レプトン数の時間発展とmee マヨラナ型位相

Time evolution of Lepton Number , mee Majorana type phase

両角　卓也 Takuya Morozumi
(広島大学, Hiroshima U.)
共同研究者
河野早紀　Saki Kawano 河村優太 Yuta Kawamura
清水勇介(広大) Yusuke Shimizu (Hiroshima)
ニコラスベンワ Nicholas James Benoit (Hiroshima)
山本 恵(広工大) Kei Yamamoto (HIT)

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概要

- 軽い3つのニュートリノの質量に関してマヨラナ型の質量の場合，レプトン数が破れる。

- レプトン数の期待値の時間発展を調べることでニュートリノ質量やマヨラナ位相に関して，どのような情報が得られるかを調べる。

- ニュートリノレス2重ベータ崩壊を決めるm_{ee}との関係も議論する。

目次

マヨラナ質量項とレプトン混合行列（PMNS 行列）

荷電レプトンの質量基底　$\alpha = e, \mu, \tau$

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu \nu_{L\alpha} W^- - \frac{1}{2}\overline{(\nu_{L\alpha})^c} m_{\nu\alpha\beta}\nu_{L\beta} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

$$\begin{pmatrix}\nu_{L\alpha}\\ l_{L\alpha}\end{pmatrix} = \left\{\begin{pmatrix}\nu_{Le}\\ l_{Le}\end{pmatrix}, \begin{pmatrix}\nu_{L\mu}\\ l_{L\mu}\end{pmatrix}, \begin{pmatrix}\nu_{L\tau}\\ l_{L\tau}\end{pmatrix}\right\}, \qquad m_{\nu\alpha\beta} = \begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} & m_{\nu e\tau} \\ m_{\nu e\mu} & m_{\nu\mu\mu} & m_{\nu\mu\tau} \\ m_{\nu e\tau} & m_{\nu\mu\tau} & m_{\nu\tau\tau} \end{pmatrix}$$

ニュートリノの質量基底　$\nu_{L\alpha} = U_{\alpha i}\nu_{Li} \quad i = 1, 2, 3$

$$\mathrm{U}^T_{i\alpha} m_{\nu\alpha\beta}\, \mathrm{U}_{\beta j} = m_i \delta_{ij}$$

$$\frac{g}{\sqrt{2}}\overline{l_{L\alpha}}\gamma_\mu \mathrm{U}_{\alpha i}\nu_{Li} W^- - \frac{1}{2}\overline{(\nu_{Li})^c}\, m_i \nu_{Li} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$$

$$l_\alpha \to e^{i\theta_\alpha} l_\alpha,\ \nu_i \not\to e^{i\theta_i}\nu_i$$

（荷電レプトンだけリフェイジングが許される。）

$$\mathrm{U}_{\alpha i}(6\ \text{phases} + 3\text{angles}) \to e^{i(\not\theta_i - \theta_\alpha)}\, \mathrm{U}_{\alpha i}(3\text{phases} + 3\text{angles})$$

マヨラナの場合のPMNS 行列のパラメトリゼーション

3つの角度と+3つの CPの破れの位相

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

Doi, Kotani, Nishiura, Okuda,Takasugi, PLB. 1981

1. α_{21} , α_{31} are called Majorana phases. They are physical when all the three neutrinos are massive.

2. If a neutrino is massless, only one Majorana phase is physical.

3. δ is a Kobayashi Maskawa type phase(Dirac Phase).

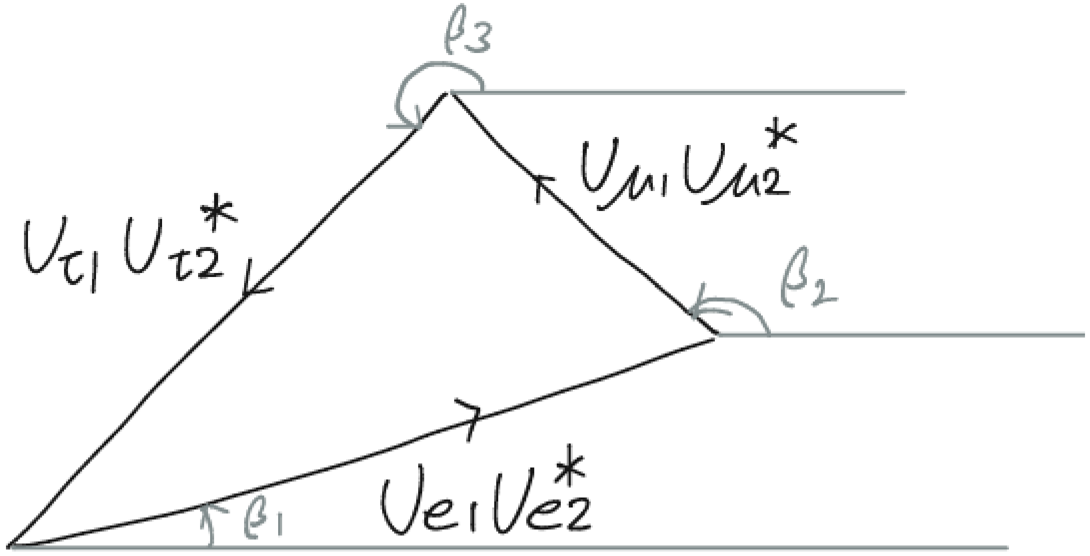
ユニタリー3角形と Majorana Type phases

$$U_{ei}\,U^*_{ej} + U_{\mu i}\,U^*_{\mu j} + U_{\tau i}\,U^*_{\tau j} = 0 \quad i \neq j$$

"Majorana type" phases.

