Necessity of experiments to theory of double-β decay

— to improve reliability of calculation of nuclear matrix elements

J. Terasaki, *Inst. of Exp. and Applied Phys., Czech Technical Univ. in Prague*.

1. Phenomenological improvement of 0νββ nuclear matrix element (NME) of the shell model and QRPA.
2. Spectrum of ^{136}Cs
3. Higher-order term of 2νββ NME, $M^{2\nu}_{\mathrm{GT\text{-}3}}$
4. Discrepancy problem of running sum of 2νββ NME of ^{136}Xe⟶^{136}Ba
5. Contribution of Δ resonance to ββ-decay NME
6. Summary

Osaka, Oct. 3, 2022

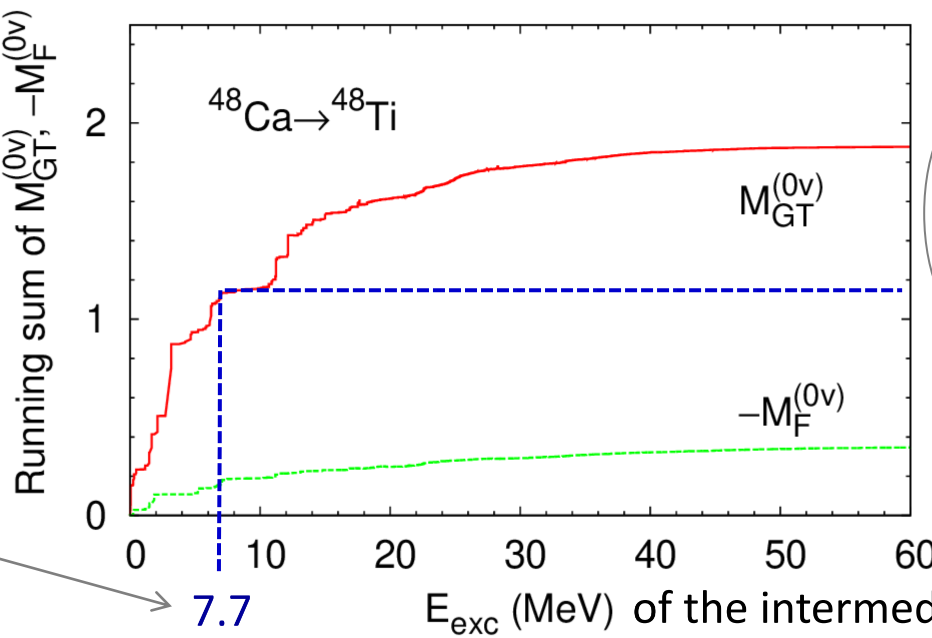
1. Phenomenological improvement of 0νββ nuclear matrix element (NME) of the shell model and QRPA

J. T. and Y. Iwata, Eur. Phys. J. Plus, **136**, 908 (2021)

Modification of the SM result

$$M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, modified}) = M_{\mathrm{GT}}^{(0\nu)}(\mathrm{SM, 1\text{-}maj.\,val.\,sh.}) \times \frac{S_2}{S_1}$$

QRPA



of the intermediate state (^{48}Sc)

