

Axion fragmentation & axion dark matter

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N. Fonseca, E. Morgante, RS, G. Servant,
E. Morgante, W. Ratzinger, RS, B.A. Stefanek,
C. Eröncel, RS, G. Servant, P. Sørensen,
C. Eröncel, RS, G. Servant, P. Sørensen,

1911.08472, JHEP 04 (2020) 010
2109.13823, JHEP 12 (2021) 037
2206.14259, JCAP 10 (2022) 053
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ALP : Axion-like particle

Axion field : ϕ

- PQ symmetry (NG boson) + $\phi G_{\mu\nu} \tilde{G}^{\mu\nu}$ -type coupling w/ gauge fields

$$\phi \rightarrow \phi + \delta\phi$$

$$\frac{1}{f} \phi G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Photon,
Gluon,
Hidden gauge boson,
...



- PQ symmetry breaking by strong dynamics

$$V(\phi) \sim -\Lambda_b^4 \cos \frac{\phi}{f}$$

Axion-like particle \rightarrow good candidate of DM

- Light and (almost) stable spin-0 particle is predicted from $\Lambda_b \ll f$.

ALP mass $m_\phi = \sqrt{V''} \sim \frac{\Lambda_b^2}{f}$

ALP lifetime $\tau_\phi \propto \frac{f^2}{m_\phi^3} \sim \frac{f^5}{\Lambda_b^6}$

Axion-like particle DM scenario

Misalignment mechanism

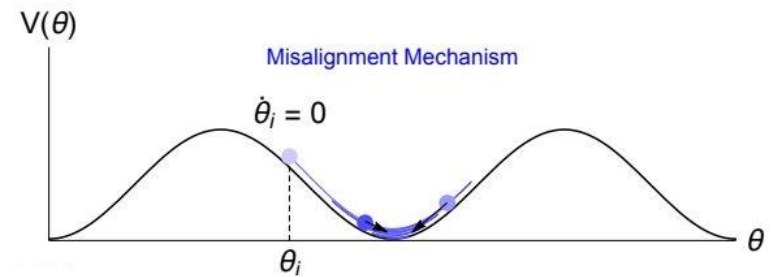
[Preskill, Wise, Wilczek (1983)]

[Abbott, Sikivie (1983)]

[Dine, Fischler (1983)]

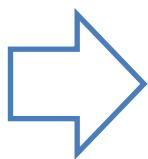
Initial condition $\phi = \phi_0 \neq 0$
 $\dot{\phi} = 0$

EOM $\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin \frac{\phi}{f} = 0$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$



$$\rho_{DM} \sim m_\phi \times \left(\frac{a(T_{osc})}{a_0} \right)^3 \times \frac{\Lambda_b(T_{osc})^4 \theta_i^2}{m_\phi(T_{osc})}$$

w/ $m_\phi(T_{osc}) \sim 3H(T_{osc})$

mass

Dilution factor

Number density at $T = T_{osc}$

Axion-like particle DM scenario

Misalignment mechanism

[Preskill, Wise, Wilczek (1983)]

[Abbott, Sikivie (1983)]

[Dine, Fischler (1983)]

Initial condition

$$\begin{aligned}\phi &= \phi_0 \neq 0 \\ \dot{\phi} &= 0\end{aligned}$$

What happens if $\dot{\phi} > \Lambda_b^2$?

EOM

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin \frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $3H(T) \sim m(T)$



$$\rho_{DM} \sim m_a \times \left(\frac{a(T_{osc})}{a_0} \right)^3 \times \frac{\Lambda_b(T_{osc})^4 \theta_i^2}{m_a(T_{osc})}$$

w/ $m_a(T_{osc}) \sim 3H(T_{osc})$

mass

Dilution factor

Number density at $T = T_{osc}$

Axion-like particle DM scenario

Kinetic Misalignment mechanism

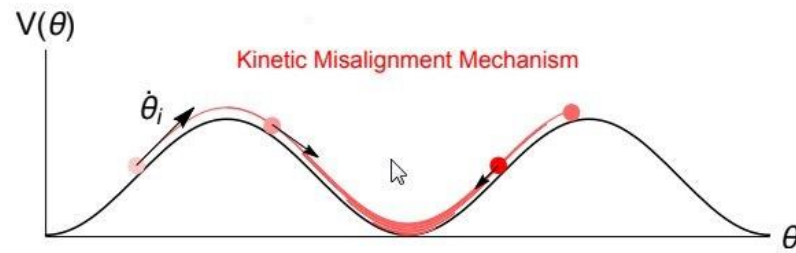
[Co, Hall, Harigaya (2019)]
[Chang, Cui (2019)]

Initial condition

$$\dot{\phi} > \Lambda_b^2$$

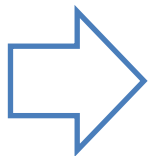
EOM

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\Lambda_b^4(T)}{f} \sin \frac{\phi}{f} = 0$$



[taken from Co, Hall, Harigaya (2019)]

The axion starts to oscillate when $\dot{\phi}^2(T) \sim \Lambda_b^4(T)$



$$\rho_{DM} \sim m_\phi \times \left(\frac{a(T_{osc})}{a_0} \right)^3 \times \frac{\Lambda_b(T_{osc})^4}{m_\phi(T_{osc})}$$

mass

Dilution factor

Number density at $T = T_{osc}$

$$\text{w/ } \dot{\phi}^2(T_{osc}) \sim \Lambda_b^4(T_{osc})$$

Delay of onset of oscillation \rightarrow larger ρ_{DM}

Why interesting?

- KMM predicts larger ρ_{DM} than conventional MM
- For given axion mass, conventional MM predicts smaller ρ_{DM} for smaller f_a



KMM tends to predicts smaller f_a (\sim larger $g_{a\gamma\gamma}$)
motivates axion detection experiments !!

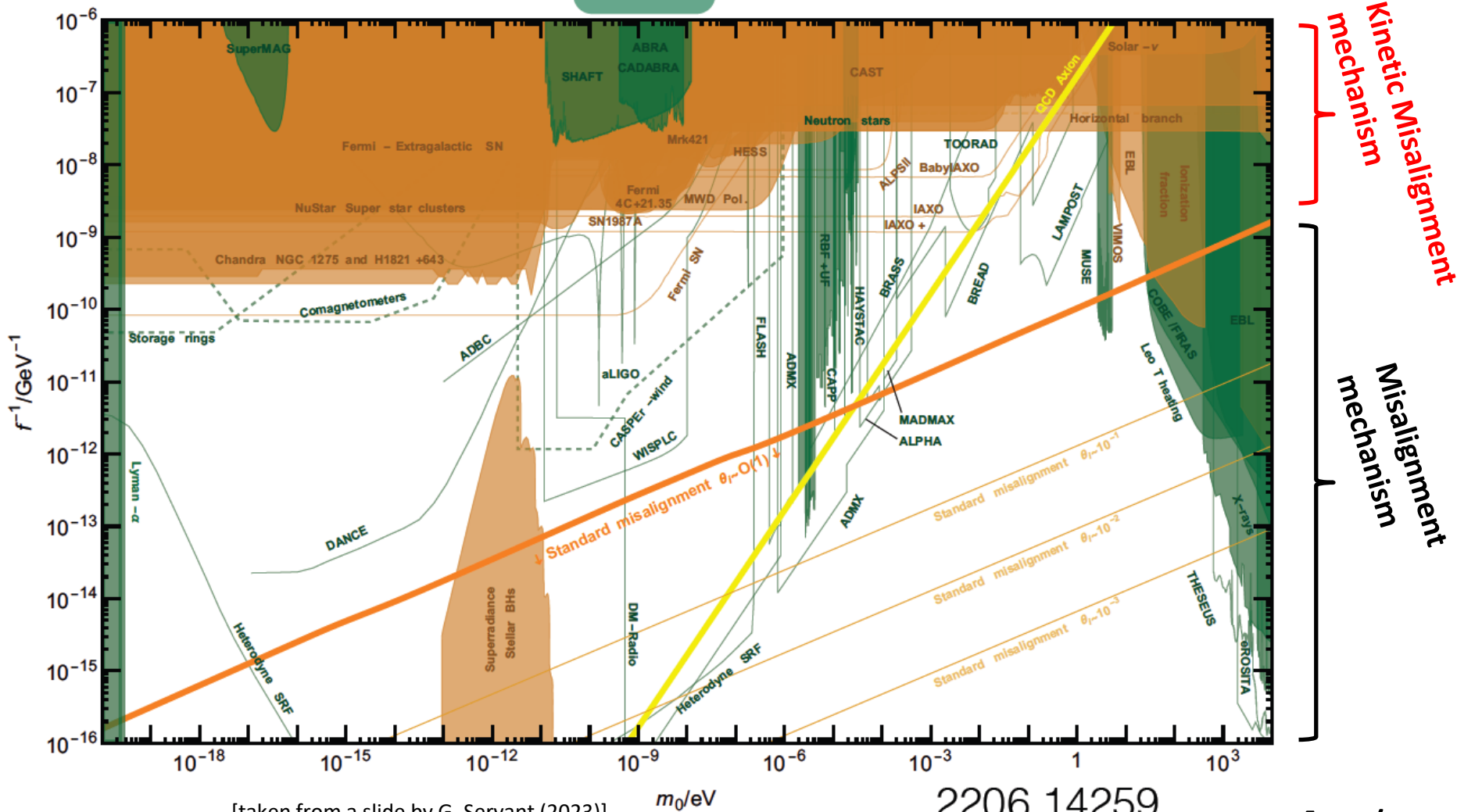
$$(L = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu})$$

Why interesting?

Any ALP

Only DM

(assuming KSVZ-like relation : $g_{a\gamma\gamma} \sim 2 \times 10^{-3}/f$)



[taken from a slide by G. Servant (2023)]

2206.14259

Axion fluctuation?

What people usually do

Solving EOM for spatially **homogeneous** field :

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

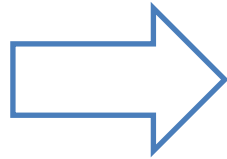
However...

Even we start from (almost) homogeneous field configuration, fluctuations **can grow** later.

Velocity as PQ charge

Velocity $\dot{\phi}$ is PQ charge : $\rho_{\text{PQ}} = f \frac{\partial L}{\partial \dot{\phi}} = f \dot{\phi}$ $\phi \rightarrow \phi + f \delta$
PQ sym. transf.

Explicit breaking of PQ sym. : $V(\phi) \sim -\Lambda_b^4 \cos \frac{\phi}{f}$



PQ charge will be lost
= axion kinetic energy will be lost
= energy dissipation

Axion fragmentation [Fonseca, Morgante, RS, Servant (2019)]

For related earlier works, see

[Green, Kofman, Starobinsky (1998)]

[Flauger, McAllister, Pajer, Westphal, Xu (2009)]

[Jaeckel, Mehta, Witkowski (2016)]

[Arvanitaki, Dimopoulos, Galanis, Lehner, Thompson, Van Tilburg (2019)]

1. Introduction

2. How axion fragmentation works

(What happen if initial $\dot{\phi} \neq 0$?)

3. ALP DM fragmentation scenario

(Who gives initial $\dot{\phi} \neq 0$?)

EOM of axion

Let us investigate the simplest case.

- $H = 0$ (no cosmic expansion)
- $V(\phi) = \Lambda_b^4 \cos(\phi/f)$

We have only **three** parameters :

$$\left\{ \begin{array}{l} \dot{\phi}_0 \\ f \\ \Lambda_b^4 \end{array} \right. \quad \begin{array}{l} : \text{initial velocity} \\ : \text{decay constant} \\ : \text{height of barrier} \end{array}$$

EOM of axion :

$$\frac{d^2 \phi}{dt^2} - \nabla^2 \phi - \frac{\Lambda_b^4}{f} \sin \frac{\phi}{f} = 0$$

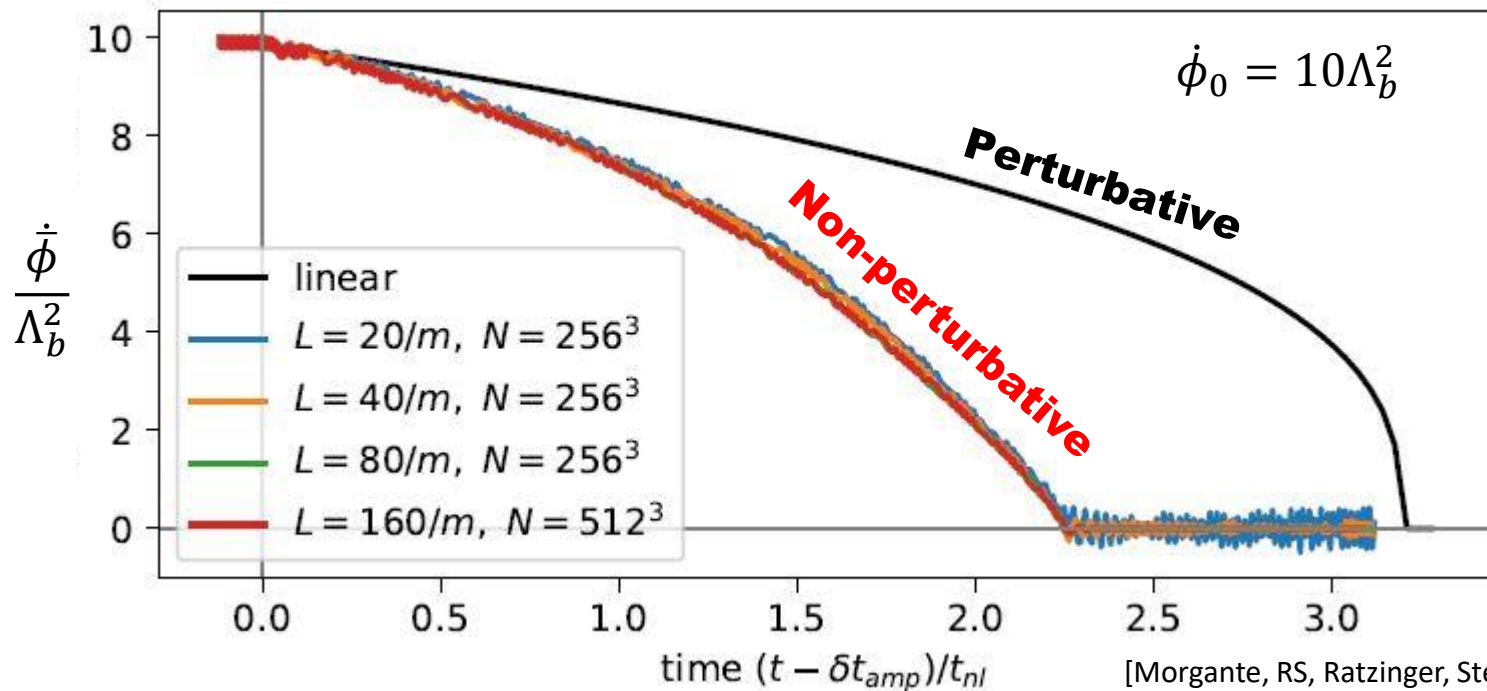
Classical lattice simulation



$$\begin{aligned} \frac{d^2 \phi_{i,j,k}}{dt^2} = & \frac{1}{a^2} (\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}) \\ & + \frac{1}{a^2} (\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}) \\ & + \frac{1}{a^2} (\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}) \\ & + \frac{\Lambda_b^4}{f} \sin \frac{\phi_{i,j,k}}{f}. \end{aligned}$$

An example of numerical result

Confirmed energy dissipation in non-perturbative calculation!