(Branco, Rebelo, arXiv:0809.2799. J. Nieves and P. B. Pal, Phys. Rev. D36(1987))

$$arg(U_{e1}\,U^*_{e2}) = \beta_1\ , arg\left(U_{\mu 1}\,U^*_{\mu 2}\right) = \beta_2, arg(U_{\tau 1}\,U^*_{\tau 2}) = \beta_3 \quad 2\,\beta_1 = -\alpha_{21}$$



$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U^0_{e2} e^{i\frac{\alpha_{21}}{2}} & U^0_{e3} e^{i\frac{\alpha_{31}}{2}} \\ U_{\mu 1} & U^0_{\mu 2} e^{i\frac{\alpha_{21}}{2}} & U^0_{\mu 3} e^{i\frac{\alpha_{31}}{2}} \\ U_{\tau 1} & U^0_{\tau 2} e^{i\frac{\alpha_{21}}{2}} & U^0_{\tau 3} e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

図：河野早紀　修論

Independent of Majorana phases
$\beta_i - \beta_j$

$$\Delta\left(\beta_i + \beta_j\right) = -\Delta\alpha_{21}:\ \Delta\beta_i = \Delta\beta_j$$

３角形の複素平面上での配向(回転)を決める。

1. Branco and Rebelo defined 6 "Majorana type" phases.
2. They define the arguments of each side of the three Unitarity triangles

$$\sum_{\alpha=e}^{\tau} U_{\alpha i}\, U^*_{\alpha j} = 0$$

$U_{\alpha i}\, U^*_{\alpha j}$

$\alpha \backslash (\mathbf{i}, \mathbf{j})$	$(\mathbf{1}, \mathbf{2})$	$(\mathbf{2}, \mathbf{3})$	$(\mathbf{3}, \mathbf{1})$
e	$U_{e1}U^*_{e2}$	$U_{e2}U^*_{e3}$	$U_{e3}U^*_{e1}$
μ	$U_{\mu 1}U^*_{\mu 2}$	$U_{\mu 2}U^*_{\mu 3}$	$U_{\mu 3}U^*_{\mu 1}$
τ	$U_{\tau 1}U^*_{\tau 2}$	$U_{\tau 2}U^*_{\tau 3}$	$U_{\tau 3}U^*_{\tau 1}$

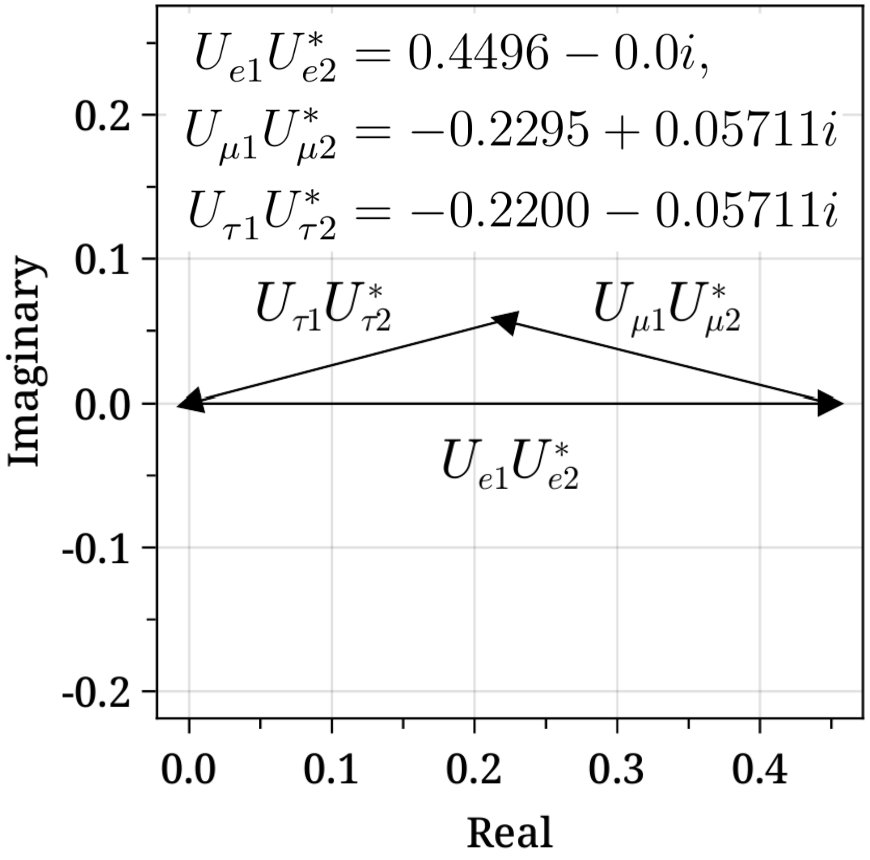
$arg\left(U_{\alpha i}\, U^*_{\alpha j}\right)$

