\begin{cases}\rho\simeq 10^{10}\mathrm{g/cc}\ ,\ T\simeq 2.698\mathrm{MeV}\ ,\ Ye\simeq 0.2447\\ (N_\epsilon,N_{\epsilon_{sub}},N_{\mu_\nu}N_{\phi_\nu})=(20,8,10,6)\quad \text{And non equilibrium distribution}\end{cases}$$

Subgrid Model

- The subgrid model can probe the small energy exchange due to this Scattering.
- The subgrid model is independent of the derivation of the reaction rate and can directly reflect the reaction rate due to physical effects.

$$R_{\rm scat}^{\rm reconst}(\epsilon,\Omega;\epsilon',\Omega')$$
$$=\frac{1}{\int_\epsilon \epsilon''^2 d\epsilon'' f(\epsilon'',\Omega)}\int_\epsilon \epsilon''^2 d\epsilon''$$
$$\times\int \epsilon'^2 d\epsilon' R_{\rm scat}(\epsilon'',\Omega;\epsilon',\Omega')$$
$$\times f(\epsilon'',\Omega)[1-f(\epsilon',\Omega')]$$



Reaction Rate Of Neutrino Nucleon Scattering

The interaction Lagrangian : $\mathcal{L}=\frac{G_F}{2}\,l_\alpha j^\alpha_{NC}$

the nucleonic neutral current

$$j^\alpha_{NC}=\overline{\Psi_4}\left\{\gamma^\alpha[G_1^N(q^2)-G_A^N(q^2)\gamma^5]+\frac{G_2^N(q^2)i\sigma^{\alpha\beta}q_\beta}{M}\right\}\Psi_2$$

The form factors are

$$G^p_{1,2}(q^2)=\frac{1}{2}[(1-4\sin^2\theta_W)F^p_{1,2}-F^n_{1,2}-F^s_{1,2}],$$
$$G^n_{1,2}(q^2)=\frac{1}{2}[(1-4\sin^2\theta_W)F^n_{1,2}-F^p_{1,2}-F^s_{1,2}],$$
$$G^p_A(q^2)=\frac{1}{2}(G_A+F^s_A)\qquad G^n_A(q^2)=\frac{1}{2}(-G_A+F^s_A)$$

Reaction Rate depend on the interaction Lagrangian based on **nucleon state**

	Recoil	Weak magnetism	Momentum Transfer	Strange -ness	Density depend
nwnd	○				
nwde	○				○
nmnd	○	○			
nmde	○	○			○
fund	○	○	○		
fude	○	○	○		○
fsnd	○	○	○	○	
fsde	○	○	○	○	○

(sugiura et al, 2020,Fischer 2016)

With

$$F_1^p=\frac{1-\frac{q^2\gamma_p}{4M^2}}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}\quad F_1^n=\frac{-\frac{q^2\gamma_n}{4M^2}}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}\quad F_1^s=\frac{F_1^s(0)q^2}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}$$

$$F_2^p=\frac{\gamma_p-1}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}\quad F_2^n=\frac{\gamma_n}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}\quad F_2^s=\frac{F_2^s(0)}{\left(1-\frac{q^2}{4M^2}\right)\left(1-\frac{q^2}{M_V^2}\right)^2}$$

$$F_A(q^2)=\frac{g_A\left(1-\frac{1}{3}\frac{m_{\rm eff}}{m}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}}\right)^{-1}}{\left(1-\frac{q^2}{M_A^2}\right)^2}\qquad F_A^s(q^2)=\frac{\Delta s}{\left(1-\frac{q^2}{M_A^2}\right)^2}$$

$$F_1^s(0)=-\frac{1}{6}\langle r_s^2\rangle=0\qquad F_2^s(0)=\mu_s=0\qquad \Delta s=-0.1$$