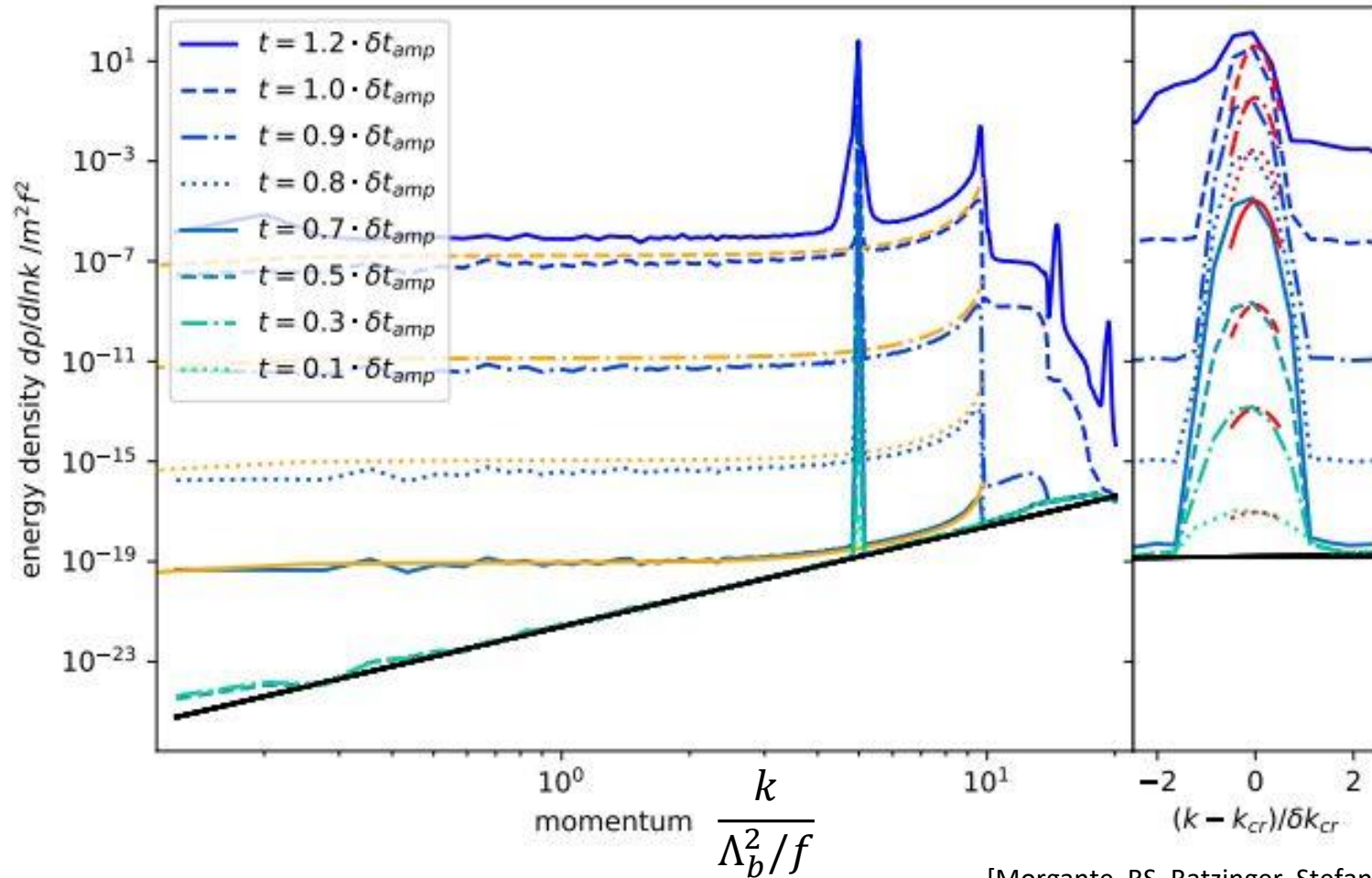


Time scale of fragmentation : $\Delta t \sim \frac{f\dot{\phi}_0^3}{\Lambda_b^8}$ $\left(t_{nl} = \frac{f\dot{\phi}_0^3}{\Lambda_b^8}\right)$

Field excursion during fragmentation : $\Delta\phi \sim \frac{f\dot{\phi}_0^4}{\Lambda_b^8}$

Growth of spectrum (early stage)

$$\dot{\phi}_0 = 10\Lambda_b^2$$

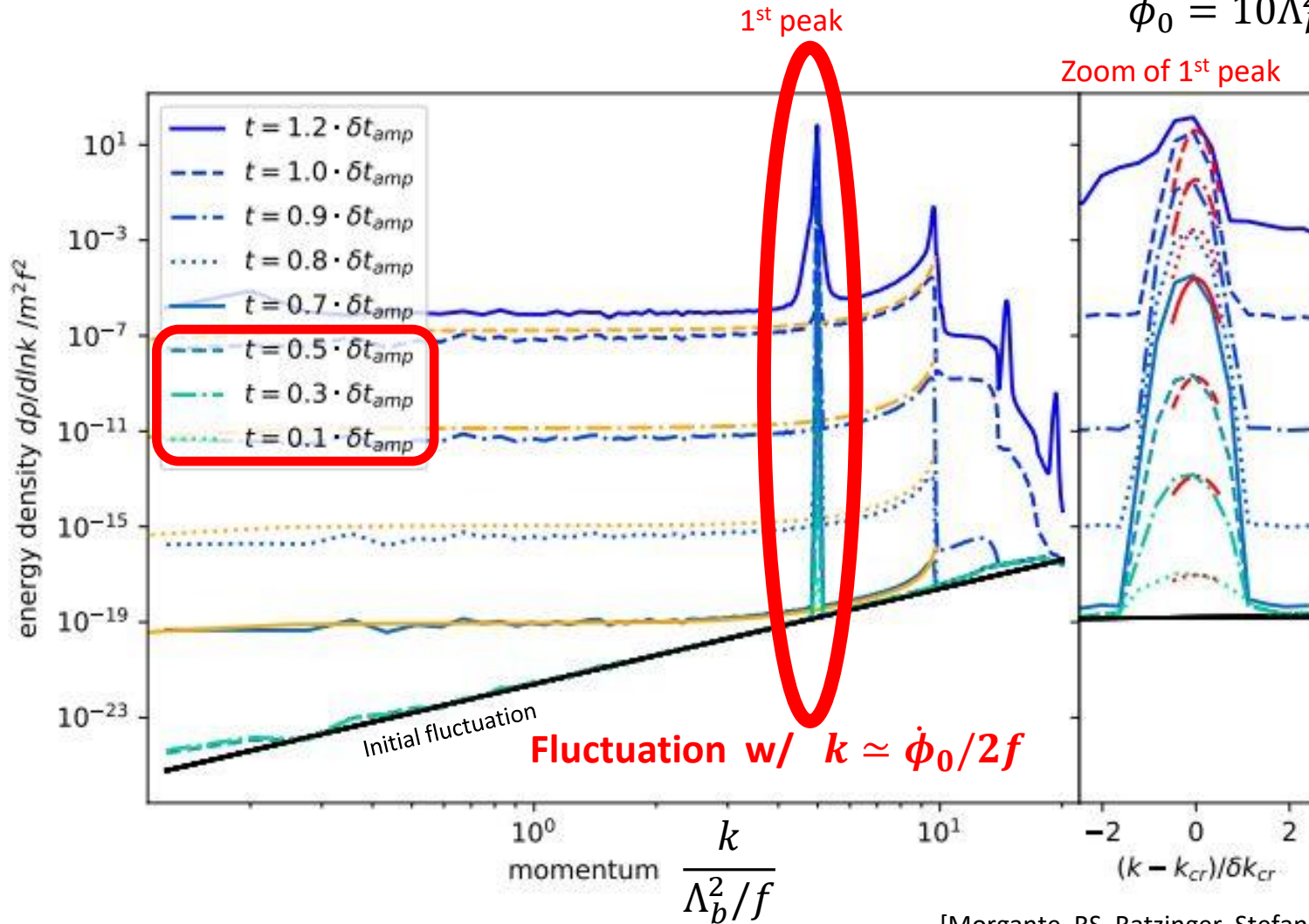


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16f^4}{\dot{\phi}^2}$$

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Growth of spectrum w/ Mathieu eq

EOM of axion :

$$\frac{d^2 \phi}{dt^2} - \nabla^2 \phi - \frac{\Lambda_b^4}{f} \sin \frac{\phi}{f} = 0$$

 $\phi(x, t) \simeq \bar{\phi}(t) + \delta\phi(x, t)$


$$O(\delta\phi^0) \quad \frac{d^2 \bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f} \sin \frac{\bar{\phi}}{f} = 0$$

$$O(\delta\phi^1) \quad \frac{d^2 \delta\phi}{dt^2} - \nabla^2 \delta\phi - \frac{\Lambda_b^4}{f^2} \cos \frac{\bar{\phi}}{f} \delta\phi = 0$$

Growth of spectrum w/ Mathieu eq

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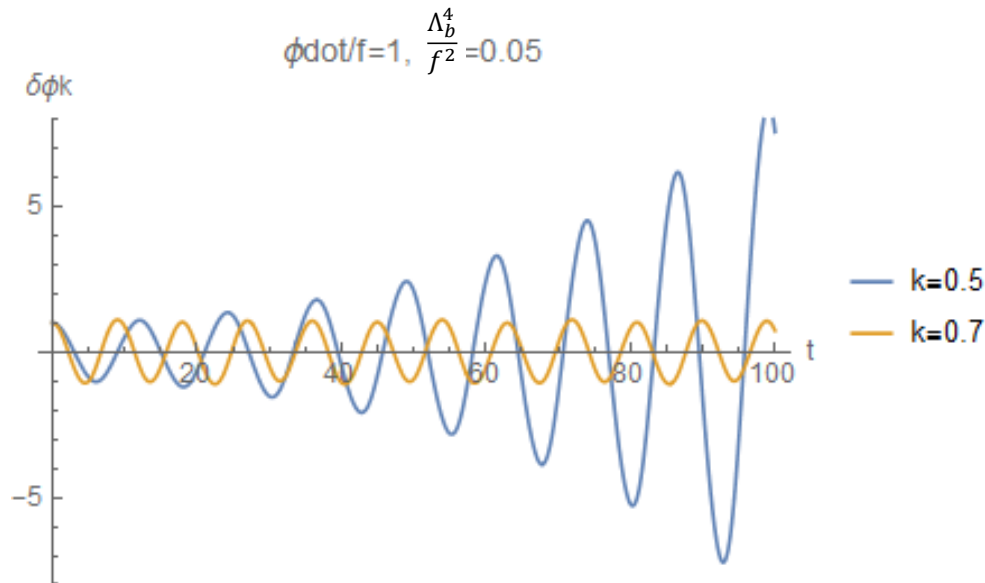
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$O(\delta\phi^0)$ $\frac{d^2 \bar{\phi}}{dt^2} - \frac{\Lambda_b^4}{f} \sin \frac{\bar{\phi}}{f} = 0$ $\overset{\dot{\bar{\phi}} \gg \Lambda_b^2}{\Rightarrow}$ $\bar{\phi}(t) \simeq \dot{\bar{\phi}}_0 t$

$O(\delta\phi^1)$ $\frac{d^2 \delta\phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2} \cos \frac{\dot{\bar{\phi}} t}{f} \right) \delta\phi_k = 0$
Mathieu equation

Growth of spectrum w/ Mathieu eq

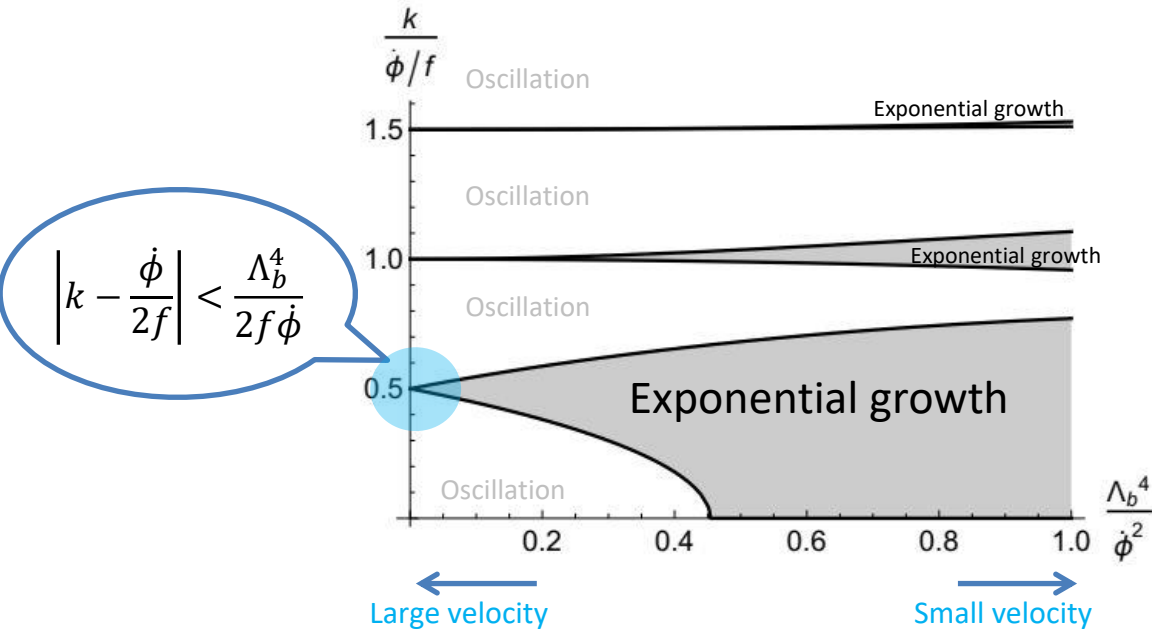


There exist resonant solutions for this.
It's like a swing!

$$\frac{d^2 \delta \phi_k}{dt^2} + \left(k^2 - \frac{\Lambda_b^4}{f^2} \cos \frac{\dot{\phi} t}{f} \right) \delta \phi_k = 0 \quad (\bar{\phi} \approx \dot{\phi} t)$$

Mathieu equation

Growth of spectrum w/ Mathieu eq



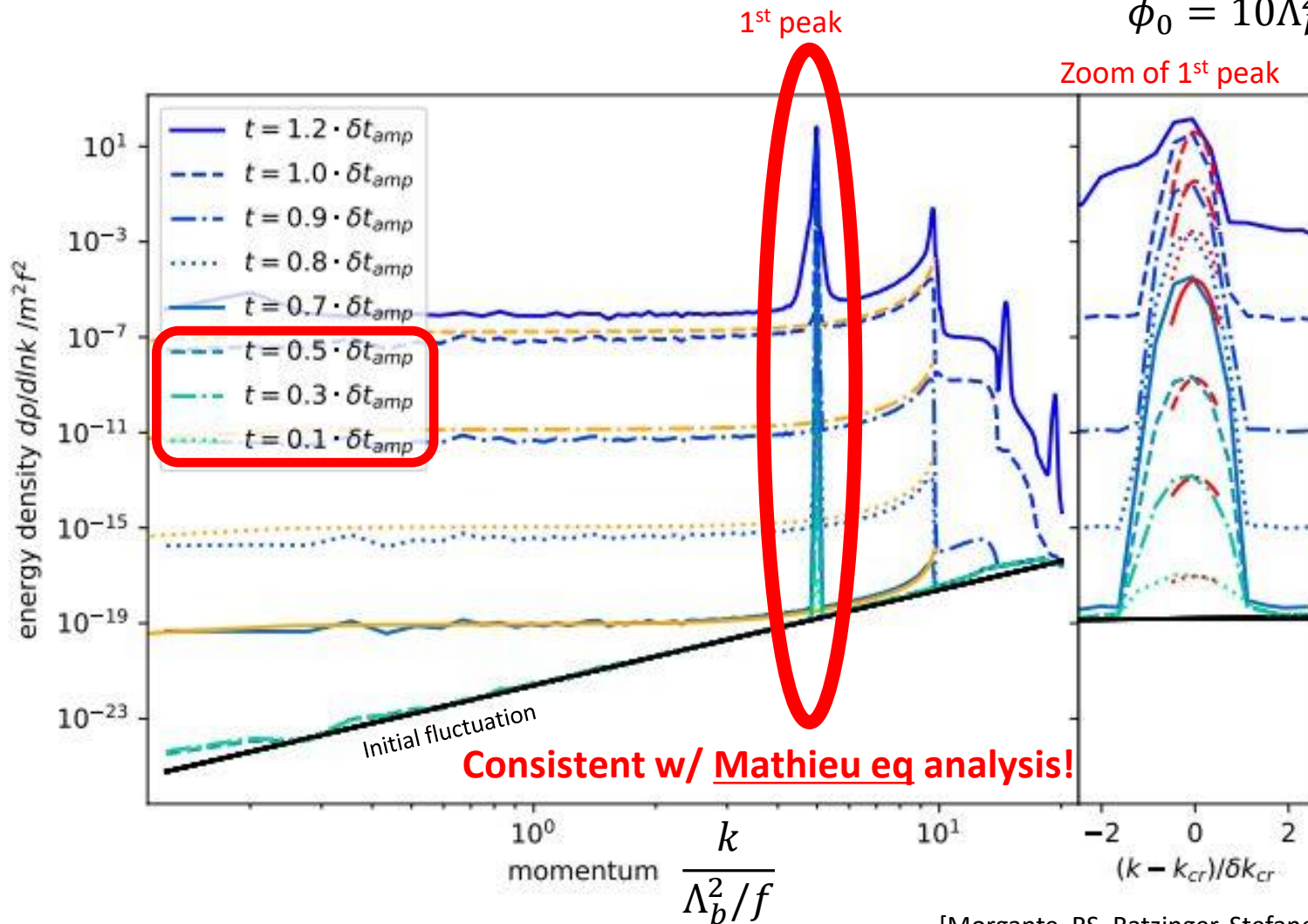
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Mathieu equation