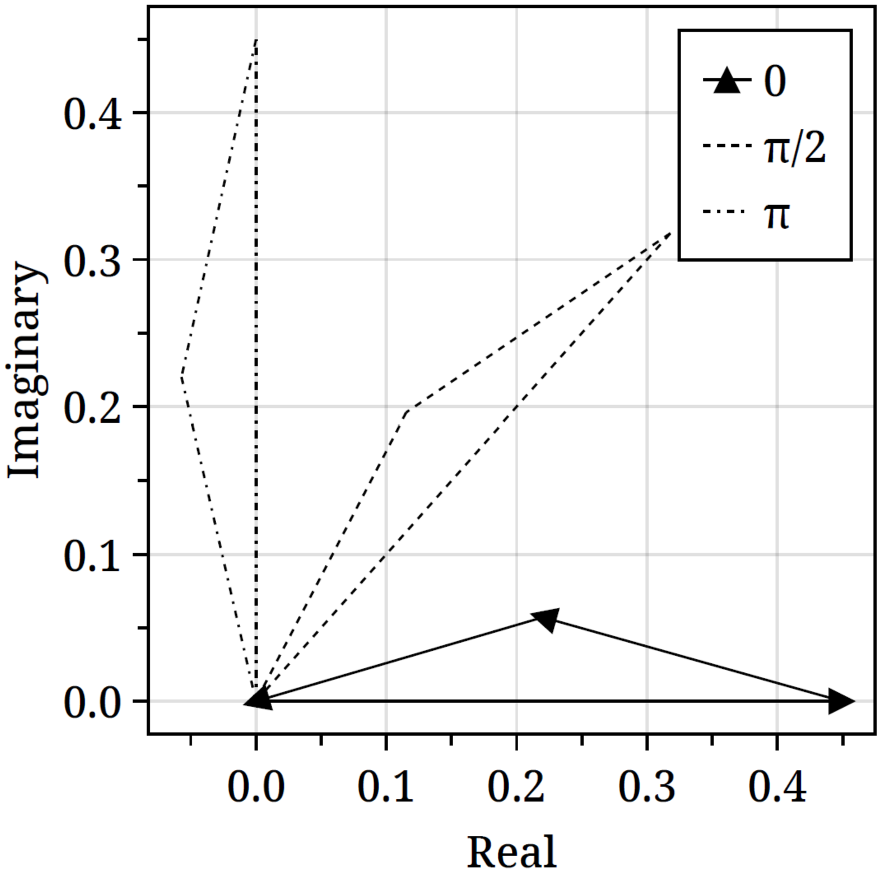
$\alpha \backslash (\mathbf{i}, \mathbf{j})$	$(\mathbf{1}, \mathbf{2})$	$(\mathbf{2}, \mathbf{3})$	$(\mathbf{3}, \mathbf{1})$
e	$\boldsymbol{\beta_1}$	$\boldsymbol{\gamma_1 - \beta_1}$	$\boldsymbol{-\gamma_1}$
μ	$\boldsymbol{\beta_2}$	$\boldsymbol{\gamma_2 - \beta_2}$	$\boldsymbol{-\gamma_2}$
τ	$\boldsymbol{\beta_3}$	$\boldsymbol{\gamma_3 - \beta_3}$	$\boldsymbol{-\gamma_3}$

$$2\,\boldsymbol{\beta_1} = -\boldsymbol{\alpha_{21}}, \quad 2\boldsymbol{\gamma_1} = 2\boldsymbol{\delta} - \boldsymbol{\alpha_{31}}$$

$$2\,\beta_1 = -\alpha_{21} = 0, \frac{\pi}{2}, \pi$$

Triangle 1 $\boldsymbol{\beta_1 = 0 = -\alpha_{21}/2}$



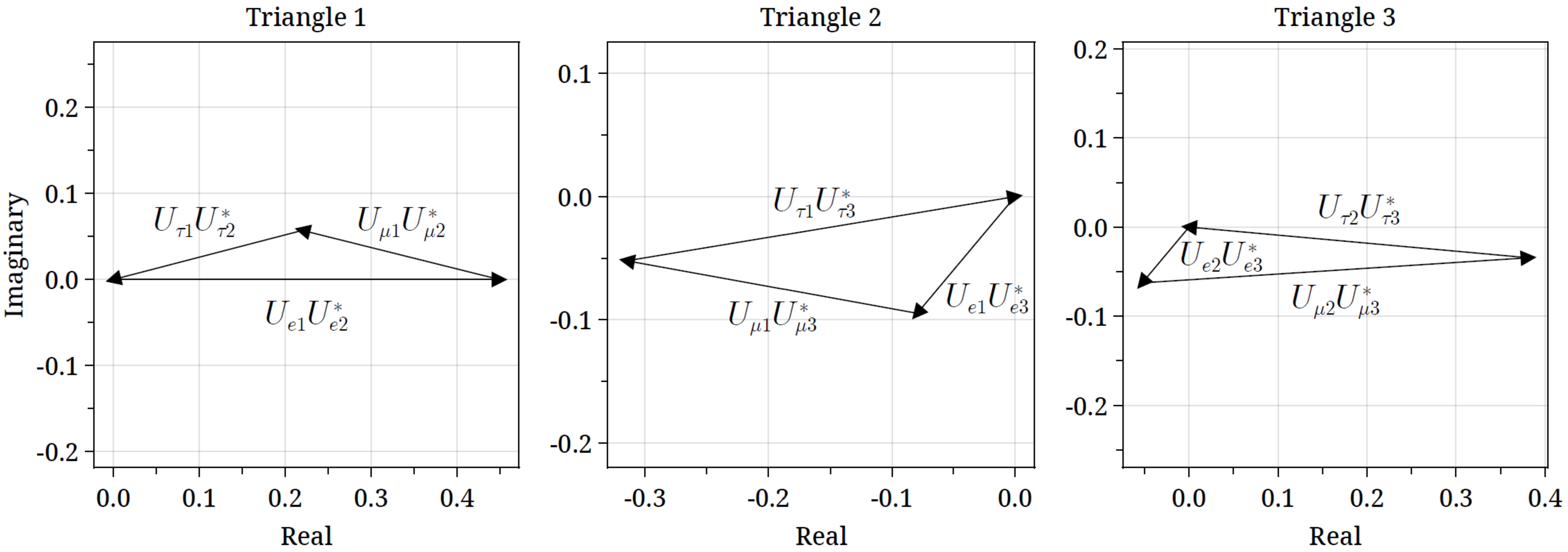


$$\boldsymbol{arg(U_{e1}\,U_{e2}^*) = \beta_1}$$
$$\boldsymbol{arg(U_{\mu1}\,U_{\mu2}^*) = \beta_2}$$
$$\boldsymbol{arg(U_{\tau1}\,U_{\tau2}^*) = \beta_3}$$