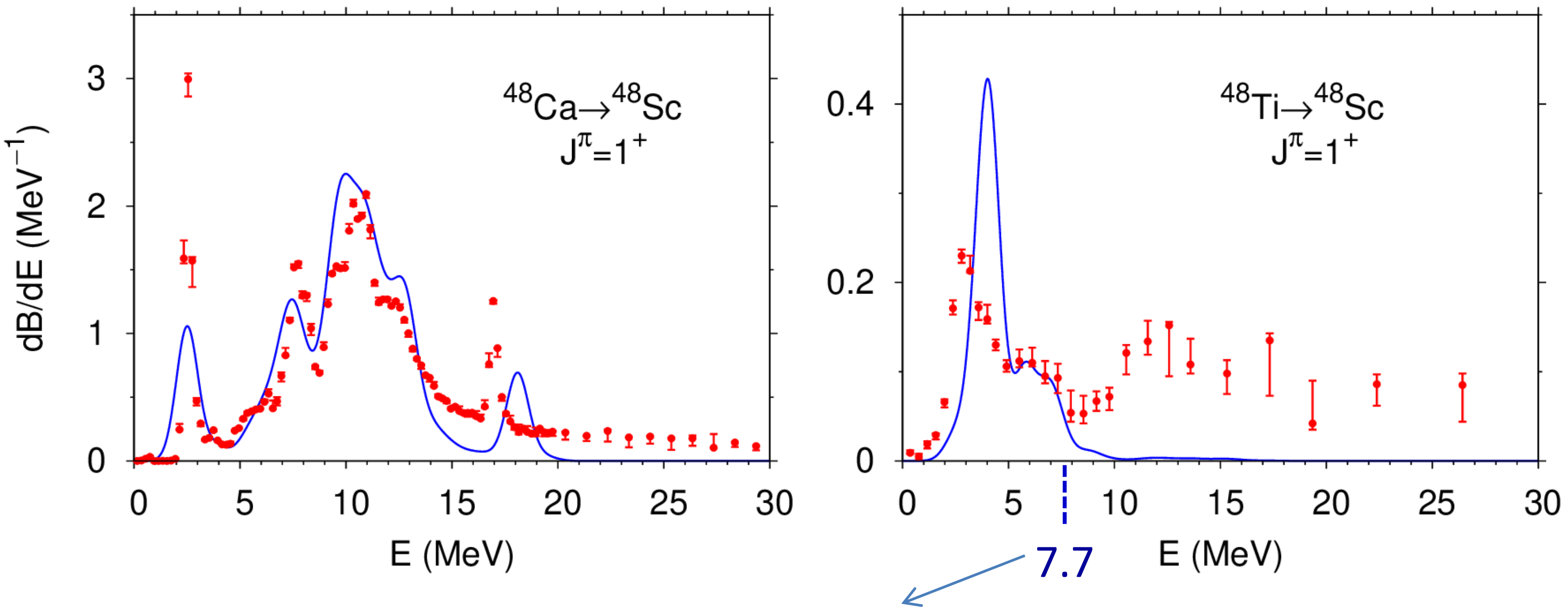
S_2

S_1

← QRPA

Corresponding to 1-maj. val. sh.; see next figure.

Exp. data of charge-change 1^+ strength function (red symbols, isovector spin monopole comp. possibly included) and SM GT strength function using 1-major valence shell (blue line)



Upper limit of E_{exc} of the intermediate states that the SM (1-maj. val. sh.) can supply for ββ NME.

Exp. data: K. Yako et al, Phys. Rev. Lett. **103**, 012503 (2009)

Evaluation of possible region of modified QRPA result

Anticoherent limit

$M_{\mathrm{GT}}^{(0\nu)}(\mathrm{QRPA})$

$+$ **Reduction in low-E rgn.**

$-$ **enhancement in high-E rgn.**

$\leq$ $M_{\mathrm{GT}}^{(0\nu)}(\mathrm{QRPA,\ modified})$ $\leq$

Coherent limit

$M_{\mathrm{GT}}^{(0\nu)}(\mathrm{QRPA})$

$+$ **Reduction in low-E rgn.**

$+$ **enhancement in high-E rgn.**

$$-\left(1-\sqrt{\frac{\text{shifted str.} + \text{original}}{\text{original str.}}}\right) M_{\mathrm{GT}}^{(0\nu)}(\mathrm{QRPA,\ high}\ E)$$