Growth of spectrum (early stage)

$$\dot{\phi}_0 = 10\Lambda_b^2$$

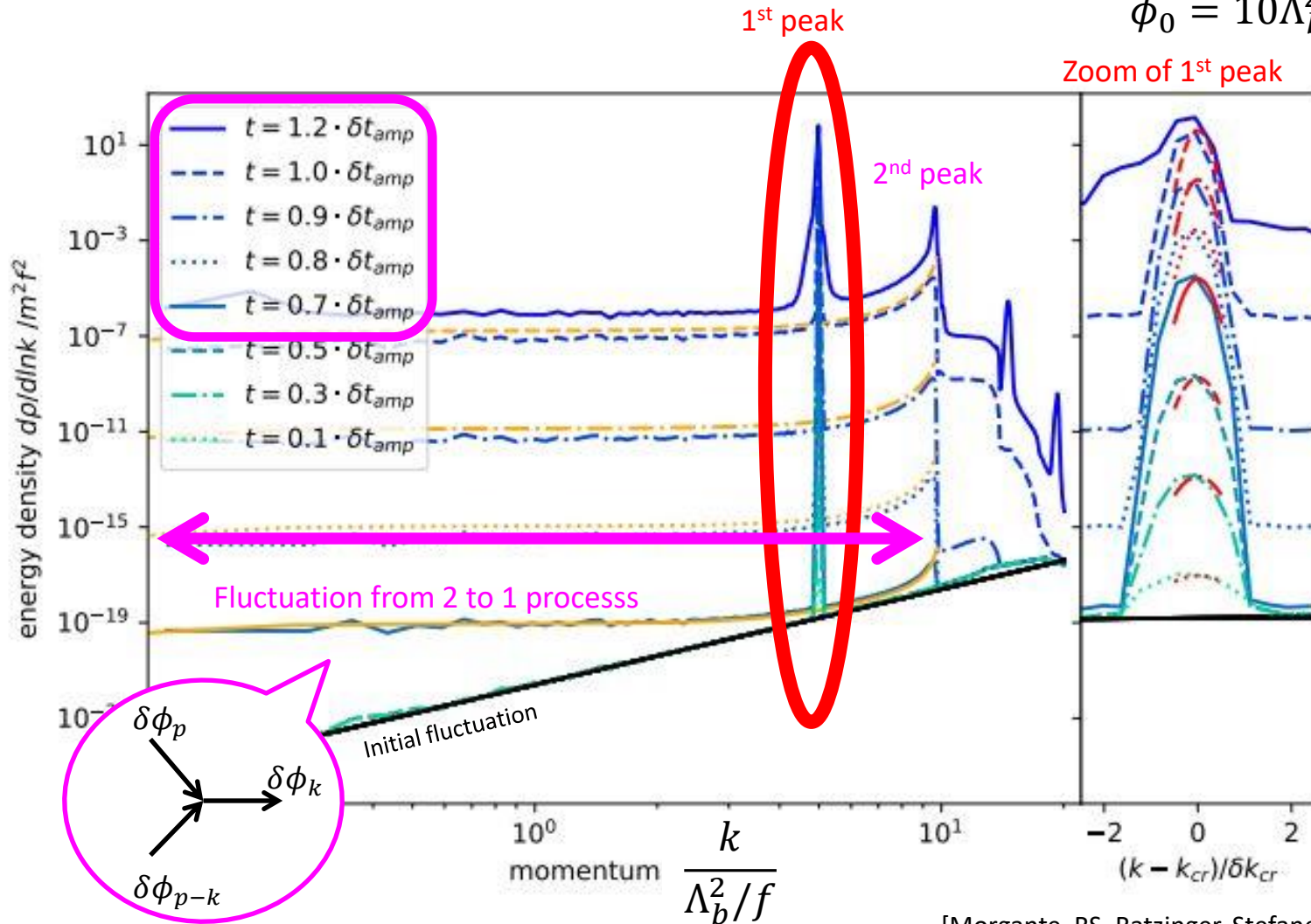


[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16f^4}{\dot{\phi}^2}$$

Growth of spectrum (early stage)

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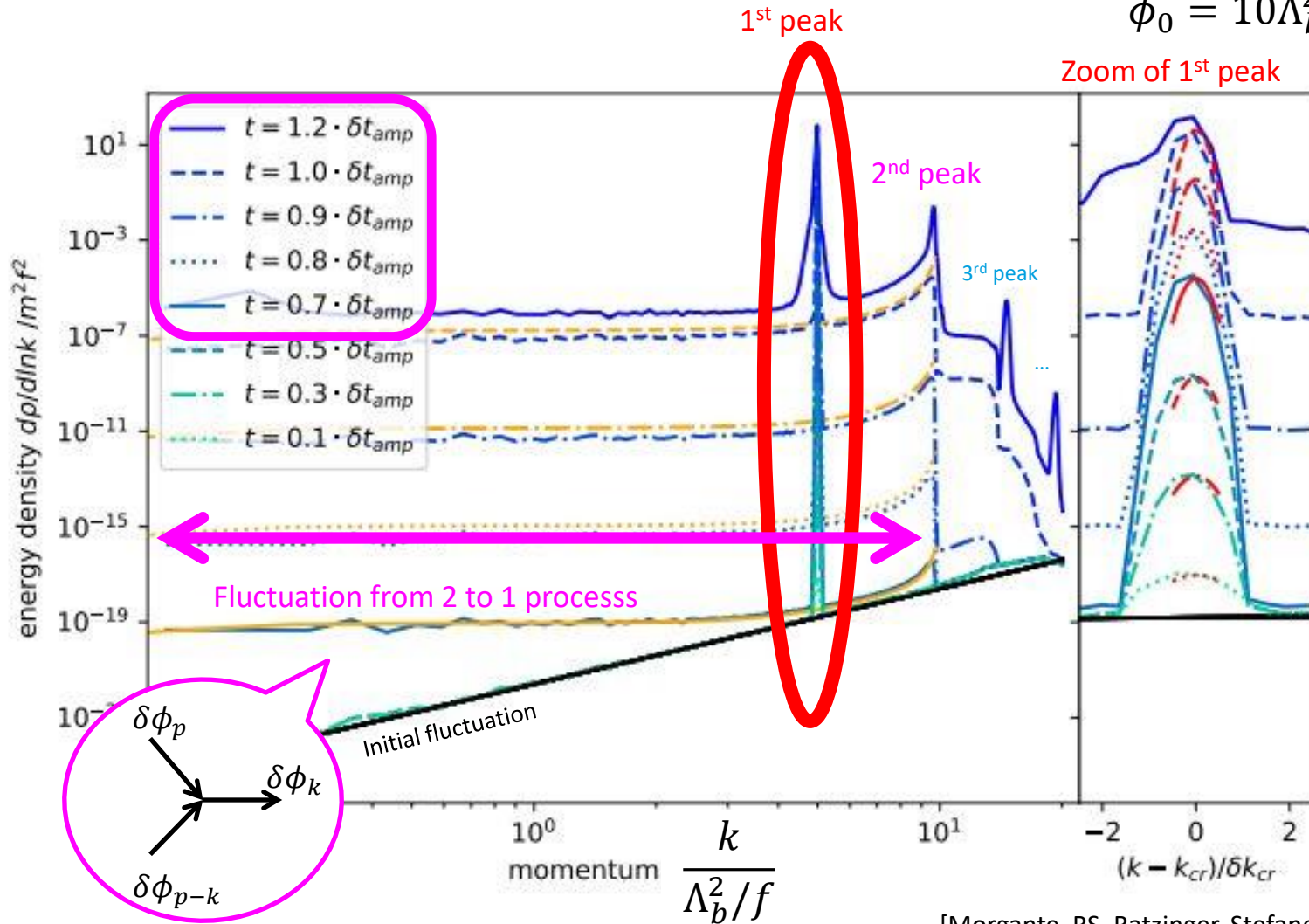


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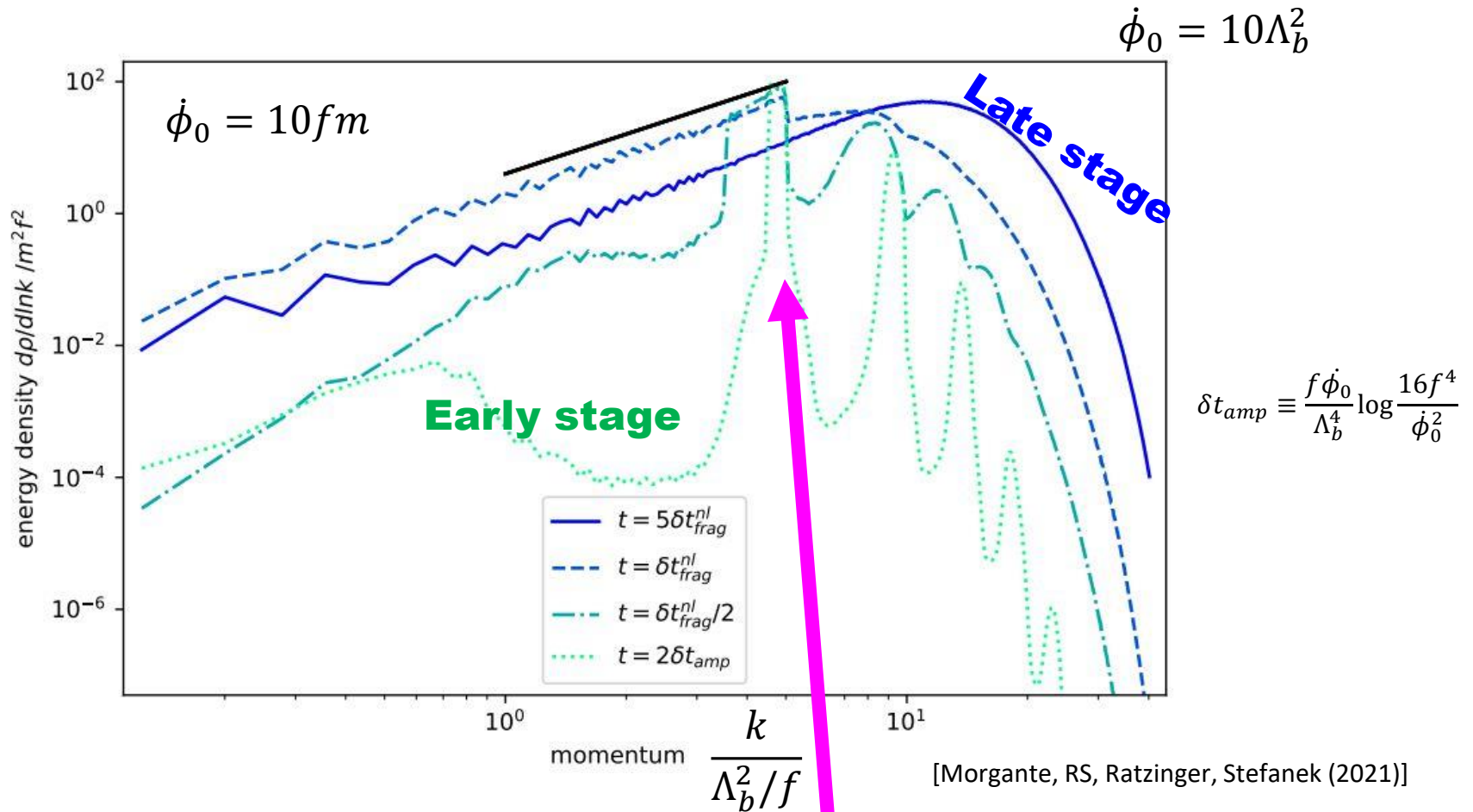
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[Morgante, RS, Ratzinger, Stefanek (2021)]

$$\delta t_{amp} \equiv \frac{f\dot{\phi}}{\Lambda_b^4} \log \frac{16f^4}{\dot{\phi}^2}$$

Growth of spectrum



Resonance from Mathieu Eq. $k = \frac{\dot{\phi}_0}{2f}$

- Parametric resonance in early stage
- Broad spectrum from non-linear effect

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2. How axion fragmentation works

(What happen if initial $\dot{\phi} \neq 0$?)

3. ALP DM fragmentation scenario

(Who gives initial $\dot{\phi} \neq 0$?)

Axion fragmentation & ALP dark matter

How do we get **initial $\dot{\phi} \neq 0$** ??

Affleck-Dine mechanism

[Affleck, Dine (1985)]

[Dine, Randall, Thomas (1996)]

$$P = \frac{1}{\sqrt{2}} s \exp\left(\frac{i\phi}{f}\right)$$

P : PQ charged scalar

s : saxion

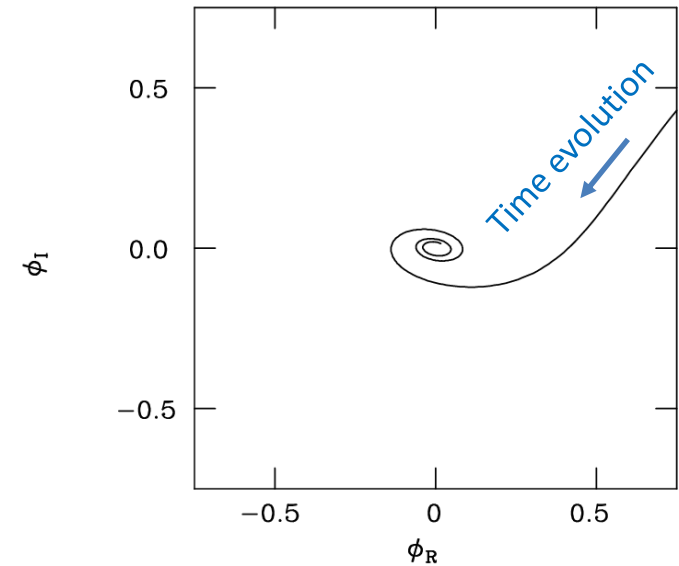
ϕ : axion

$$V(P) = V_{PQ}(|P|) + V_{PQ}(P)$$

PQ invariant term

PQ violating term

$$\text{e.g., } V_{PQ} \sim \frac{P^n}{\Lambda^{n-4}} + h.c.$$



Taken from [Dine, Randall, Thomas (1996)]

In addition, we also need

- large initial $|P|$
- Elimination of saxion oscillation (to avoid overclosure)

A model

[Co, Fernandez, Ghalsasi, Hall, Harigaya (2020)]

$$V = m^2 |P|^2 \left(-\frac{1}{2} + \frac{1}{2} \log \frac{2|P|^2}{f^2} \right) - c_H H^2 |P|^2 + \left(A \frac{P^n}{M^{n-3}} + h.c. \right) + \frac{|P|^{2n-2}}{M^{2n-6}}$$

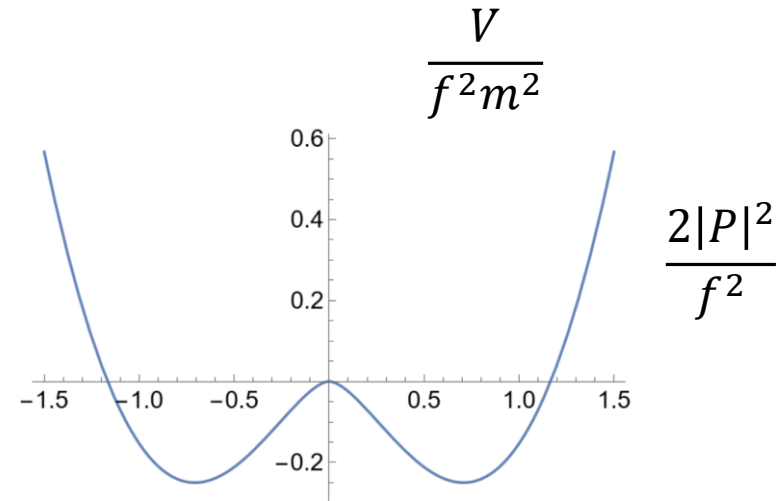
A model

[Co, Fernandez, Ghalsasi, Hall, Harigaya (2020)]

PQ breaking [Moxhay, Yamamoto (1985)]

$$V = m^2 |P|^2 \left(-\frac{1}{2} + \frac{1}{2} \log \frac{2|P|^2}{f^2} \right)$$

$$-c_H H^2 |P|^2 + \left(A \frac{P^n}{M^{n-3}} + h.c. \right) + \frac{|P|^{2n-2}}{M^{2n-6}}$$



How to get kick ($\dot{\phi} \neq 0$)

→ Affleck-Dine mechanism

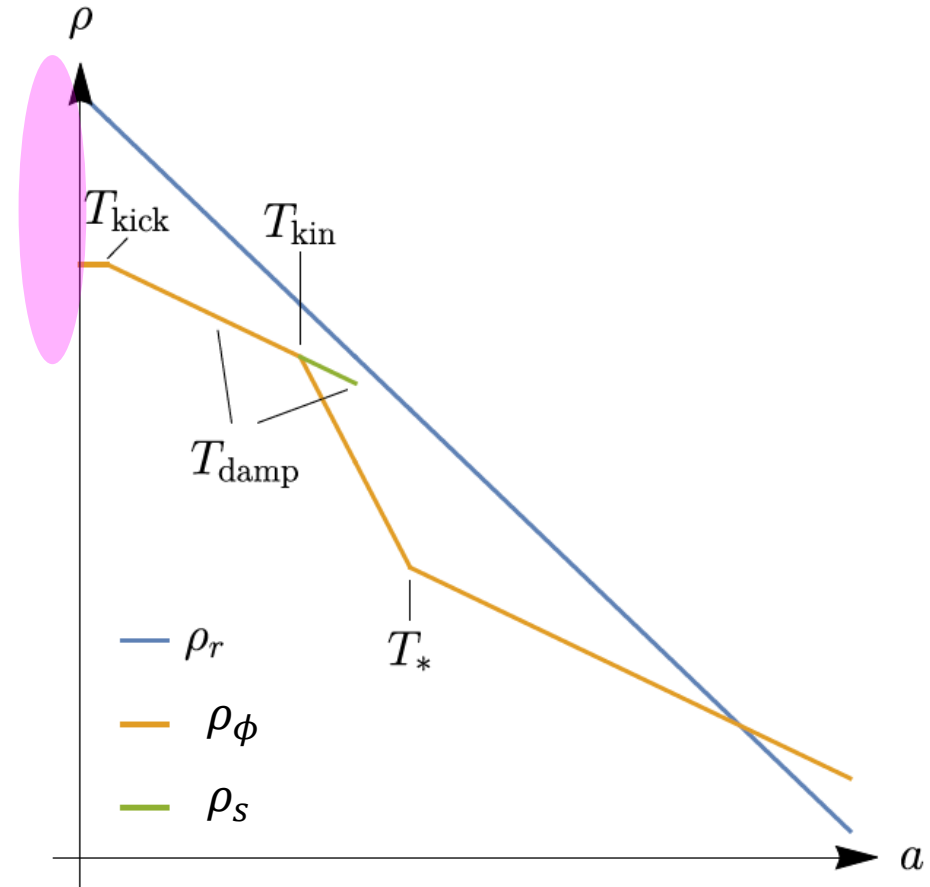
[Affleck, Dine (1985)]