$$U_{e1}U_{e2}^* = 0.4496 - 0.0i,$$
$$U_{\mu1}U_{\mu2}^* = -0.2295 + 0.05711i$$
$$U_{\tau1}U_{\tau2}^* = -0.2200 - 0.05711i$$

$$U_{e1}U_{e3}^* = -0.07945 - 0.09469i$$
$$U_{\mu1}U_{\mu3}^* = -0.2354 + 0.04261i,$$
$$U_{\tau1}U_{\tau3}^* = 0.3149 + 0.05208i;$$

$$U_{e2}U_{e3}^* = -0.05251 - 0.06258i$$
$$U_{\mu2}U_{\mu3}^* = 0.4339 + 0.02816i,$$
$$U_{\tau2}U_{\tau3}^* = -0.3814 + 0.03442i.$$



$$\alpha_{21} = 0$$
$$2\,\beta_1 = 0$$

$$\alpha_{31} = 0$$
$$\mathbf{2\gamma_1 = 2\delta}$$

$$\alpha_{31} - \alpha_{21} = 0$$
$$\mathbf{2\gamma_1 - 2\,\beta_1 = 2\delta}$$

The goal for the case of 3 massive Majorana neutrinos

1. Determine two Majorana phases α_{21} , α_{31}

   Equivalently determine the orientation of unitarity triangles

   two of β_1, γ_1 (β_1- γ_1)

2. Determine the lightest neutrino mass including the ordering “normal or inverted”.

有効マヨラナ質量$m_{\nu ee}$とマヨラナタイプ位相

$$m^*_{\nu\alpha\beta} = \sum_{i=1}^{3} U_{\alpha i} m_i U_{\beta i}$$

$$= \begin{pmatrix} U_{e1}^2 m_1 + U_{e2}^2 m_2 + U_{e3}^2 m_3 & U_{e1} m_1 U_{\mu 1} + U_{e2} m_2 U_{\mu 2} + U_{e3} m_3 U_{\mu 3} \\ & \\ & \end{pmatrix}$$

$$|m_{\nu ee}|^2 = |U_{e1}|^4 m_1^2 + |U_{e2}|^4 m_2^2 + |U_{e3}|^4 m_3^2$$
$$+2\,|U_{e1} U_{e2}|^2\, m_1 m_2 \cos(2\beta_1)$$
$$+2\,|U_{e2} U_{e3}|^2\, m_2 m_3\, \cos\big(2(\beta_1 - \gamma_1)\big)$$
$$+2\,|U_{e1} U_{e3}|^2\, m_1 m_3\, \cos(2\gamma_1)$$

有効マヨラナ質量$m_{\nu ee}$とマヨラナタイプ位相

$$|m_{\nu ee}|^2 = |U_{e1}|^4 m_1^2 + |U_{e2}|^4 m_2^2 + |U_{e3}|^4 m_3^2$$
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$$+2\,|U_{e2}U_{e3}|^2\, m_2 m_3\, \cos\big(2(\beta_1 - \gamma_1)\big)$$
$$+2\,|U_{e1}U_{e3}|^2\, m_1 m_3\, \cos(2\gamma_1)$$

- $|m_{\nu ee}|$ は３つの未知パラメーターに依存
- $\beta_1, \gamma_1, m_1(Normal)$or m_3(Inverted)

  -2β_1=α_{21}, -2$\gamma_1 = \alpha_{31} - 2\delta$
- これらをすべて決めるには, 十分ではない。

The Time evolution of Lepton Number for Majorana neutrinos
マヨラナニュートリノのレプトン数の時間発展

$$L_\alpha(t) = \int \frac{d^3p}{(2\pi)^3 2|p|}\left[a_\alpha^\dagger(p,t)a_\alpha(p,t) - b_\alpha^\dagger(p,t)b_\alpha(p,t)\right]$$

$$|\boldsymbol{\sigma}(\boldsymbol{q})\rangle = \boldsymbol{a}_\sigma^\dagger(\boldsymbol{q})|\mathbf{0}> \qquad \boldsymbol{E}_i(\boldsymbol{q}) = \sqrt{\boldsymbol{q}^2 + \boldsymbol{m}_i^2}$$

- depends on Majorana phases
- depends on the combinations of $\boldsymbol{\beta}_i + \boldsymbol{\beta}_j / \boldsymbol{\gamma}_i + \boldsymbol{\gamma}_j$

$$\langle\sigma| L_\alpha^M(t) |\sigma\rangle$$

$$= \sum_{i,j}^{3}\Bigg[\mathrm{Re}(U_{\alpha i}^* U_{\sigma i} U_{\alpha j} U_{\sigma j}^*)\left(\cos\{E_i(\mathbf{q})t\}\cos\{E_j(\mathbf{q})t\} + \frac{\mathbf{q}^2}{E_i(\mathbf{q})E_j(\mathbf{q})}\sin\{E_i(\mathbf{q})t\}\sin\{E_j(\mathbf{q})t\}\right)$$

$$- \mathrm{Im}(U_{\alpha i}^* U_{\sigma i} U_{\alpha j} U_{\sigma j}^*)\left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})}\sin\{E_i(\mathbf{q})t\}\cos\{E_j(\mathbf{q})t\} - \frac{|\mathbf{q}|}{E_j(\mathbf{q})}\cos\{E_i(\mathbf{q})t\}\sin\{E_j(\mathbf{q})t\}\right)$$

$$- \mathrm{Re}(U_{\alpha i}^* U_{\sigma i}^* U_{\alpha j} U_{\sigma j})\frac{m_i}{E_i(\mathbf{q})}\frac{m_j}{E_j(\mathbf{q})}\sin\{E_i(\mathbf{q})t\}\sin\{E_j(\mathbf{q})t\}\Bigg] \quad (17)$$