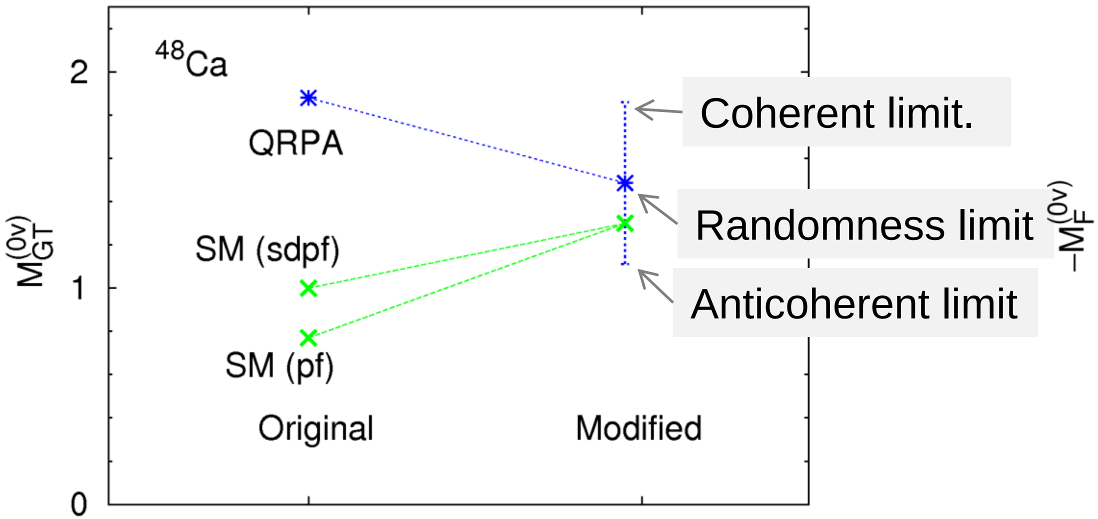
1.27 from $n \rightarrow p$; 1.44 from $p \rightarrow n$

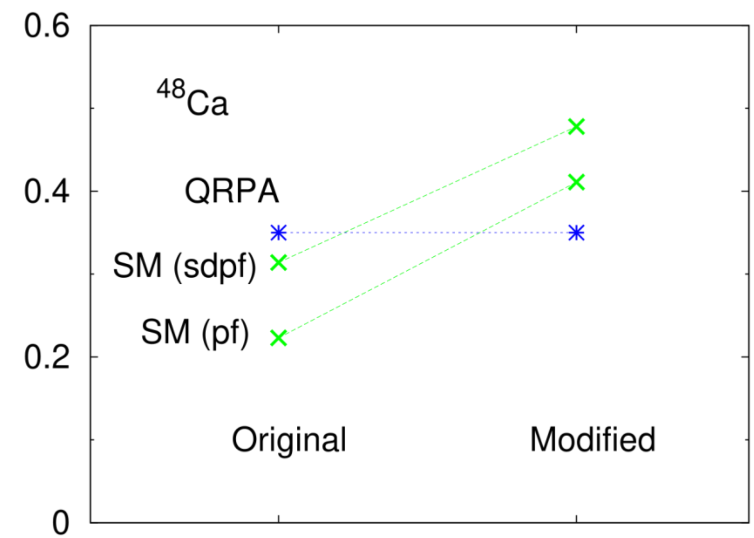
0.92 from $n \rightarrow p$

0.80 from $p \rightarrow n$

$$-(1-R_q) M_{\mathrm{GT}}^{(0\nu)}(\mathrm{QRPA,\ low}\ E)$$

Modified 0νββ NME of ^{48}Ca





My speculation

The randomness limit is closer to the true value than the coherent and anticoherent limits.

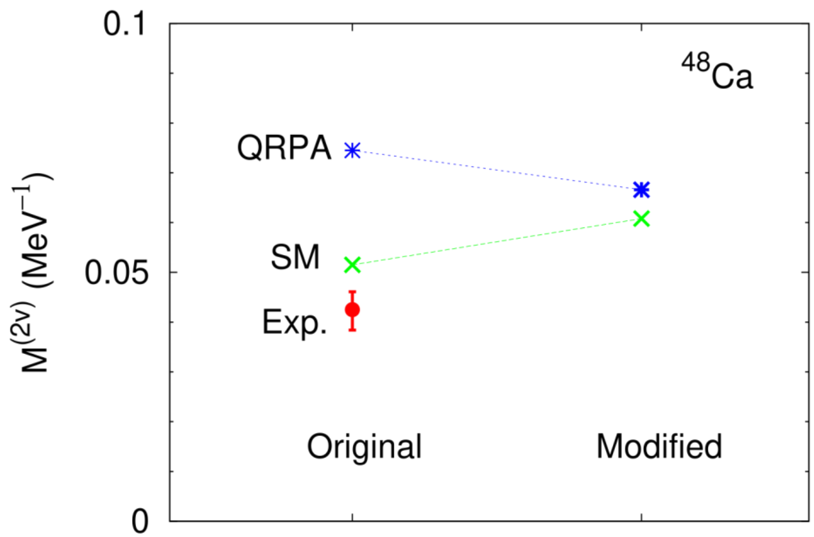
Sign of correction terms unknown

No modification for QRPA

- Fermi transition
- Coordinate operators do not cause quenching

2vββ NME of ^{48}Ca

Quenching factors included in the original calculations.