A thermal history

Tachyonic mass drives Large VEV

$$\phi_{early} \sim (HM^{n-3})^{\frac{1}{n-2}}$$

$$V \simeq -c_H H^2 |P|^2 + \left(A \frac{\rho^n}{M^{n-3}} + h.c. \right) + \frac{|P|^{2n-2}}{M^{2n-6}}$$



[Eröncel, RS, Sørensen, Servant, in preparation]

A thermal history

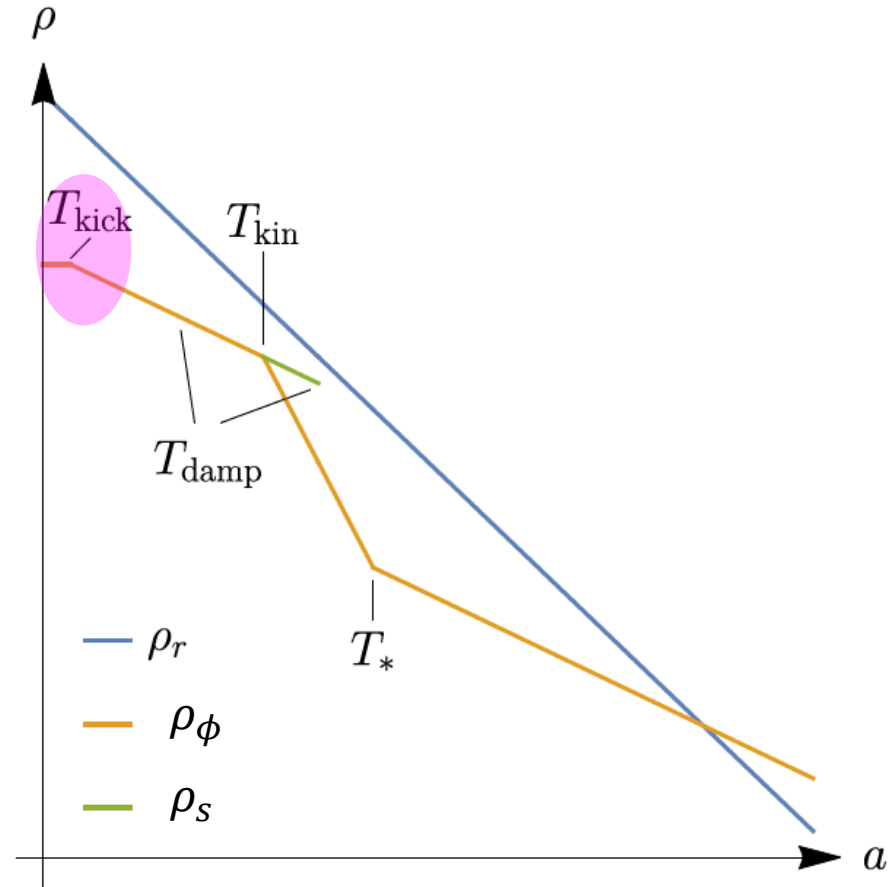
Tachyonic mass drives Large VEV

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ϕ starts to roll

$$m \sim H, \quad T_{\text{kick}} \sim \sqrt{mM_{\text{pl}}}$$

$$V \simeq -c_H H^2 |P|^2 + \left(A \frac{\rho^n}{M^{n-3}} + h.c. \right) + \frac{|P|^{2n-2}}{M^{2n-6}}$$



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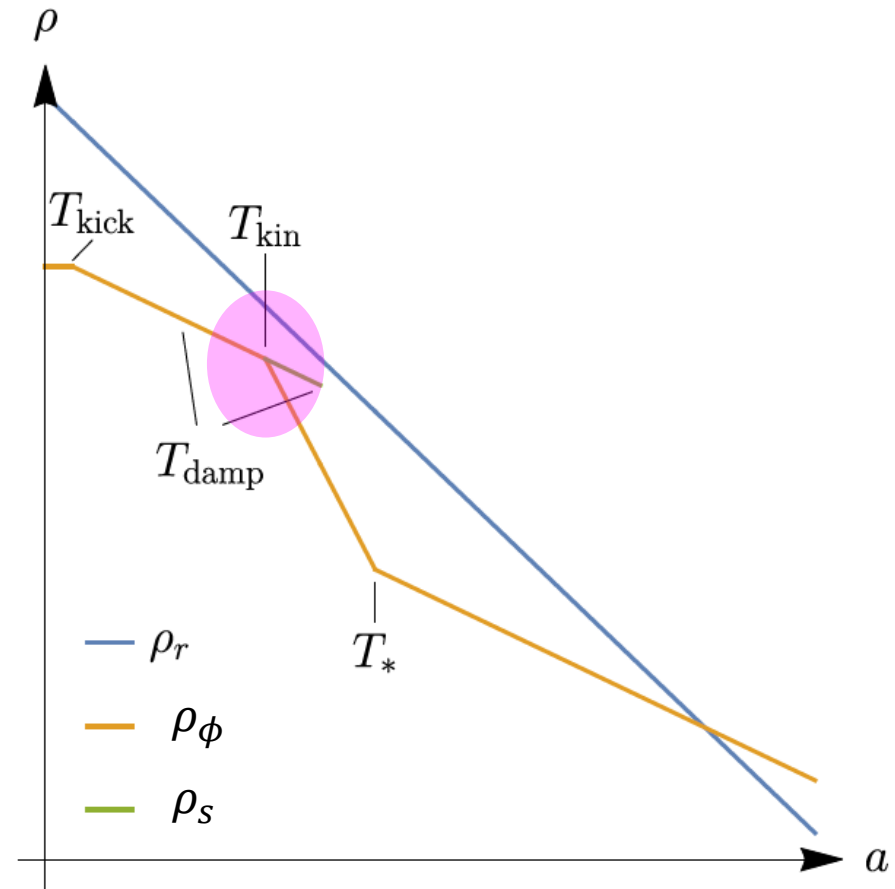
ϕ starts to roll

$$m \sim H, \quad T_{kick} \sim \sqrt{mM_{pl}}$$

Energy in s dissipates (via, e.g., $L \sim s\bar{\chi}\chi$)

Nonzero $\dot{\phi} \propto a^{-3}$ survives

$$V \simeq m^2|P|^2 \left(-\frac{1}{2} + \frac{1}{2} \log \frac{2|P|^2}{f^2} \right)$$



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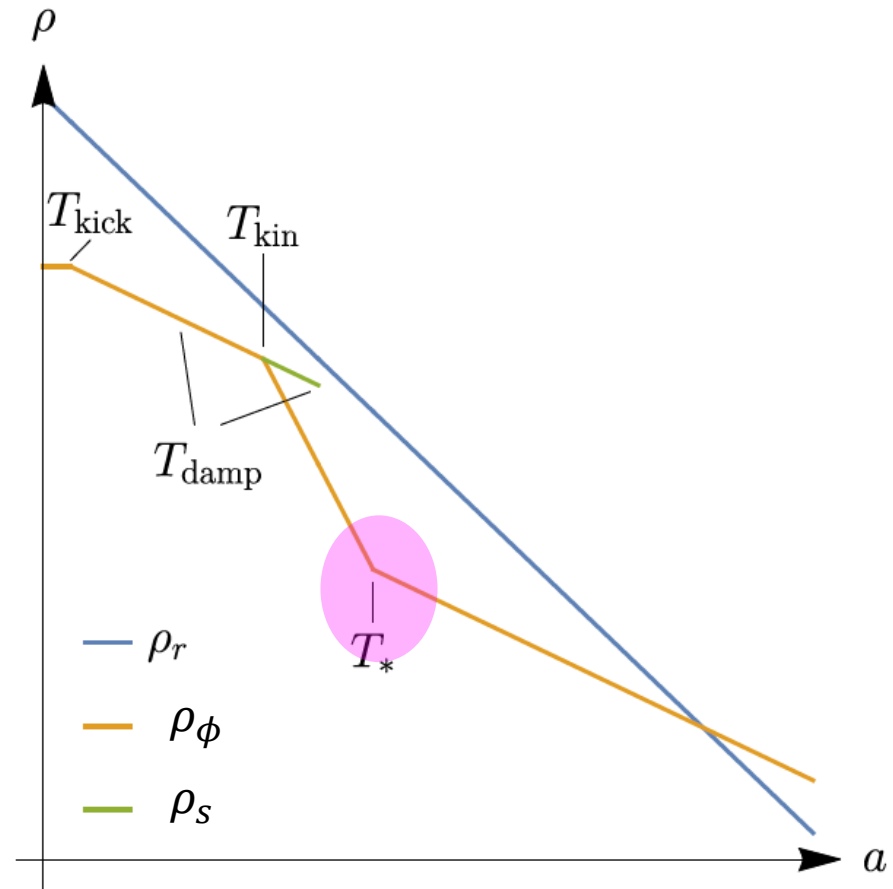
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Trapping by potential, fragmentation, etc

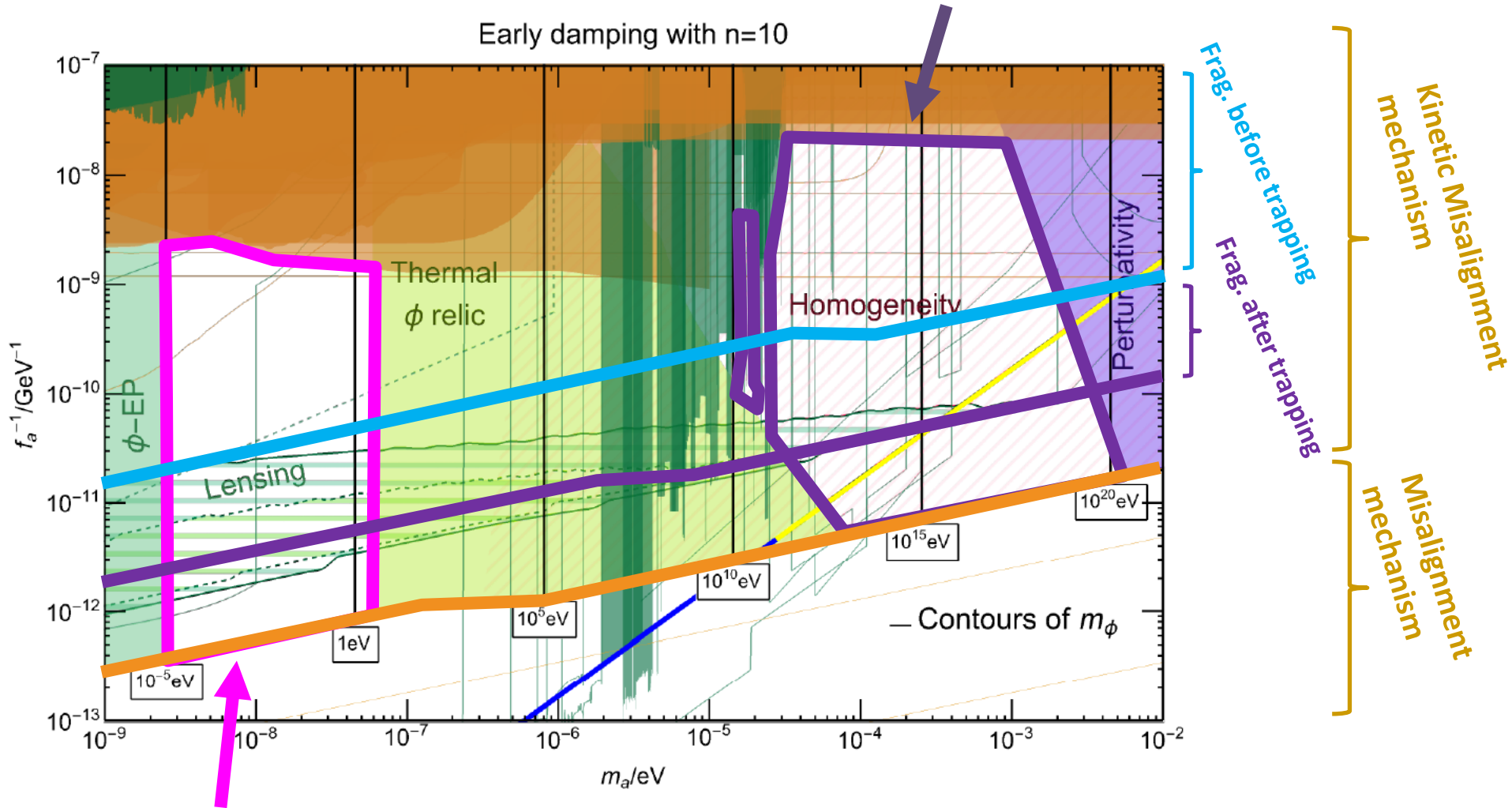
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[Eröncel, RS, Sørensen, Servant, in preparation]

A parameter space

Initial adiabatic fluctuation could dominate (depending on inflation scenario)



Kinetic misalignment (+ fragmentation)

Summary

- ALP DM w/ kinetic misalignment is an interesting experimental target
- Zeromode kinetic energy dissipates into fluctuations
- **Generic phenomena w/ periodic potential and large velocity**
- Fragmentation can happen in ALP DM scenario
- Other theoretical applications
 - Relaxion scenario ([1911.08473](#), [Fonseca-Morgante-Sato-Servant](#))
Relaxion fragmentation can be a source of friction to stop relaxion.
 - Any other interesting idea?