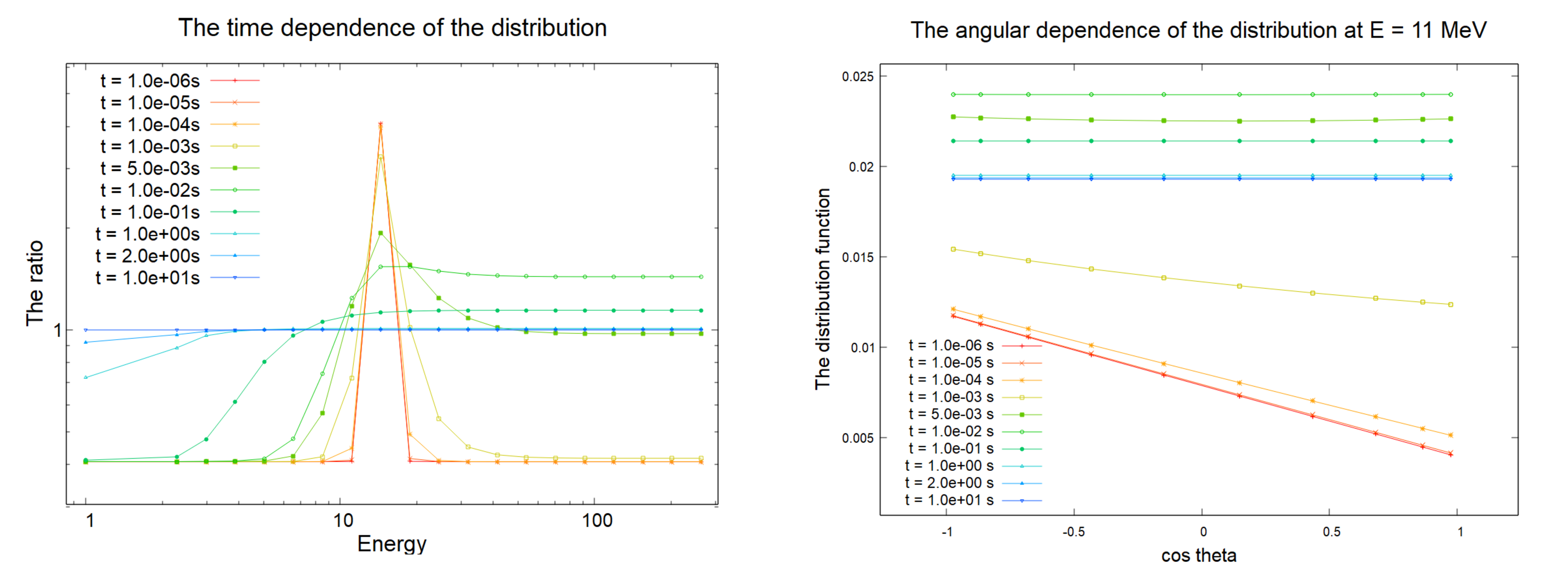
Pseudo Advection effects in OneZone Calculation

The source term $\boldsymbol{S}$ is represented as the other effects, like advection and other reaction. If the source term is decided precisely, without calculating full Boltzmann equation, we can calculate the distribution function with pseudo advection and other reaction.

$$\frac{1}{c}\frac{\partial f}{\partial t}=\frac{1}{c}\int\frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' R_{scat}(\epsilon,\Omega;\epsilon',\Omega')\,f(\epsilon,\Omega)\,[1-f(\epsilon',\Omega')]$$
$$-\frac{1}{c}\int\frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' R_{scat}(\epsilon',\Omega';\epsilon,\Omega)\,f(\epsilon',\Omega')\,[1-f(\epsilon,\Omega)]+S$$