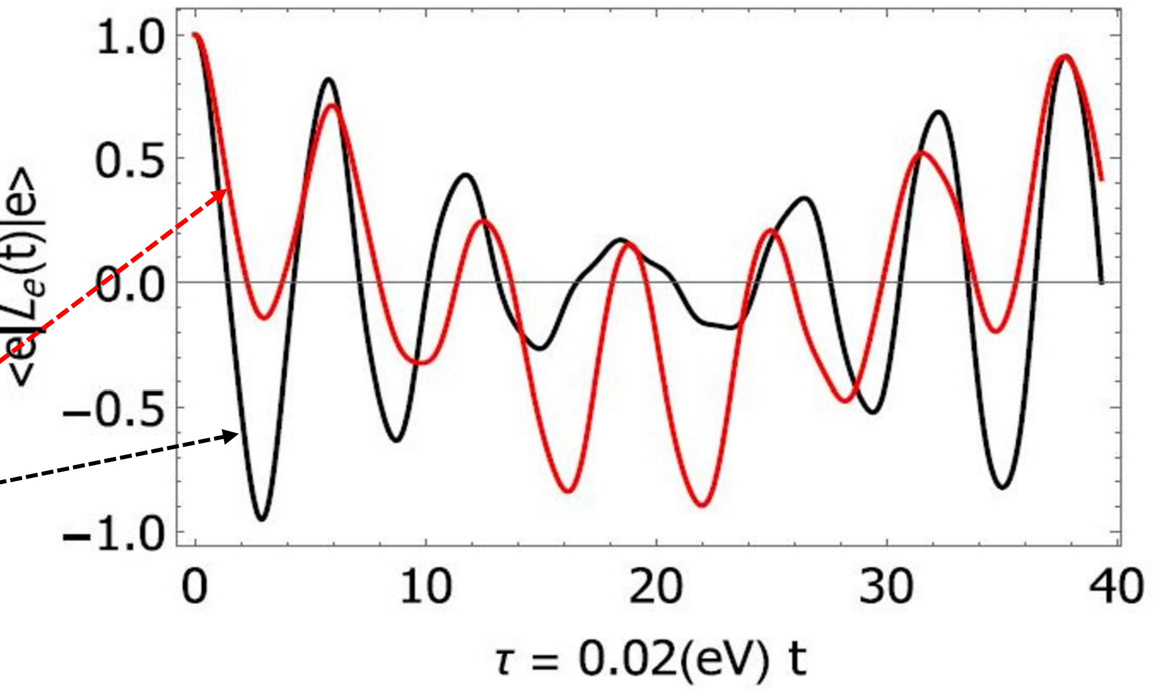
- Independent on Majorana phases

The extraction of Majorana phases from the second time derivatives of Lepton family numbers
レプトン数の期待値の２階時間微分を使ったマヨラナ位相と最も軽いニュートリノ質量の決定

The sharp decrease of the electron number for the largest $|m_{ee}|$(black curve) and the $|m_{ee}|$ is the smallest for（red curve).

$(\alpha_{21}, \alpha_{31} - 2\delta)$
$m_{ee}^{max}, m_{ee}^{min}$
$(0,0), (\pi, \pi)$
$0.012, 0.0018$