Backup

Necessity of non-linear analysis

Naïve dimensional analysis

Initial kinetic energy : $\dot{\phi}_0^2/2$

Typical wavenumber : $\dot{\phi}_0/f$

Energy conservation : $(\delta\phi)^2 \times (\dot{\phi}_0/f)^2 \sim \dot{\phi}_0^2$



Typical field variation : $\delta\phi \sim f$ **NOT SMALL !!**

Classical lattice simulation

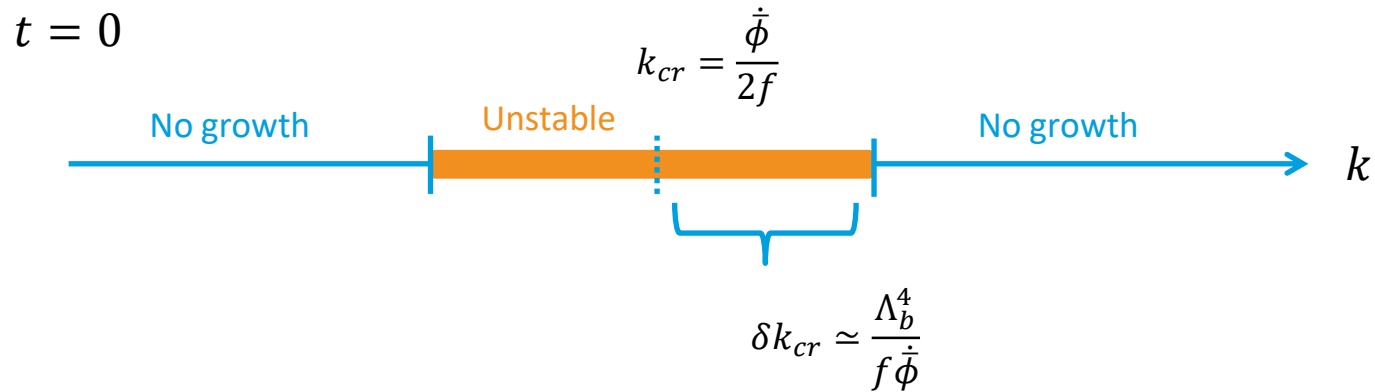
$$\ddot{\phi} = \nabla^2 \phi + \frac{\Lambda_b^4}{f} \sin \frac{\phi}{f}$$



$$\begin{aligned} \frac{d^2 \phi_{i,j,k}}{dt^2} = & \frac{1}{a^2} (\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}) \\ & + \frac{1}{a^2} (\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}) \\ & + \frac{1}{a^2} (\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}) \\ & + \frac{\Lambda_b^4}{f} \sin \frac{\phi_{i,j,k}}{f}. \end{aligned}$$

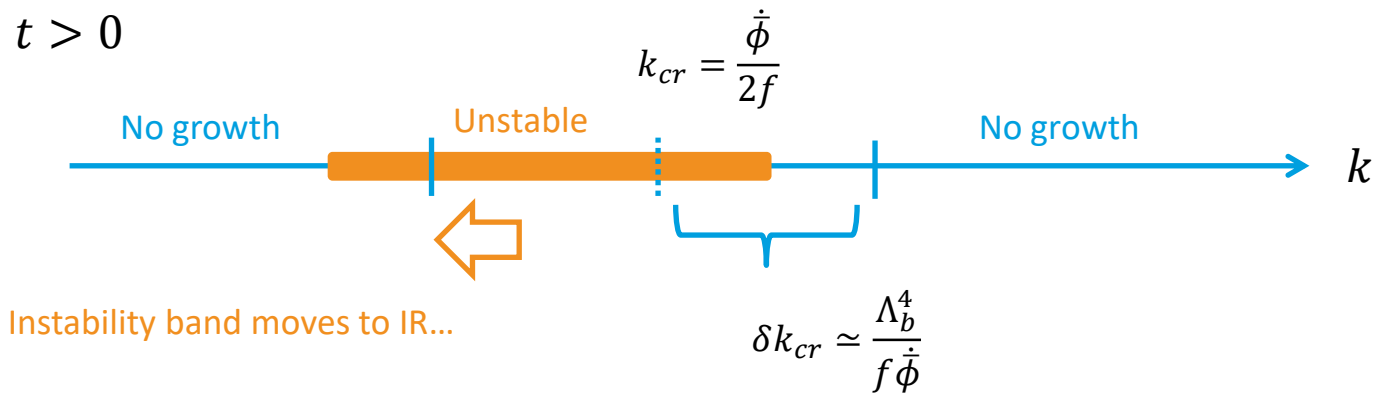
Naïve estimation on back reaction

As long as $\dot{\phi}$ is constant, $\delta\phi_k \sim \exp\left(\frac{\Lambda_b^4 t}{f\dot{\phi}}\right)$ for $\left|k - \frac{\dot{\phi}}{2f}\right| < \frac{\Lambda_b^4}{2f\dot{\phi}}$



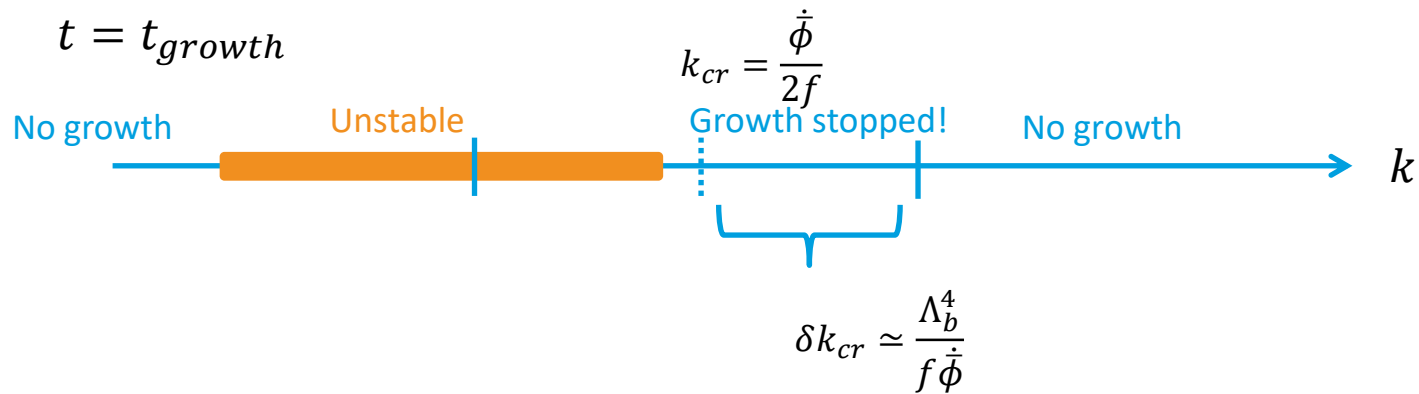
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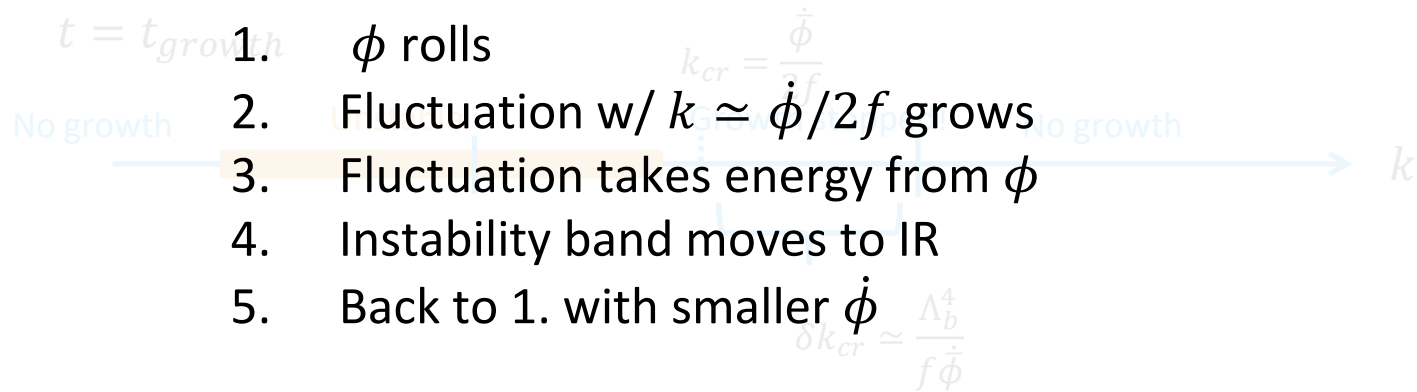
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This process repeats
and ϕ loses its kinetic energy!

Naïve estimation on back reaction

Time scale of growth of single mode :

$$t_{stop} \sim \frac{f \dot{\phi}}{\Lambda_b^4} \log \frac{f^4}{\dot{\phi}^2}$$

Energy stored in fluctuations :

$$\rho_{fluc}(t_{stop}) \sim \dot{\phi}^2 \times \frac{\delta k_{cr}}{k_{cr}},$$

$$\Rightarrow \frac{d}{dt} \dot{\phi}^2 \sim - \frac{\rho_{fluc}(t_{stop})}{t_{stop}} \sim - \frac{\Lambda_b^8}{f \dot{\phi}} \left(\log \frac{f^4}{\dot{\phi}^2} \right)^{-1}$$

$$\Rightarrow \frac{d}{dt} \dot{\phi} \sim - \frac{\Lambda_b^8}{f \dot{\phi}^2} \left(\log \frac{f^4}{\dot{\phi}^2} \right)^{-1}$$

c.f.) WKB approx. with $\dot{\phi} \gg \Lambda_b^2$ gives $\frac{d\dot{\phi}}{dt} = - \frac{\pi \Lambda_b^8}{2 f \dot{\phi}^2} \left(\log \frac{32\pi^2 f^4}{\dot{\phi}^2} \right)^{-1}$

(see 1911.08472 for details)

Time scale of fragmentation :

$$\Delta t_{frag} \sim f \frac{\dot{\phi}_0^3}{\Lambda_b^8} \log \frac{f^4}{\dot{\phi}_0^2}$$

Field excursion:

$$\Delta \phi_{frag} \sim \dot{\phi}_0 \Delta t_{frag} \sim f \frac{\dot{\phi}_0^4}{\Lambda_b^8} \log \frac{f^4}{\dot{\phi}_0^2}$$

Non-zero slope & Hubble expansion

What happens for non-zero μ^3 & non-zero H ?

- Fragmentation
 - Acceleration by slope
 - Hubble expansion
- $$\ddot{\phi}_{frag} = -\frac{\pi\Lambda_b^8}{2f\dot{\phi}^2} \left(\log \frac{32\pi^2 f^4}{\dot{\phi}^2} \right)^{-1}$$
- $$\mu^3$$
- $$3H\dot{\phi}$$

Fragmentation works if

- During inflation ($3H\dot{\phi} \sim \mu^3$)

$$3H\dot{\phi} < \sim |\ddot{\phi}_{frag}| \quad \text{If not, axion keeps rolling with slow-roll velocity}$$

- Not during inflation ($3H\dot{\phi} \ll \mu^3$)

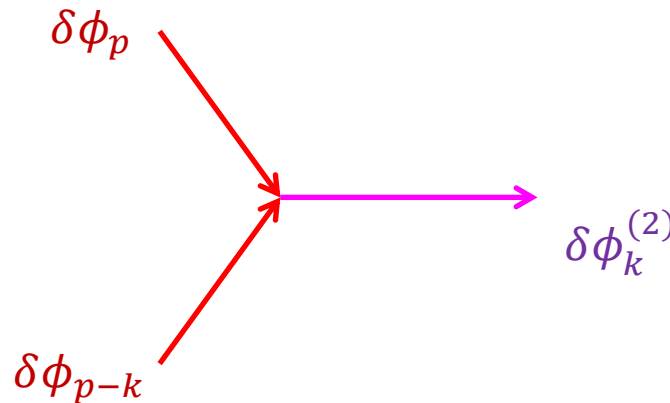
$$\mu^3 < \sim |\ddot{\phi}_{frag}| \quad \text{If not, axion is just accelerated by slope}$$

2 to 1 process

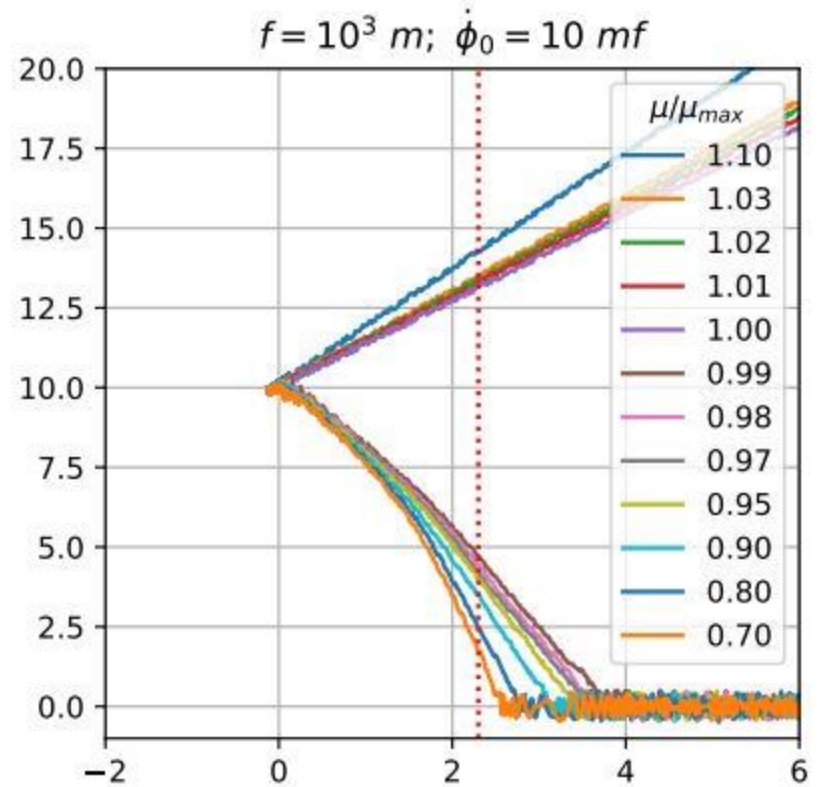
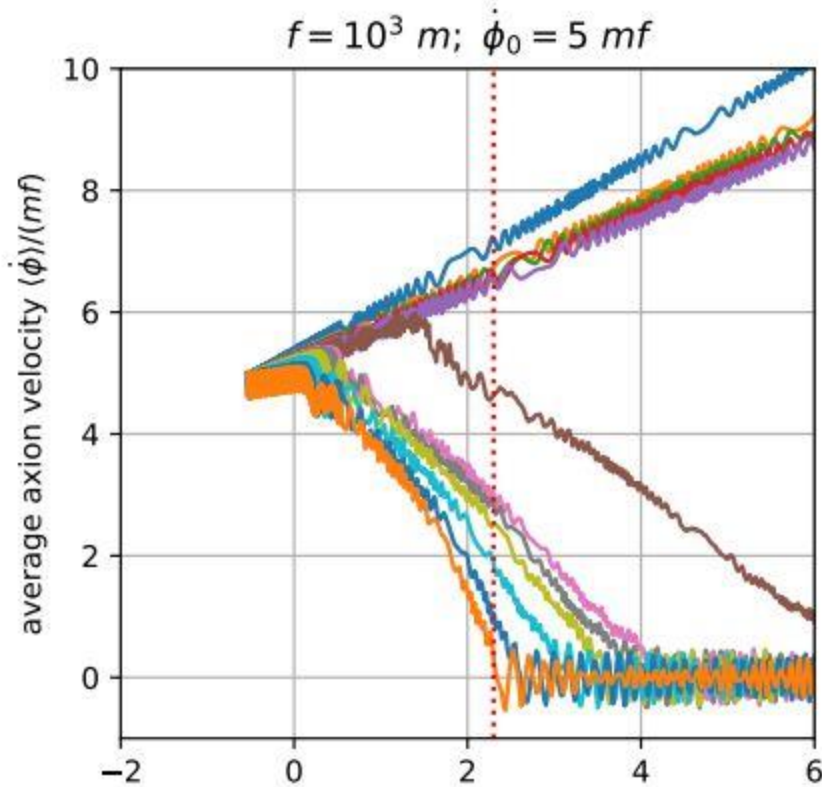
$$\phi(x, t) = \phi(t) + \delta\phi(x, t) + \delta\phi^{(2)}(x, t) + \dots$$

$$\ddot{\phi} - \nabla^2\phi = V'(\phi) \quad \Rightarrow \quad \delta\ddot{\phi}^{(2)} + (k^2 + V'')\delta\phi^{(2)} = -\frac{1}{2}V''' \int d^3p \delta\phi_p \delta\phi_{k-p}$$

- $\delta\phi_p$ with $|p| = \dot{\phi}/2f$ is amplified by resonance
- $\delta\phi$ becomes source term for $\delta\phi^{(2)}$



Lattice calc. w/ slope term



[Morgante, RS, Ratzinger, Stefanek (2021)]

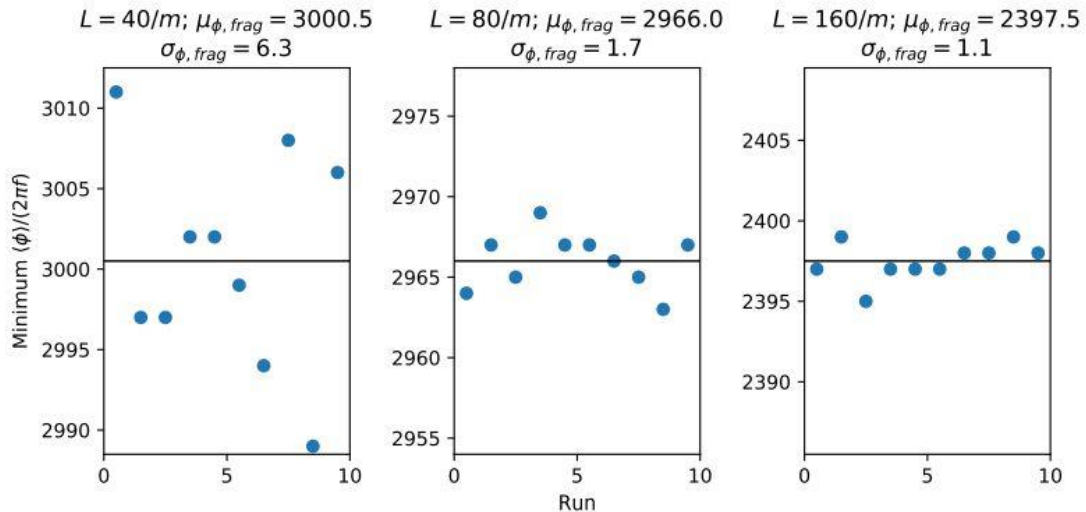
Domain wall?