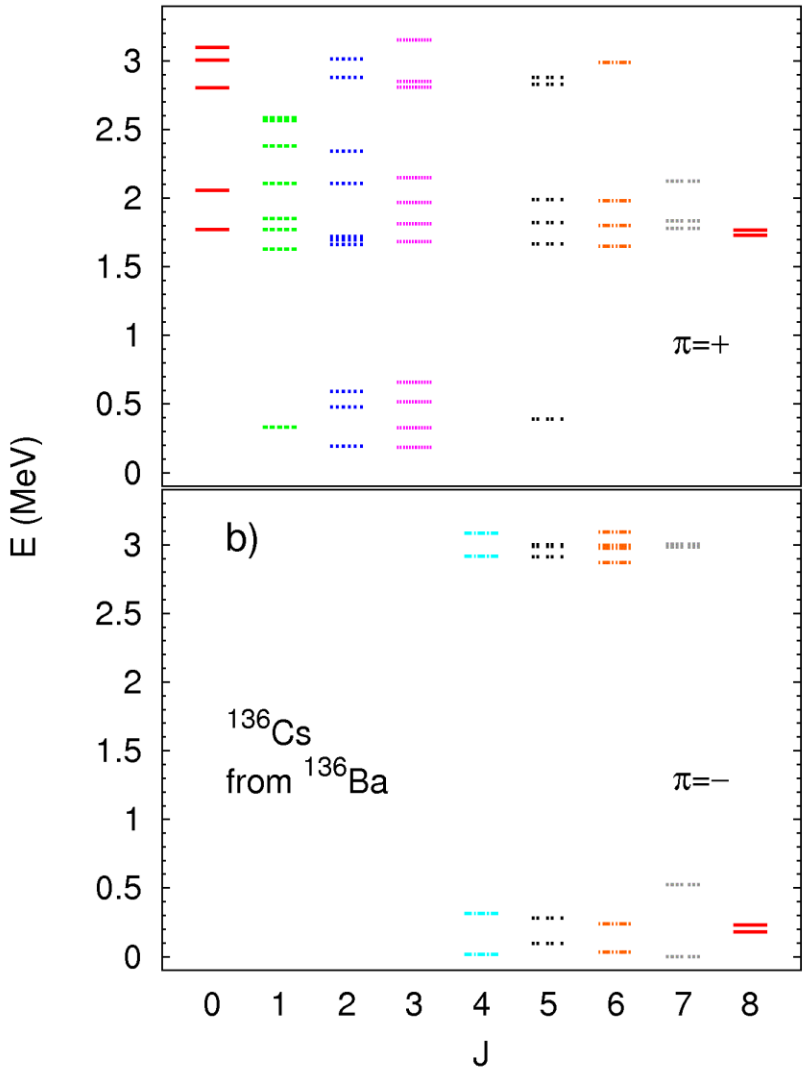


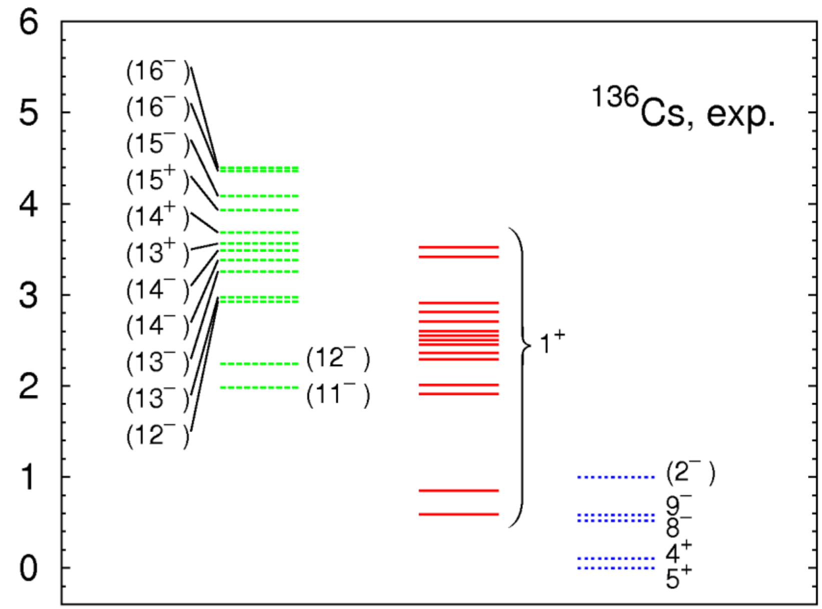
The increase in $M^{(2\nu)}$ ← Giant resonance

Exp. $M^{(2\nu)}$ ← Exp. half-life (A.S. Barabash, MEDEX2019 Proc.) and $G_{2\nu}$ (J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998))

Spectrum of intermediate nucleus

$^{136}Xe \rightarrow {}^{136}Cs \rightarrow {}^{136}Ba$