q=0.0002(eV) $m_1 = 0.01(ev)$



Electron number for normal hierarchy

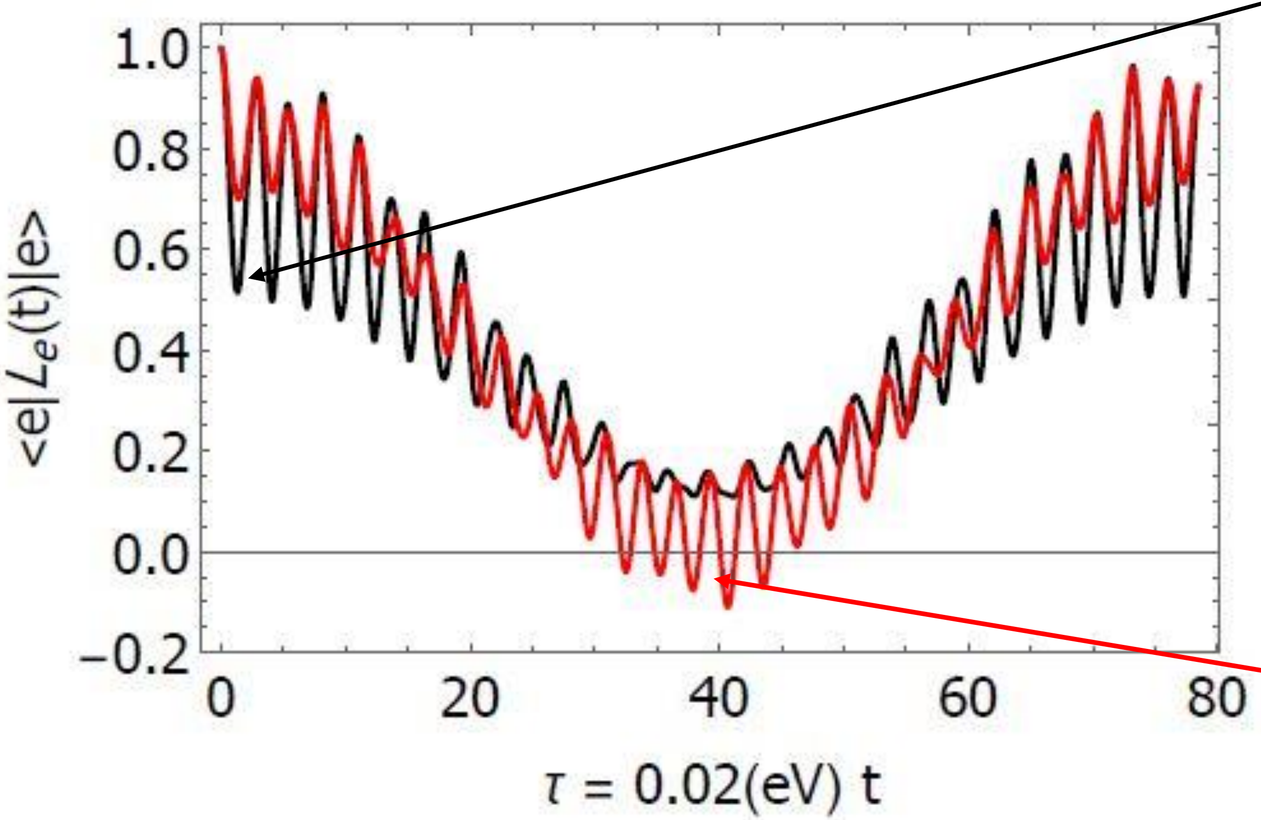
Figures from

A. S. Adam, N. J. Benoit, Y. Kawamura, Y. Matsuo, T. Morozumi, Y. Shimizu, Y. Tokunaga and N. Toyota, PTEP **2021**, 5 (2021) doi:10.1093/ptep/ptab025 [arXiv:2101.07751 [hep-ph]].

The dependence on Majorana phase:

$m_1 \approx m_2$=**0.01 < q=0.02**$< m_3 = 0.03$

Figure3 $\langle e|L_e(t)|e\rangle$ electron number.



The sharp decrease of the electron number for the largest $|m_{ee}|$(black curve). $|m_{ee}|$ is the smallest for (red curve).

$\nu_e \rightarrow \bar{\nu}_e$

レプトン数の２階微分

$$L^M(t) = \sum_{a=e}^{\tau} L^M_\alpha(t)$$ （全レプトン数）

$$\frac{d^2}{dt^2}\langle\sigma|\,L^M_\alpha(t)\,|\sigma\rangle\,|_{t=0} = -2\sum_i^3 \delta_{\alpha\sigma}|U_{\sigma i}|^2 m_i^2 - 2\sum_{i,j}^3 \mathrm{Re}(U^*_{\alpha i}U^*_{\sigma i}U_{\alpha j}U_{\sigma j})m_i m_j$$

$$\frac{d^2}{dt^2}\langle\sigma|\,L^M_\alpha(t)\,|\sigma\rangle\,|_{t=0} = \delta_{\alpha\sigma}\frac{1}{2}\frac{d^2}{dt^2}\langle\sigma|\,L^M(t)\,|\sigma\rangle\,|_{t=0} - 2\sum_{i,j}^3 \mathrm{Re}(U^*_{\alpha i}U^*_{\sigma i}U_{\alpha j}U_{\sigma j})m_i m_j.$$

$$\frac{d^2}{dt^2}\langle\sigma|\,L^M(t)\,|\sigma\rangle\,|_{t=0} = -4\sum_i^3 m_i^2|U_{\sigma i}|^2.$$

全レプトンの２階微分と最も軽いニュートリノ質量

$$m_1^2 = -\frac{1}{4}\frac{d^2}{dt^2}\langle\sigma|\,L^M(t)\,|\sigma\rangle\,|_{t=0} - \Delta m^2_{21}|U_{\sigma 2}|^2 - \Delta m^2_{31}|U_{\sigma 3}|^2 \qquad \text{Normal,}$$

$$m_3^2 = -\frac{1}{4}\frac{d^2}{dt^2}\langle\sigma|\,L^M(t)\,|\sigma\rangle\,|_{t=0} - \Delta m^2_{13}|U_{\sigma 1}|^2 - \Delta m^2_{23}|U_{\sigma 2}|^2 \qquad \text{Inverted.}$$

エレクトロン数（電子ファミリー数）の期待値の2階時間微分と有効マヨラナ質量$|m_{vee}|$の関係

Relation bet. 2nd derivative of electron number and effective Majorana mass

$$\frac{d^2}{dt^2}\langle e|\, L_e^M(t)\,|e\rangle\,|_{t=0} = \frac{1}{2}\frac{d^2}{dt^2}\langle e|\, L^M(t)\,|e\rangle\,|_{t=0} - 2\sum_{i=1}^{3} m_i^2|U_{ei}|^4 - 4m_1m_2|U_{e1}U_{e2}^*|^2\cos(2\beta_1)$$

$$- 4m_2m_3|U_{e2}U_{e3}^*|^2\cos(2\beta_1 - 2\gamma_1) - 4m_3m_1|U_{e1}U_{e3}^*|^2\cos(2\gamma_1). \quad (32)$$

$$= \frac{1}{2}\frac{d^2}{dt^2}\langle \sigma|\, L^M(t)\,|\sigma\rangle\,|_{t=0} \; -2|m_{vee}|^2$$

$$|m_{vee}|^2 = |U_{e1}|^4 m_1^2 + |U_{e2}|^4 m_2^2 + |U_{e3}|^4 m_3^2$$
$$+2\,|U_{e1}U_{e2}|^2\, m_1m_2\cos(2\beta_1)$$
$$+2\,|U_{e2}U_{e3}|^2\, m_2m_3\,\cos\big(2(\beta_1-\gamma_1)\big)$$
$$+2\,|U_{e1}U_{e3}|^2\, m_1m_3\,\cos(2\gamma_1)$$