Field variance after fragmentation is not so small :

$$\delta\phi \sim f$$

Multiple run with finite size box

- $\delta\phi$ in multiple run = $\delta\phi$ of causally disconnected area
- Extrapolation to $V^{1/3} \approx \delta t_{frag}$



[Morgante, RS, Ratzinger, Stefanek (2021)]



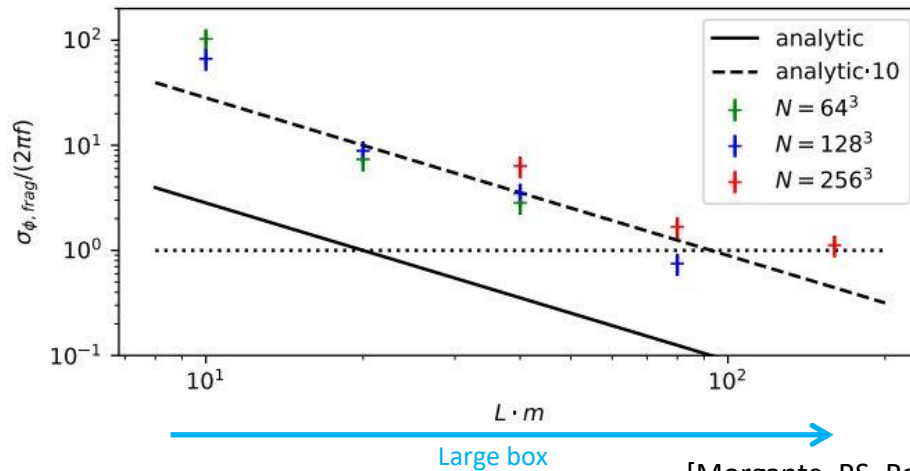
Domain wall?

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Multiple run with finite size box

- $\delta\phi$ in multiple run = $\delta\phi$ of causally disconnected area
- Extrapolation to $V^{1/3} \approx \delta t_{\text{frag}}$



Empirical formula of variance:

$$\frac{\delta\phi}{2\pi f} \sim O(10) \times V^{-1/2} \times \left(\frac{f\phi_0}{\Lambda_b^2} \right)^{3/2}$$

[Morgante, RS, Ratzinger, Stefanek (2021)]

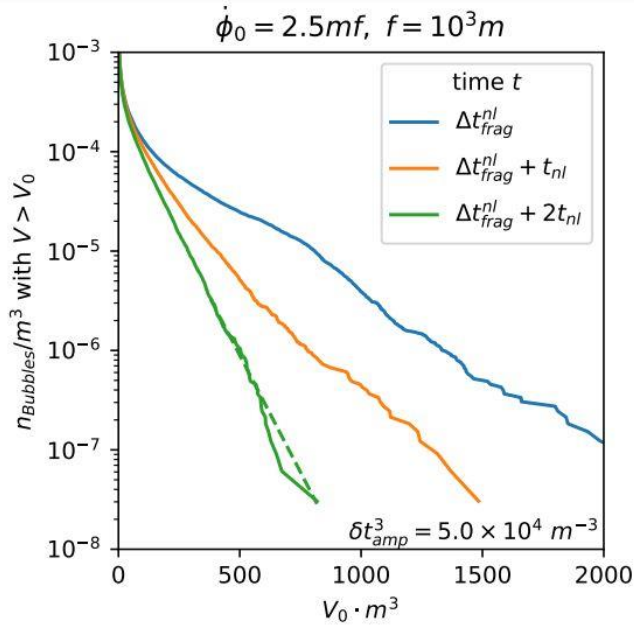
Naïve extrapolation to $V^{1/3} \sim t_{\text{amp}}$:

$$\frac{\sigma}{2\pi f} \sim O(10) \times \left(\log \frac{8\pi f^2}{\phi_0} \right)^{-\frac{3}{2}} \sim 0.01 - 0.1$$

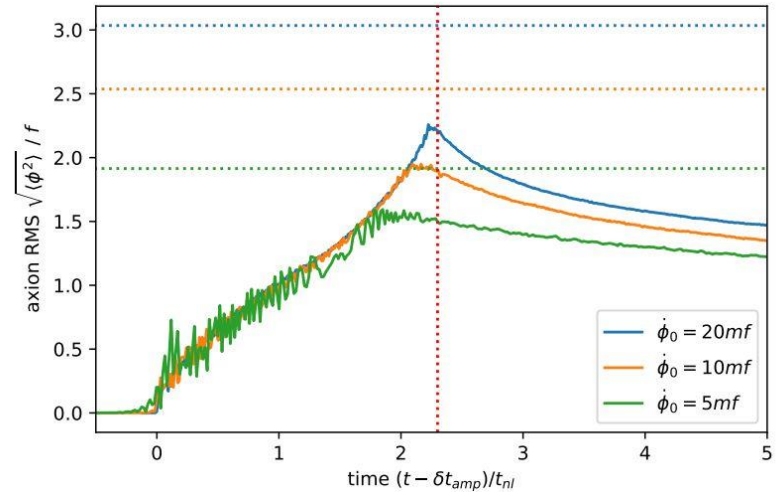
Domain wall formation probability is $\sim e^{-100} - e^{-10}$

Energy cascade into UV

Number counting of “bubble”



Time evolution of variance $\langle \delta \phi^2 \rangle$



[Morgante, RS, Ratzinger, Stefanek (2021)]

- Fluctuation with long wave-length is exponentially suppressed.
- The size of variance decreases in time.

Possible signals

Axion mini-cluster

See Eröncel-Servant (2207.10111)

Gravitational Wave (tensor perturbation in metric)

$$\nu \sim \frac{k}{a_{emit}} \frac{a_{emit}}{a_0} \quad (\text{Typically, } k \sim m)$$

Wave number at emission Redshift

$$\Omega_{GW}^{peak} \sim \frac{64\pi^2}{3M_{pl}^4 H_{emit}^2} \frac{\rho_{\theta,emit}^2}{(k_{peak}/a_{emit})^2} \frac{\alpha^2}{\beta} \quad (\text{Typically, } \alpha < 1, \beta > 1)$$

See [Chatrchyan, Jaeckel (2020)]

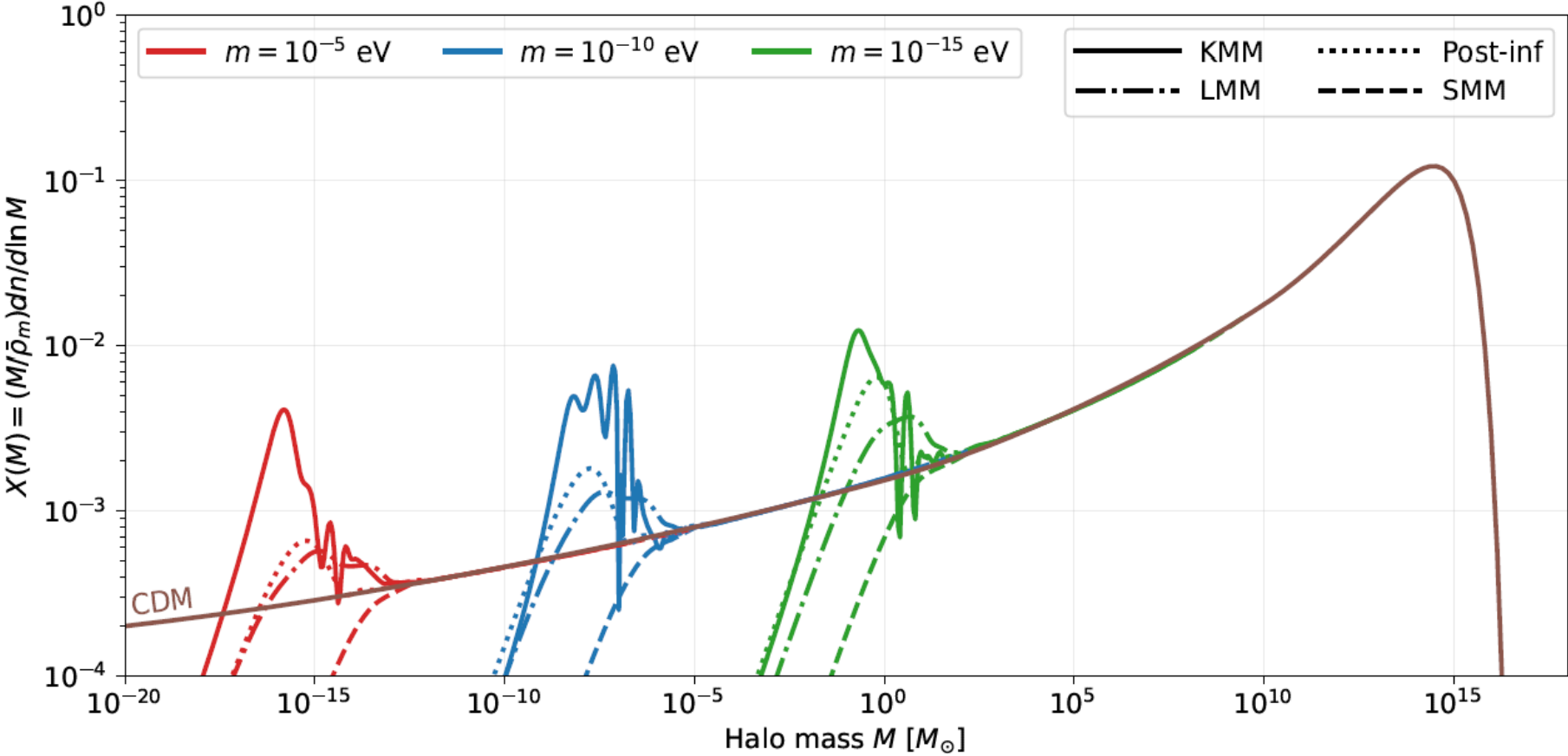
$$\text{c.f.) } \ddot{h} + 3H\dot{h} \sim \frac{1}{M_{pl}^2} \rho_\phi, \quad \rho_{GW} \sim M_{pl}^2 \dot{h}^2$$

Possible signals : ALP mini-cluster

clump of axion DMs

Small $m \rightarrow$ Large mini-cluster

Perturbative analysis + Press-Schechter formalism



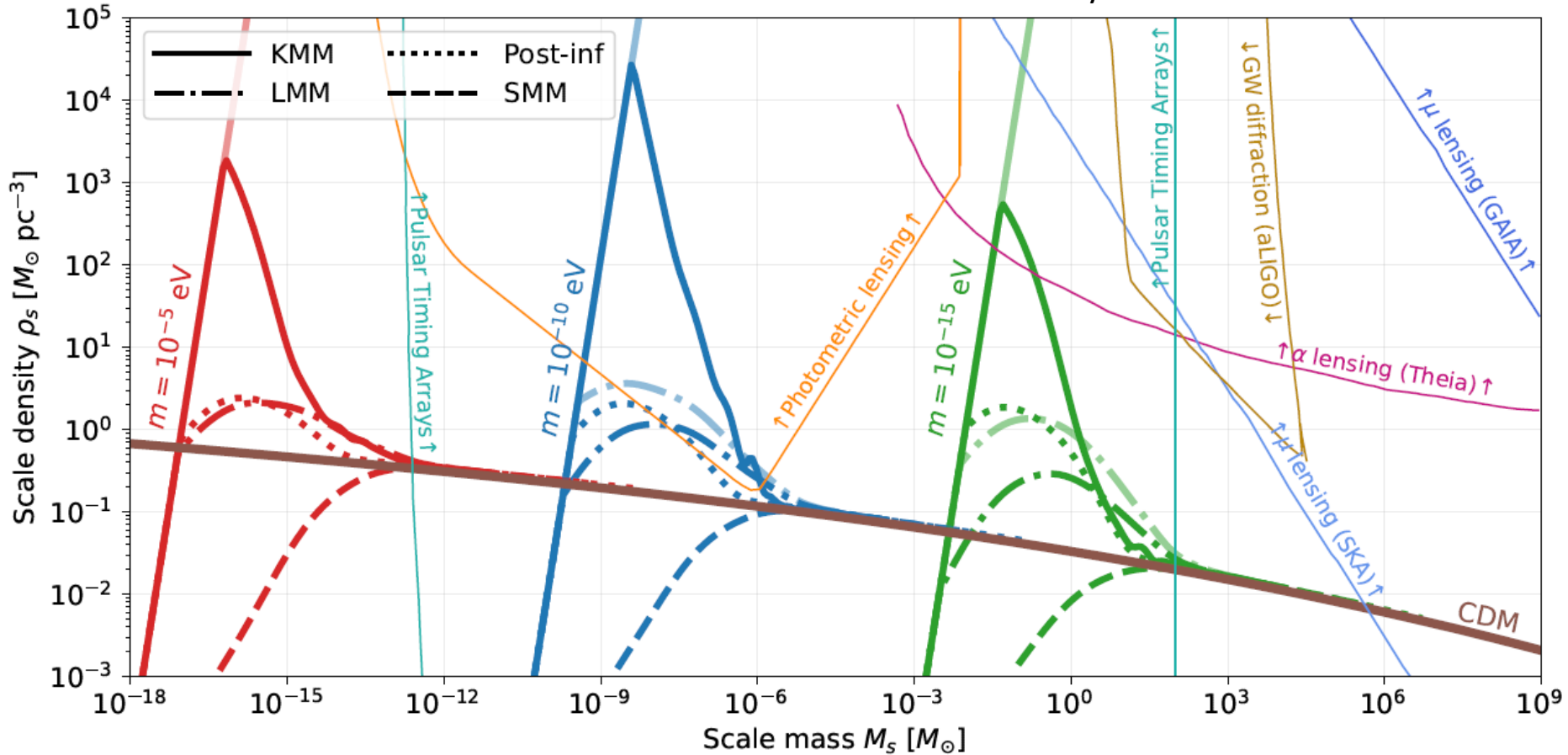
[Eröncel, Servant (2022)]

Possible signals : ALP mini-cluster

clump of axion DMs

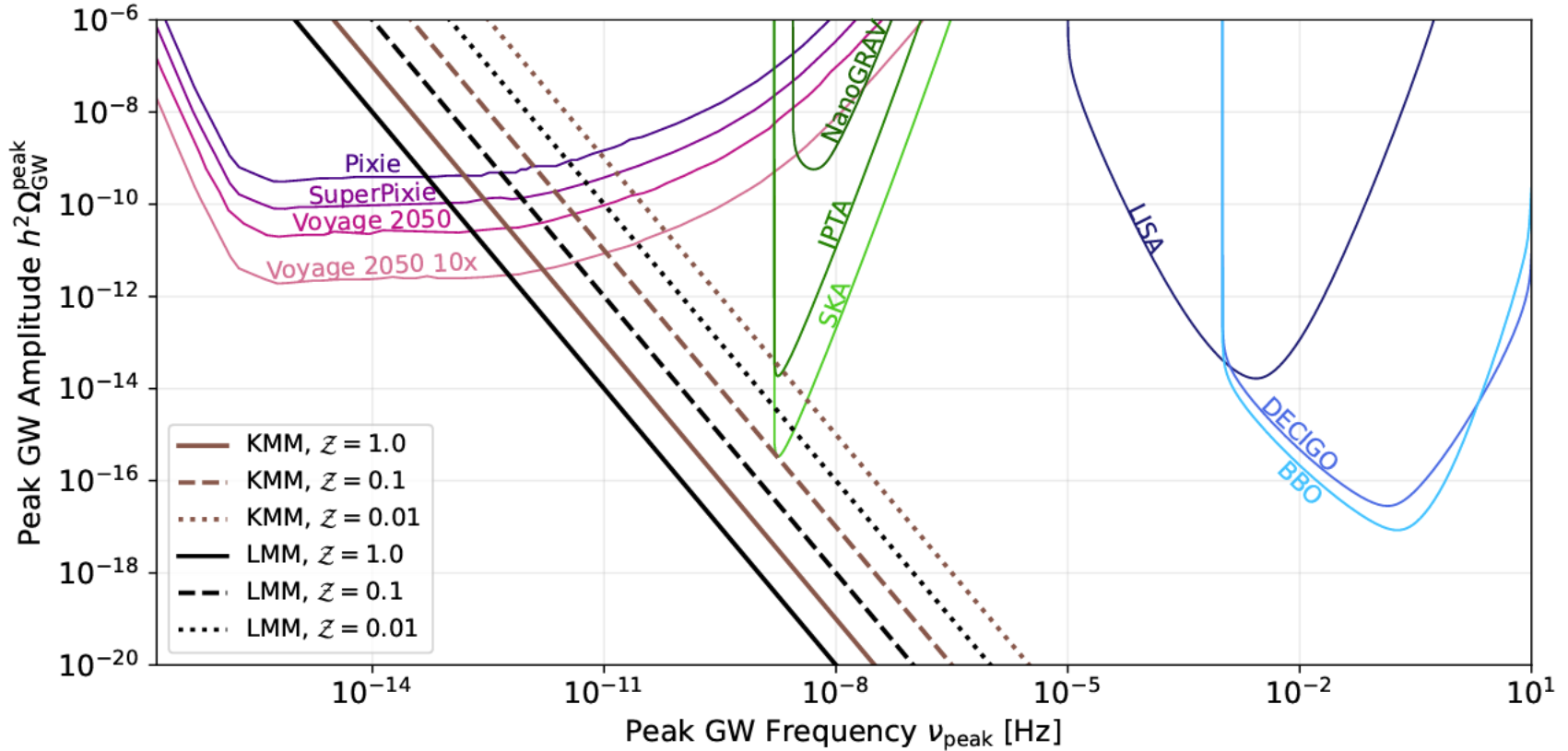
Small $m \rightarrow$ Large mini-cluster