www.nndc.bnl.gov

J. T. PRC **100**, 034325 (2019)

3. Higher-order term of $2\nu\beta\beta$ NME

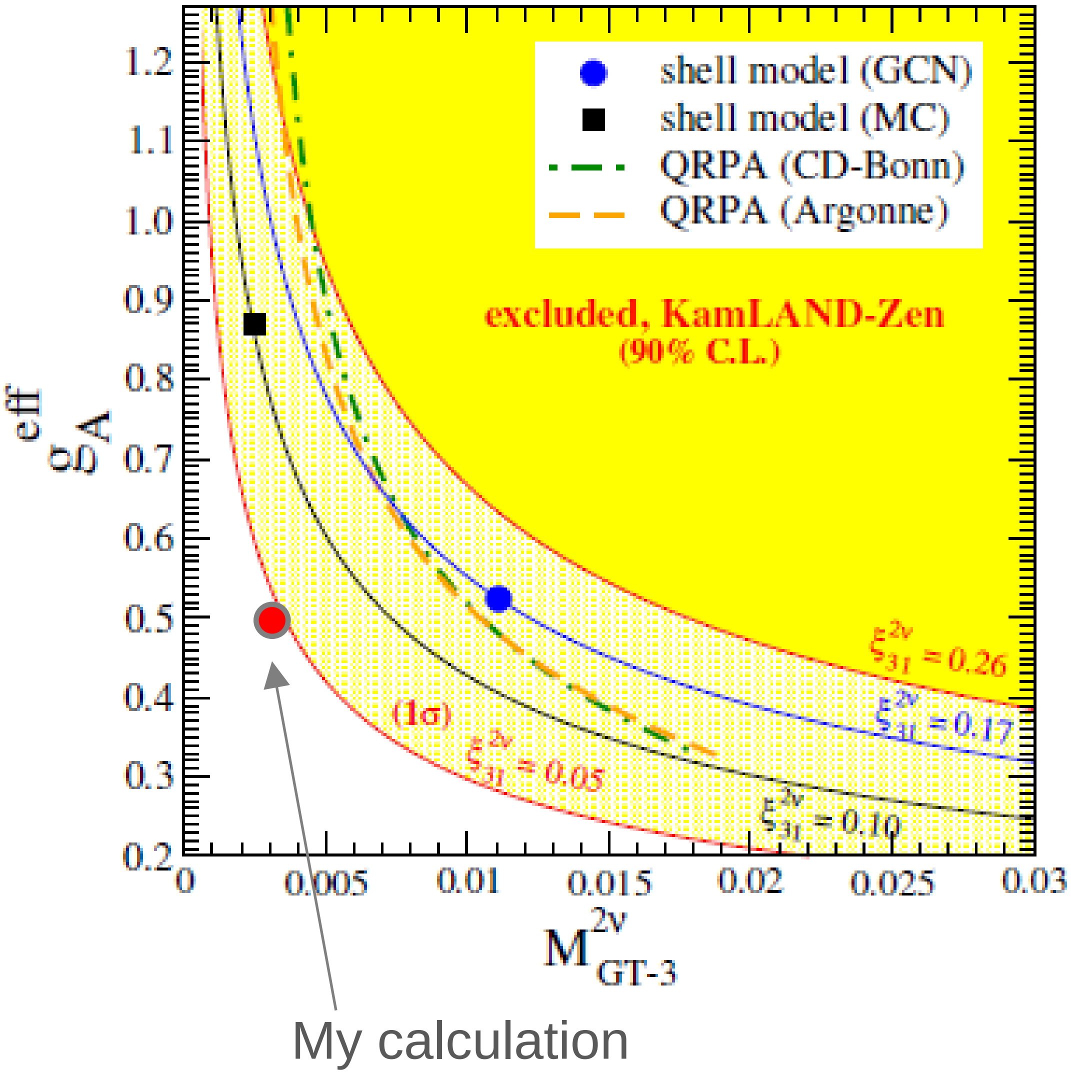
$$M^{2\nu}_{\text{GT-3}} = \sum_j \frac{4}{\{E_j - (E_i + E_f)/2\}^3} \langle 0_f^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 1_j^+ \rangle \times \langle 1_j^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 0_i^+ \rangle.$$