To determine β_1 γ_1 two Majorana type phases we need additional info. e.g., muon number

$$\frac{d^2}{dt^2}\langle e|\, L_\mu^M(t)\,|e\rangle\,|_{t=0} = -2\sum_i^3 |U_{\mu i}|^2|U_{ei}|^2 m_i^2 - 4\left|U_{\mu 1}^* U_{\mu 2} U_{e1}^* U_{e2}\right| \cos(\beta_1+\beta_2) m_1 m_2$$
$$- 4\left|U_{\mu 2}^* U_{\mu 3} U_{e2}^* U_{e3}\right| \cos(\beta_1-\gamma_1+\beta_2-\gamma_2) m_2 m_3$$
$$- 4\left|U_{\mu 3}^* U_{\mu 1} U_{e3}^* U_{e1}\right| \cos(\gamma_1+\gamma_2) m_3 m_1,$$

$$= -2\left|\boldsymbol{m_{\nu\, e\mu}}\right|^2$$

$$\left|\boldsymbol{m_{\nu\, e\mu}}\right| = \left|\boldsymbol{U_{e1} m_1 U_{\mu 1} + U_{e2} m_2 U_{\mu 2} + U_{e3} m_3 U_{\mu 3}}\right|$$

Numerical Analysis :two generation case

- $\begin{pmatrix} \nu_{L\alpha} \\ l_{L\alpha} \end{pmatrix} = \left\{ \begin{pmatrix} \nu_{Le} \\ l_{Le} \end{pmatrix}, \begin{pmatrix} \nu_{L\mu} \\ l_{L\mu} \end{pmatrix} \right\}$, $\quad m_{\nu\alpha\beta} = \begin{pmatrix} m_{\nu ee} & m_{\nu e\mu} \\ m_{\nu e\mu} & m_{\nu\mu\mu} \end{pmatrix}$
- $U_{\alpha i} = \begin{pmatrix} c_{12} & s_{12} e^{i\frac{\alpha_{21}}{2}} \\ -s_{12} & c_{12} e^{i\frac{\alpha_{21}}{2}} \end{pmatrix}$ $s_{12} = \sin\theta_{12}$ (we take $\theta_{12} = \frac{\pi}{4}$)
- $\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.42 \times 10^{-5} (eV^2)$
- The remaining parameters are the lightest neutrino mass m_1 and a Majorana phase $\alpha_{21} = -2\beta_1$. ($\beta_2 = \beta_1 + \pi$)

Time evolution of lepton number(2 gene. Case)

$$
\begin{aligned}
\langle \nu_e | L_e(t) | \nu_e \rangle = {} & c_{12}^4 \left(1 - \frac{2m_1^2 \sin^2(E_1 t)}{E_1^2}\right) + s_{12}^4 \left(1 - \frac{2m_2^2 \sin^2(E_2 t)}{E_2^2}\right) \\
& + s_{12}^2 c_{12}^2 \left\{ \left(1 + \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 - E_2)t\} \right. \\
& \left. + \left(1 - \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 + E_2)t\} \right\}
\end{aligned}
$$

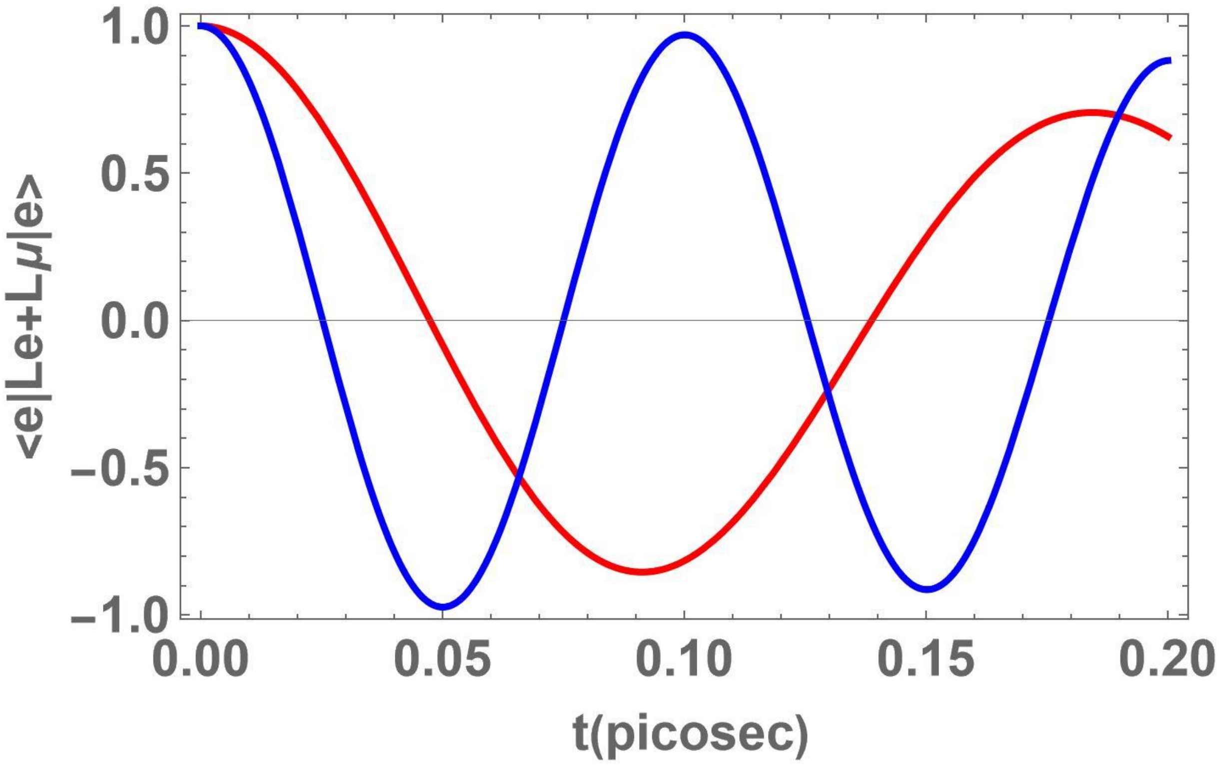
$$
\begin{aligned}
\langle \nu_e | L_\mu(t) | \nu_e \rangle = {} & c_{12}^2 s_{12}^2 \left(\left(1 - \frac{2m_1^2 \sin^2(E_1 t)}{E_1^2}\right) + \left(1 - \frac{2m_2^2 \sin^2(E_2 t)}{E_2^2}\right) \right) \\
& - s_{12}^2 c_{12}^2 \left\{ \left(1 + \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 - E_2)t\} \right. \\
& \left. + \left(1 - \frac{q^2 - m_1 m_2 \cos(\alpha_{21})}{E_1 E_2}\right) \cos\{(E_1 + E_2)t\} \right\}.
\end{aligned}
$$