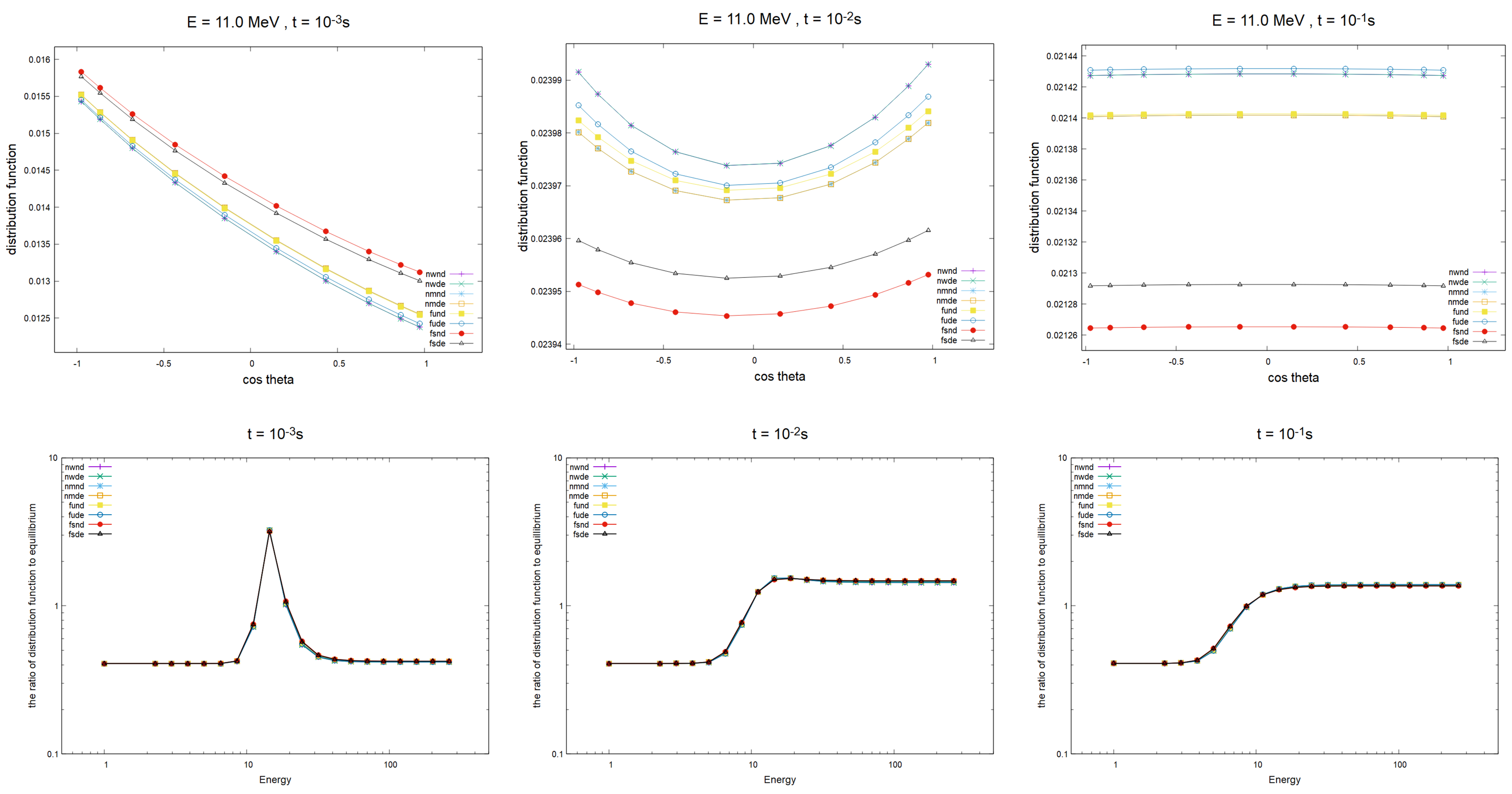
Result 1. Thermalization

The result of the Onezone calculation shows that **the sub-grid model can probe the small energy exchange** due to the scattering and the initial nonequilibrium distribution **finally reached the equilibrium state**.



Result 2. Reaction Dependence

The reaction rate and time development of the distribution depend on **the physical state of the nucleon**. The results are shown for different reaction rates and all calculated under the same initial conditions. As Result 1, **in all case, the distribution reached the equilibrium one**.



Result 3 Angular Dependence

-Result 3-1 Analysis

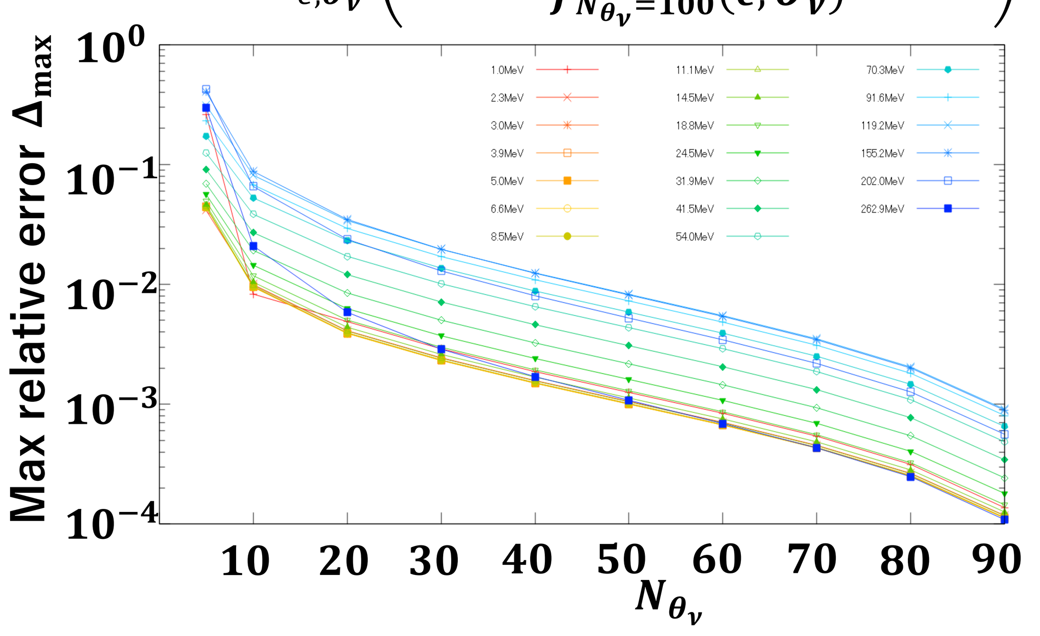
Condition :