Perturbative analysis + Press-Schechter formalism



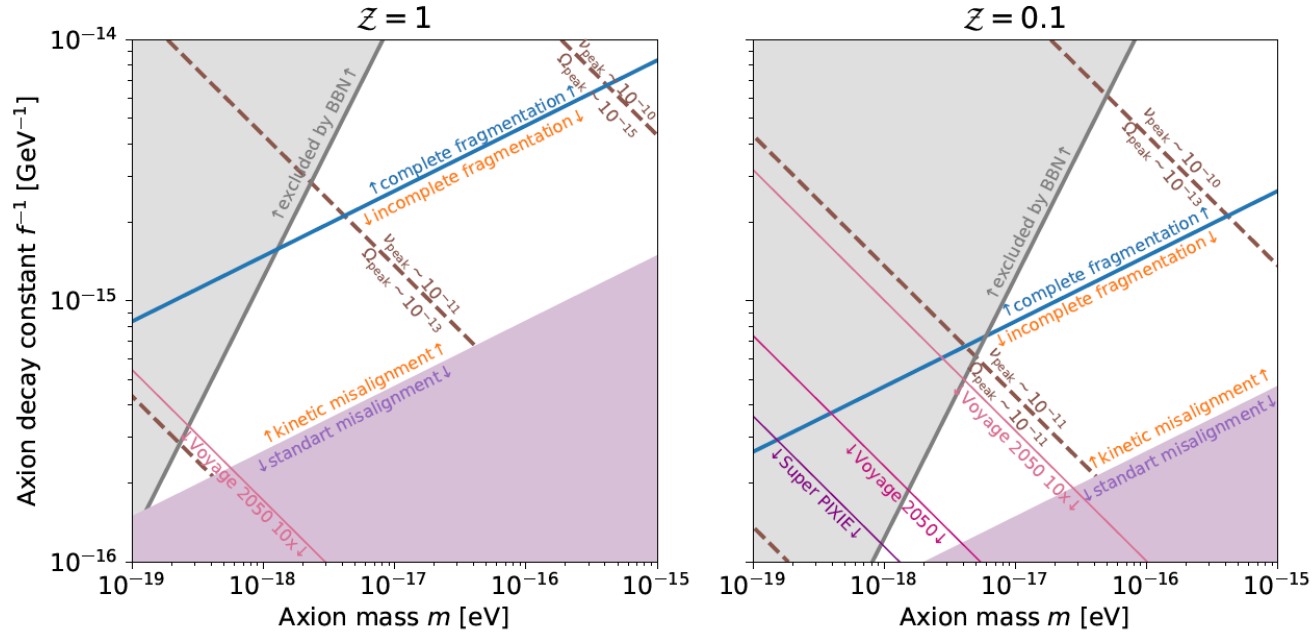
[Eröncel, Servant (2022)]

Possible signals : gravitational waves



[Eröncel, RS, Sørensen, Servant (2022)]

Possible signals : gravitational waves



Detailed analysis is future work

[Eröncel, RS, Sørensen, Servant (2022)]

$$\nu_{\text{peak}} \sim 8 \times 10^{-11} \text{ Hz} \left(\frac{m_*}{m_0} \right)^{2/3} \left(\frac{m_0}{10^{-16} \text{ eV}} \right)^{1/3} \left(\frac{f}{10^{14} \text{ GeV}} \right)^{-2/3} \mathcal{Z}^{-1/3}.$$

$$\frac{a_*}{a_0} = \left(\frac{3\pi}{8} \frac{\Omega_{\text{DM}}}{\mathcal{Z}} \frac{M_{\text{pl}}^2 H_0^2}{m_0 m_* f^2} \right)^{1/3}.$$

$$\Omega_{\text{GW},0}^{\text{peak}} \sim 1.5 \times 10^{-15} \left(\frac{m_*}{m_0} \right)^{2/3} \left(\frac{m_0}{10^{-16} \text{ eV}} \right)^{-2/3} \left(\frac{f}{10^{14} \text{ GeV}} \right)^{4/3} \mathcal{Z}^{-4/3}.$$

Fate of saxion

Saxion oscillation can overclose universe.

WHEN

before saxion dominates \rightarrow no entropy production
after saxion dominates \rightarrow entropy production

HOW

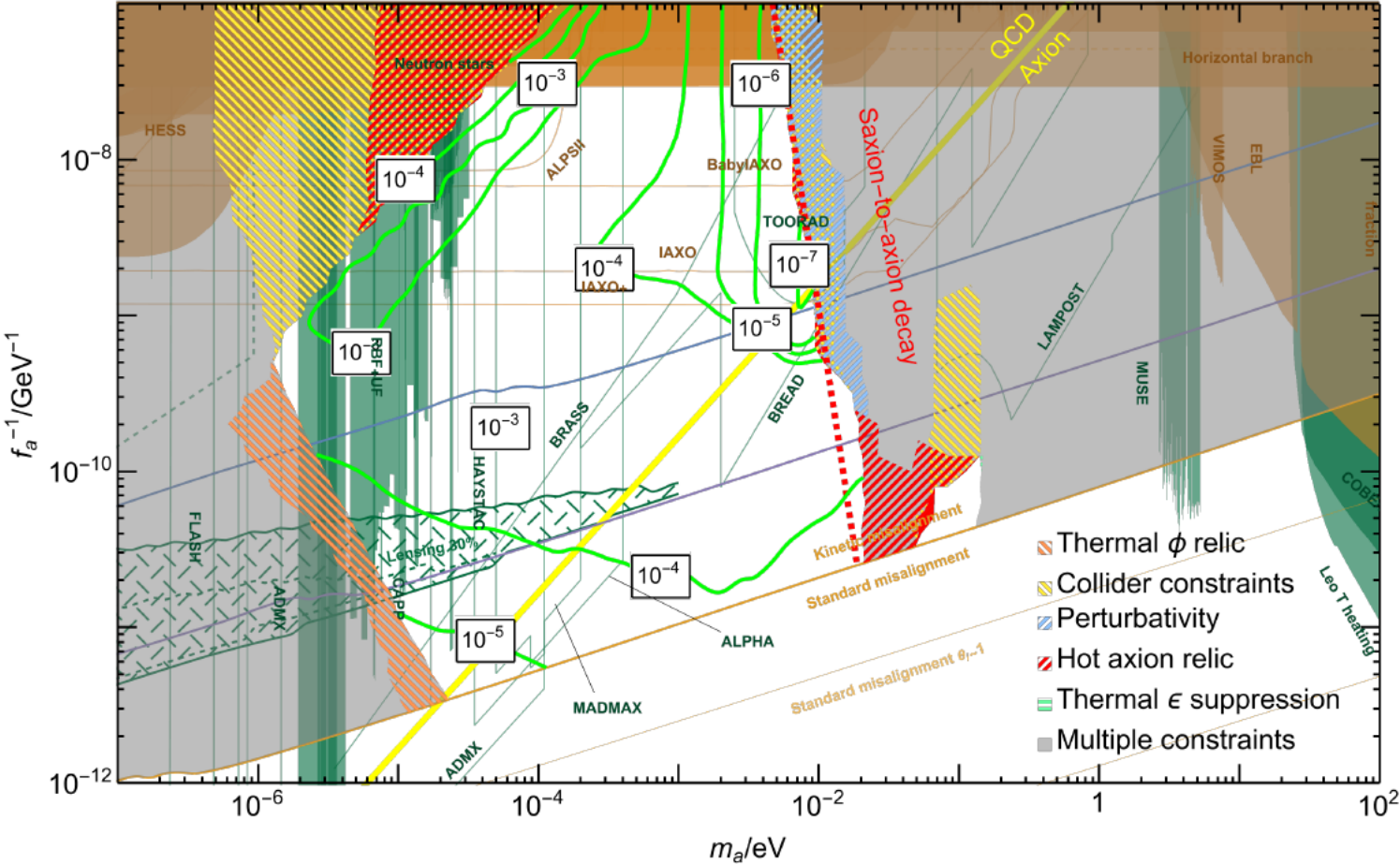
Coupling with fermion χ : $L \sim P \bar{\chi} \chi$

Coupling with Higgs : $L \sim |P|^2 |H|^2$

Something else?

Necessary suppression of A_s

Yukawa: Necessary $A_s(k_{\text{kin}})$ suppression for $M=m_{\text{Pl}}$ and $n=13$



[Eröncel, RS, Sørensen, Servant, in preparation]

Implication to ALP dark matter

Possible two approaches:

Model independent analysis

- $V = -\Lambda_b^4 \cos(\phi/f)$
- Initial value of $Y = n_{PQ}/s = f\dot{\phi}/s$
($\dot{\phi}$ scales as $a^{-3} \propto s$ at the beginning)

Model dependent analysis

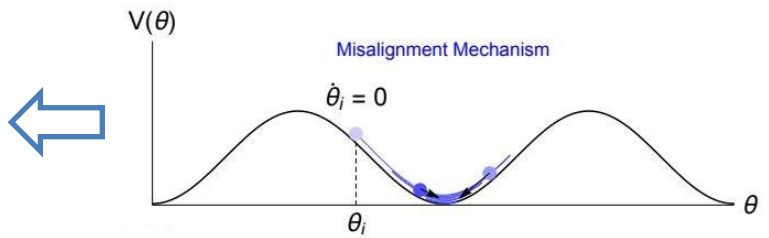
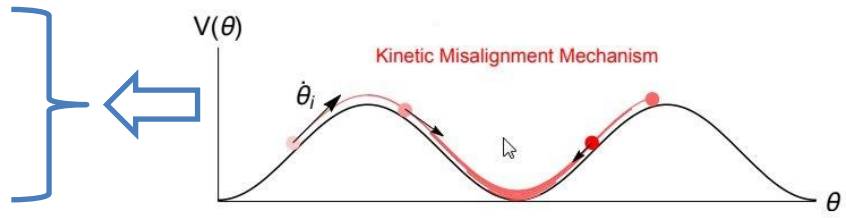
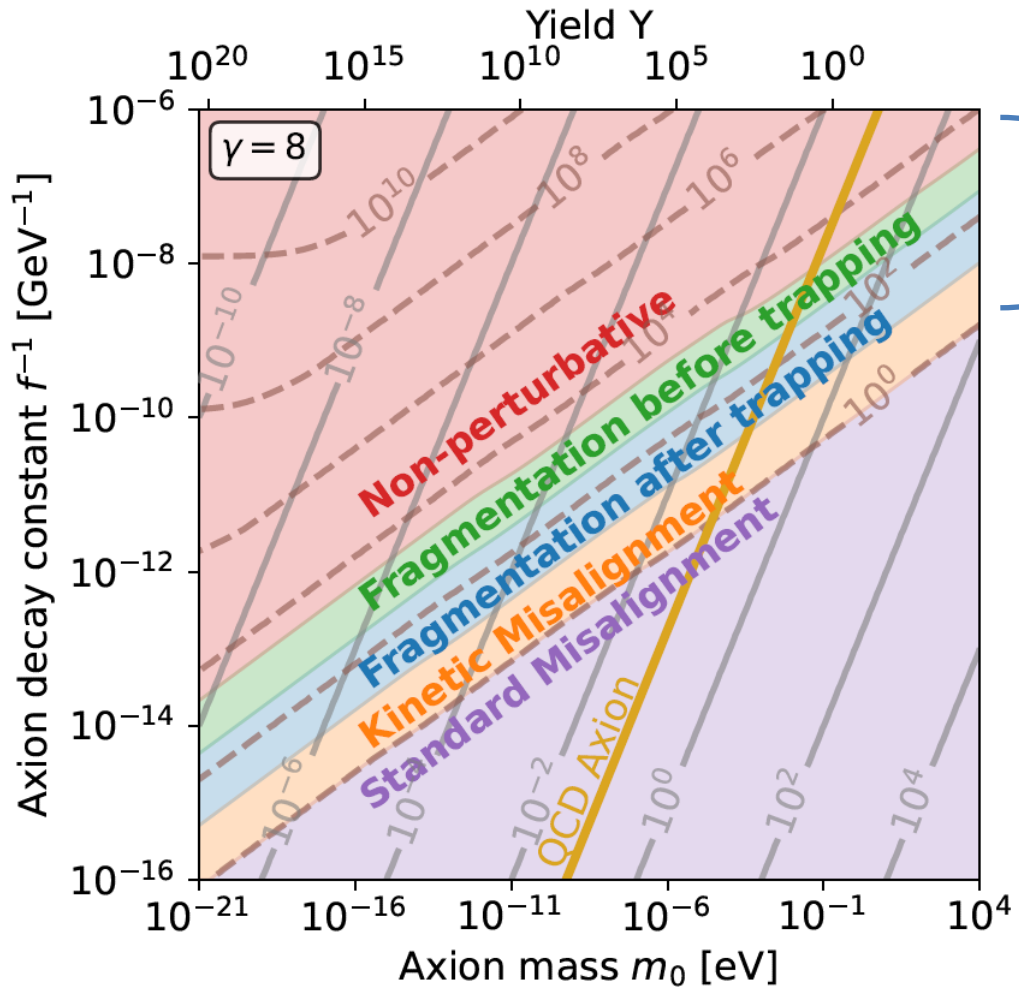
$$P = \frac{1}{\sqrt{2}} s \exp\left(\frac{i\phi}{f}\right) \quad \begin{array}{l} P : \text{PQ charged scalar} \\ s : \text{saxion} \\ \phi : \text{axion} \end{array}$$

There are many possibility of **UV models** and **thermal scenarios**.

- Potential for PQ breaking
- How to get $\dot{\phi} \neq 0$?
- How to eliminate saxion oscillation?
- etc