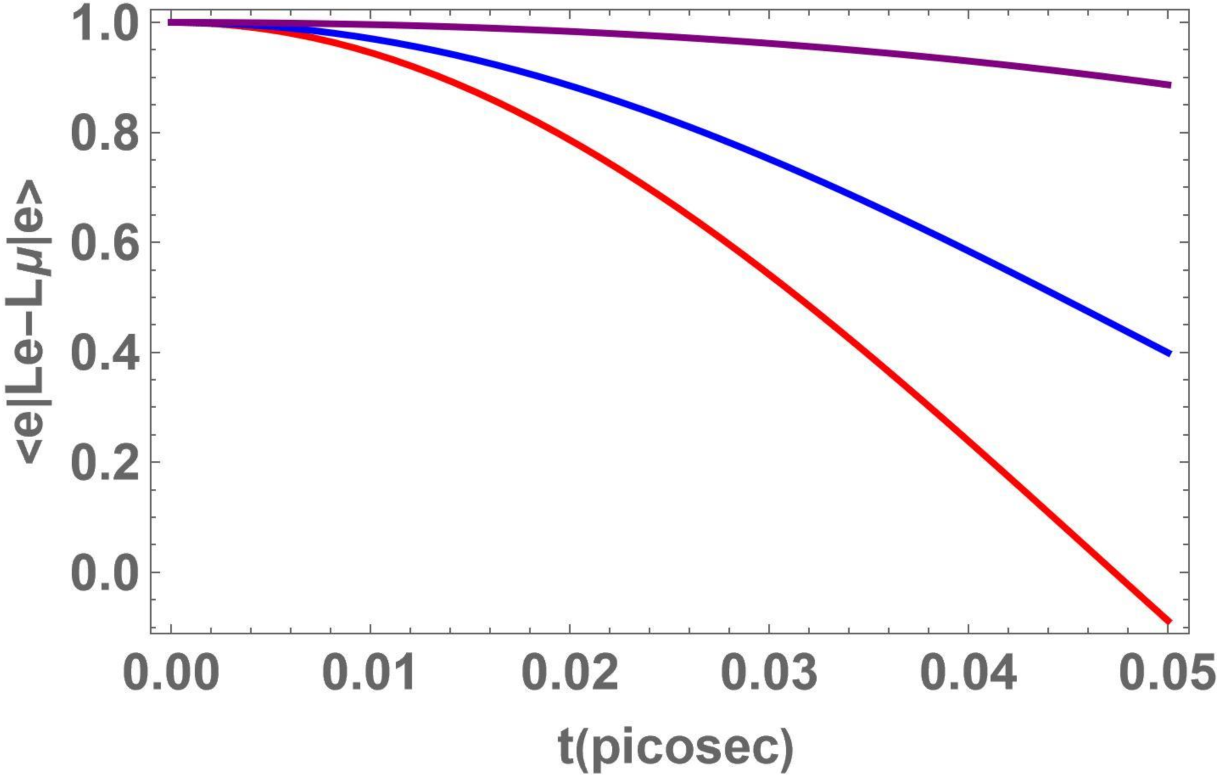
Total Lepton Number

$$
\langle e | L_e(t) + L_\mu(t) | e \rangle
$$
$$
= \langle \nu_e | L_e(t) | \nu_e \rangle + \langle \nu_e | L_\mu(t) | \nu_e \rangle
$$

electron-muon number

$$
\langle e | L_{e-\mu}(t) | e \rangle
$$
$$
= \langle \nu_e | L_e(t) | \nu_e \rangle - \langle \nu_e | L_\mu(t) | \nu_e \rangle
$$





Red : m_1=**0.01(eV)** **Blue** : m_1=**0.02(eV)**

Red **:** $\boldsymbol{\alpha_{21}=0}$ **Blue** : $\boldsymbol{\alpha_{21}=\frac{\pi}{2}}$ **Purple**: $\boldsymbol{\alpha_{21}=\pi}$

$$\frac{d^2}{\mathrm{d}t^2}\langle\nu_e|L(t)|\nu_e\rangle\Big|_{t=0} = -4(m_1^2c_{12}^2 + m_2^2s_{12}^2)$$

$$\frac{d^2}{\mathrm{d}t^2}\langle\nu_e|L_{e-\mu}(t)|\nu_e\rangle\Big|_{t=0} = -4|m_{ee}|^2$$

$$|m_{ee}|^2 = |m_1c_{12}^2 + m_2s_{12}^2e^{-i\alpha_{21}}|^2 = m_1^2c_{12}^4 + m_2^2s_{12}^4 + 2m_1m_2c_{12}^2s_{12}^2\cos(\alpha_{21})$$

結論

- マヨラナニュートリノの場合の**Unitary 3**角形は３角形の配向（各辺の実軸からの回転角）に物理的な意味がある。
  配向はマヨラナ型位相を図ることできまる。

- マヨラナニュートリノのレプトン数の時間発展はニュートリノが非相対論的なとき最小ニュートリノ質量やマヨラナ型位相に依存する。
  Mee も同様の依存性を持つ。

- ニュートリノのフレーバーが同定されたときの
  レプトン数の初期時間についての２回微分を用いて
  最小ニュートリノ質量やマヨラナ型位相を決める方法を
  説明した。