$$S\propto\cos\theta_\nu\ ,\ \frac{f_{min}(\epsilon)}{f_{max}(\epsilon)}\sim\frac{1}{3}$$

$$N_{\theta_\nu}=\{5,10,20,30,40,50,60,70,80,90,100\}$$

If the distribution $N_{\theta_\nu}=100$ is true distribution, at each N_{θ_ν}, the maximum relative error is calculated. The trend of the relative error is same and the result says that the error includes only ~1% around $N_{\theta_\nu}=40\sim50$

$$\Delta_{\max}=\max_{\epsilon,\theta_\nu}\left(\frac{|f_{N_{\theta_\nu}}(\epsilon,\theta_\nu)-f_{N_{\theta_\nu}=100}(\epsilon,\theta_\nu)|}{f_{N_{\theta_\nu}=100}(\epsilon,\theta_\nu)}\right)$$



-Result 3-2 Data of SNe

Progenitor : $15M_\odot$

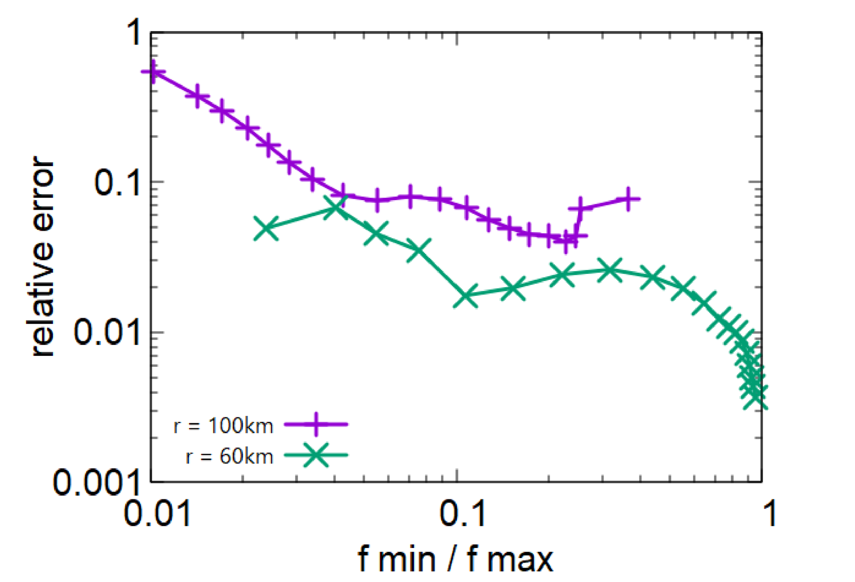
Data : t = 100ms at $r=60km,100km$

$(N_e,N_{\theta_\nu},N_{\phi_\nu})=(20,10,6)$

Comparison :

$(N_e,N_{\theta_\nu},N_{\phi_\nu})=(20,10,6)\ ,\ (20,40,6)$

The result says that the more degree to which the distribution is concentrated forward is, the more relative error is.



Summury

I developed the subgrid model to treat the small energy exchange due to neutrino-nucleon scattering. By inputting non-equilibrium distribution, it was found that the distribution finally reaches equilibrium. I also performed the tests with the source term. The result were found to converge with increasing angular resolution. It is shown that the relative error is based on the ratio of minimum distribution to maximum distribution with respect to θ_ν.
In the future, the subgrid model implement the supernovae simulation code.