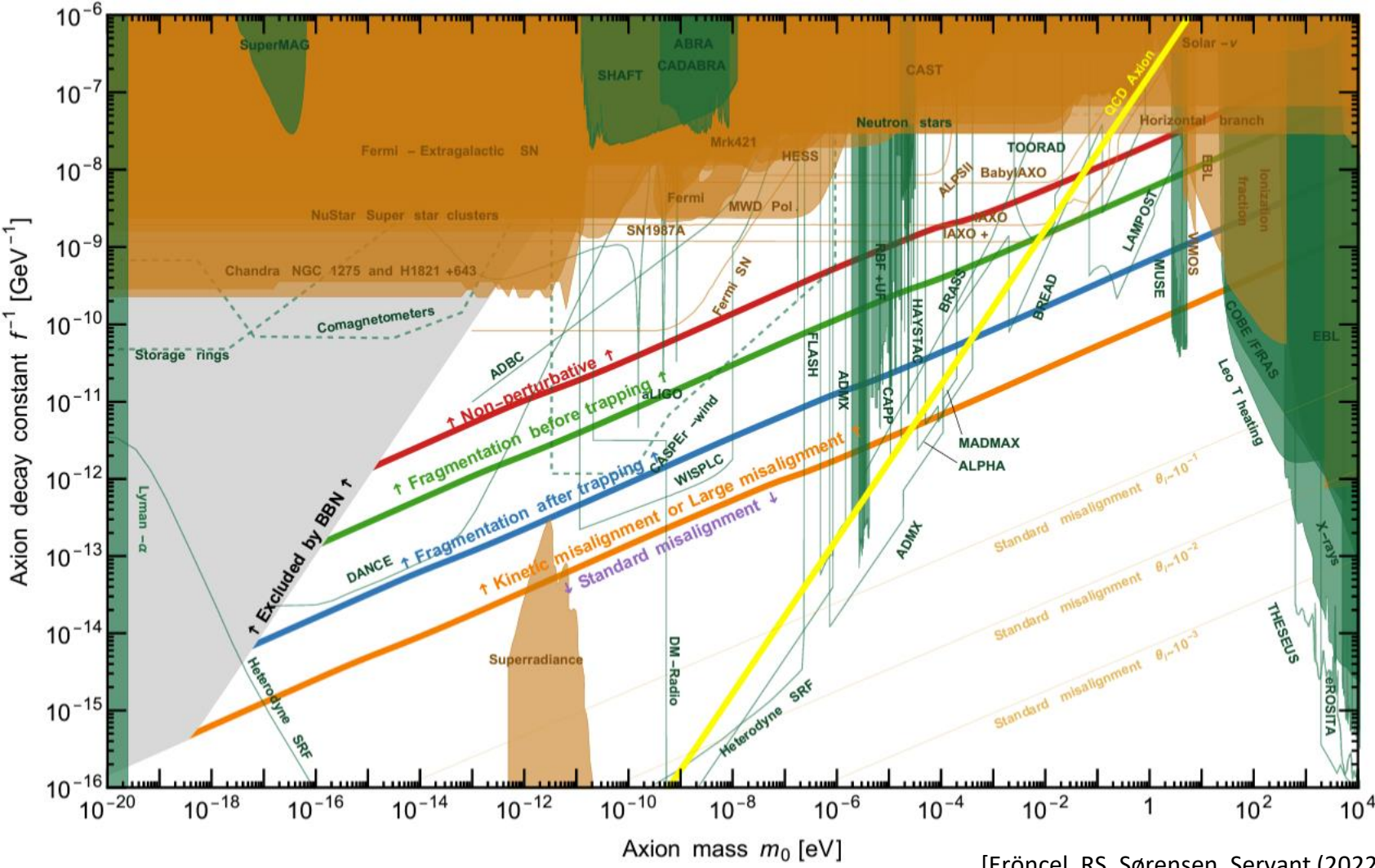
Model independent analysis

ALP dark matter : Fragmentation could happen before axion starts to oscillate

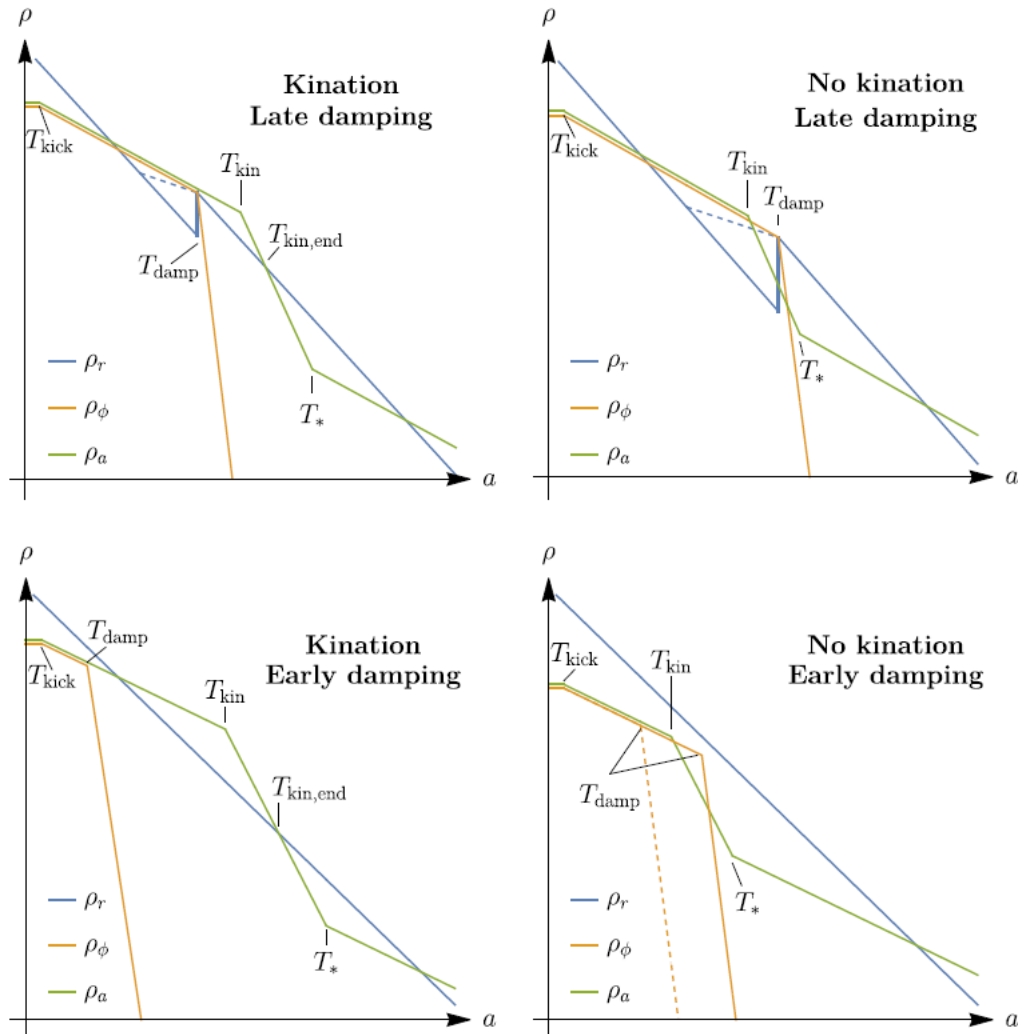


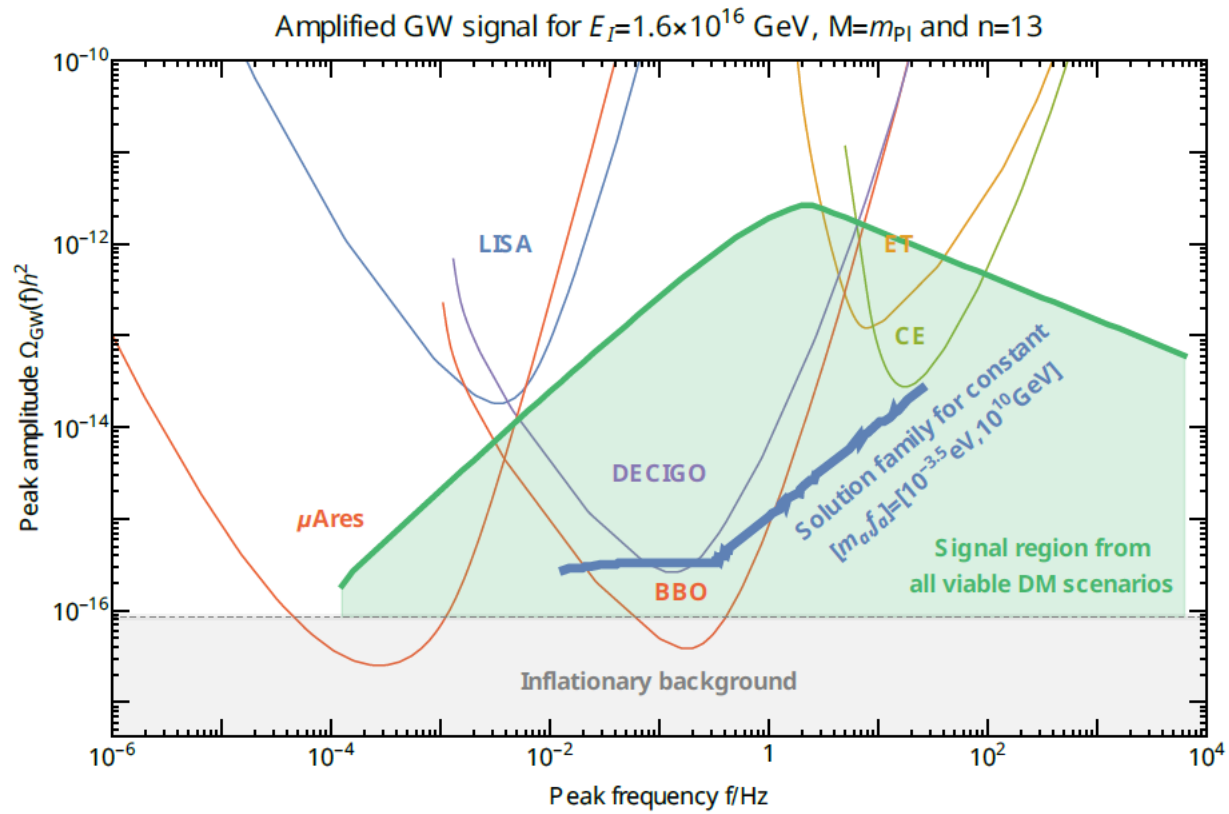
[Eröncel, RS, Sørensen, Servant (2022)]

Model independent analysis

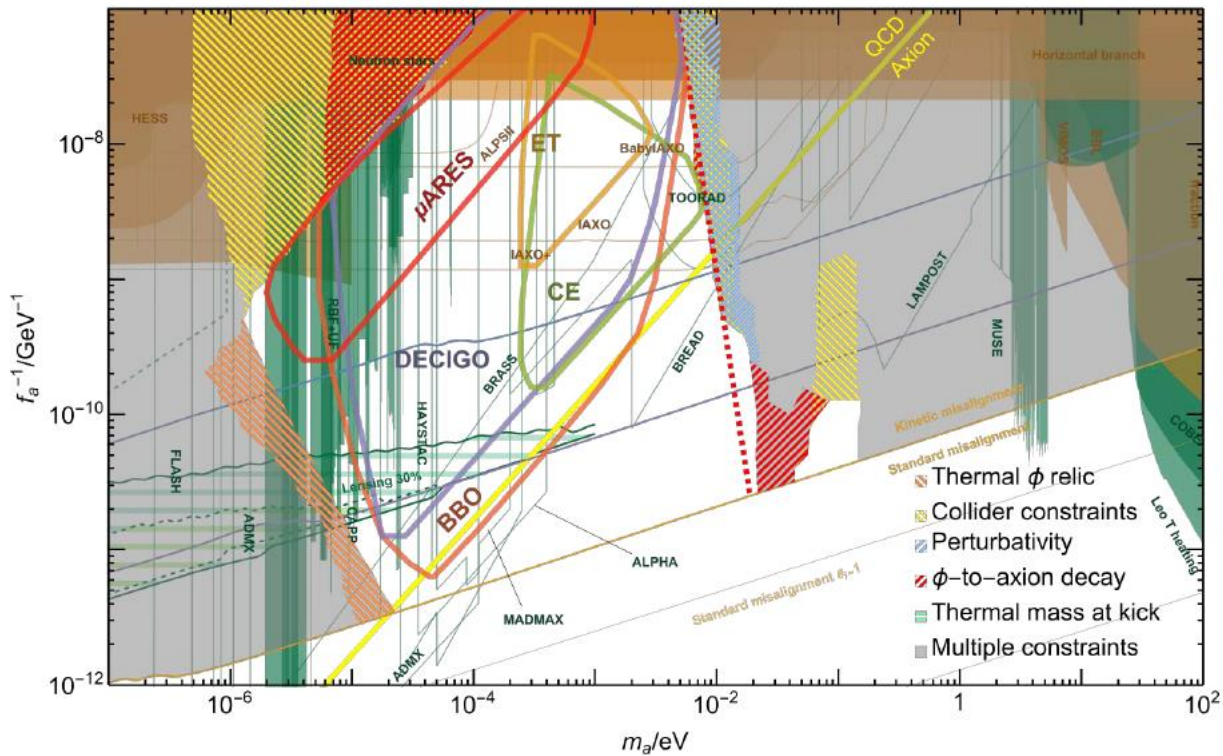


[Eröncel, RS, Sørensen, Servant (2022)]





Potential reach of GW experiments for $E_f=1.6 \times 10^{16}$ GeV, $M=m_{Pl}$ and $n=13$



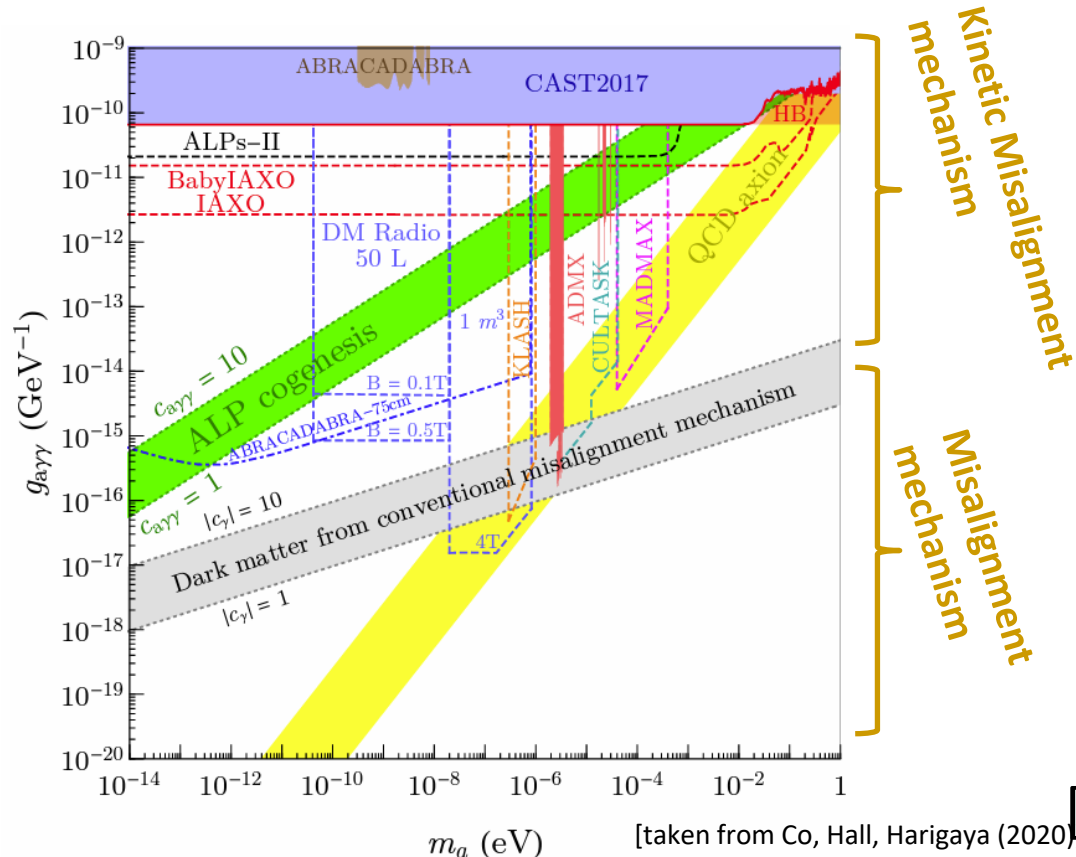
Why interesting?

For given axion mass,

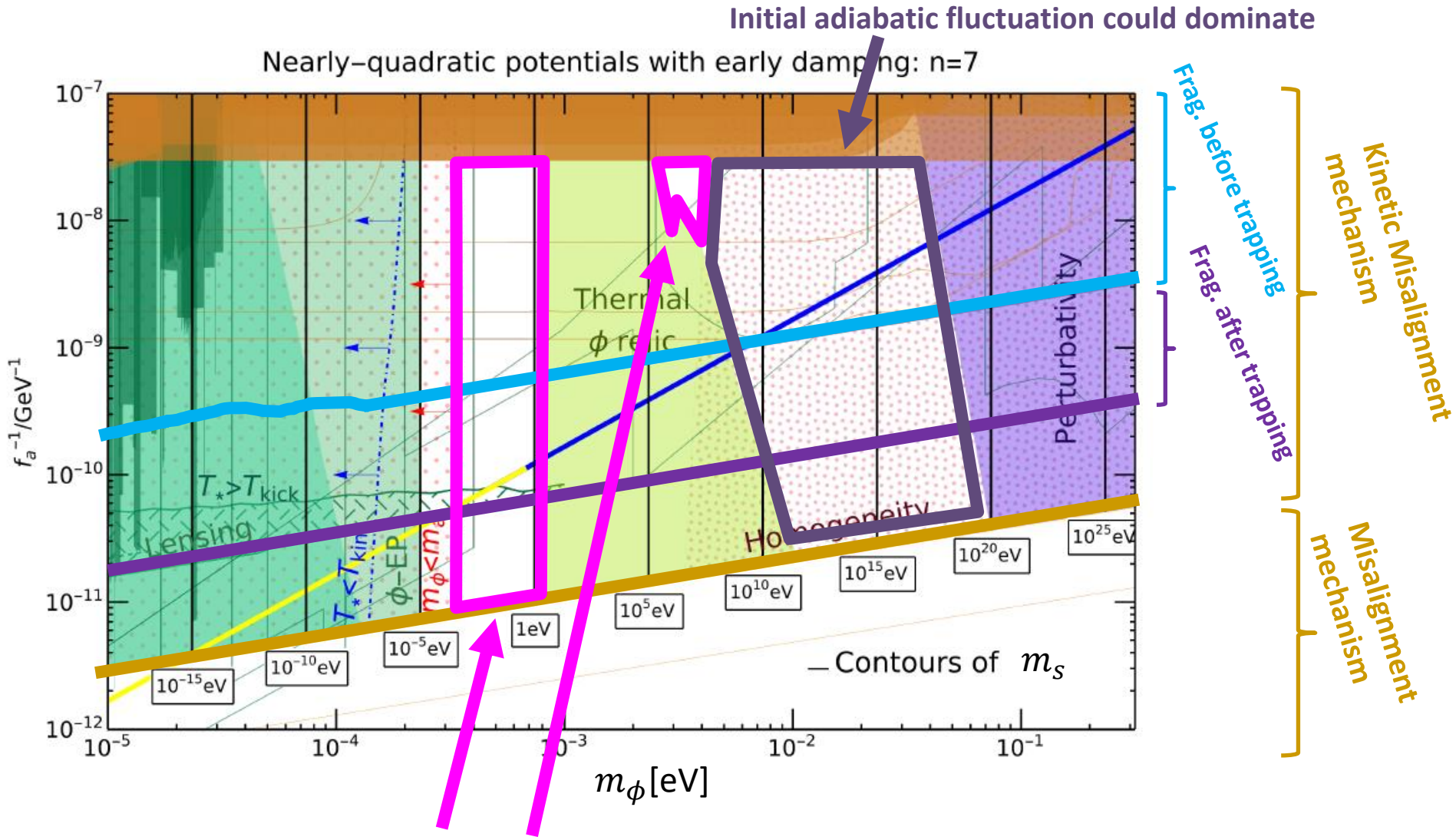
- Conventional MM predicts smaller ρ_{DM} for smaller f_a
- KMM predicts larger ρ_{DM} than conventional MM



KMM tends to predicts **smaller f_a** (\sim larger $g_{a\gamma\gamma}$)
motivates experiments!!



A parameter space



Kinetic misalignment (+ fragmentation)

[Eröncel, RS, Sørensen, Servant, in preparation]

Temperature –dependent axion mass with $\gamma=8.16$