F. Šimkovic *et al.*, PRC **97**, 034315 (2018)

E_j: intermediate-state energy; E_f: final-state energy,
E_i: initial-state energy

$$M^{2\nu}_{\text{GT}} = \sum_j \frac{1}{E_j - (E_i + E_f)/2} \langle 0_f^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 1_j^+ \rangle \times \langle 1_j^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 0_i^+ \rangle.$$

Higher-order term of $2\nu\beta\beta$ NME



Statistical analysis with the exp. data of ββ spectrum of ^{136}Xe

A. Gando et al., PRL **122**, 192501 (2019)
+ my cal.

$$\xi_{31}^{2\nu} = M_{\mathrm{GT}}^{2\nu}/M_{\mathrm{GT\text{-}3}}^{2\nu}$$

4. Discrepancy problem of running sum of 2νββ NME of $^{136}Xe \rightarrow ^{136}Ba$

Sequential accumulation of the components of 2νββ NME w.r.t. exc. energy of the intermediate nucleus.

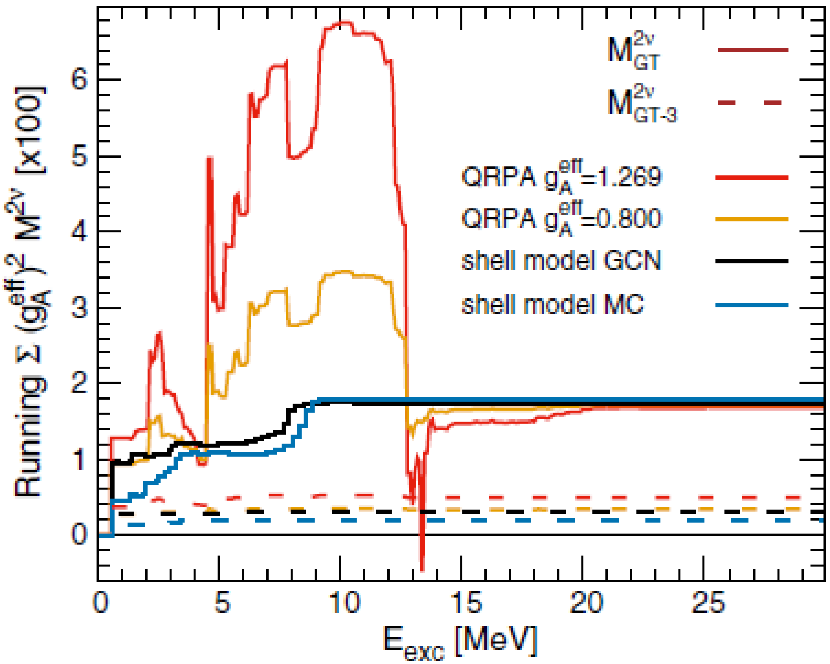
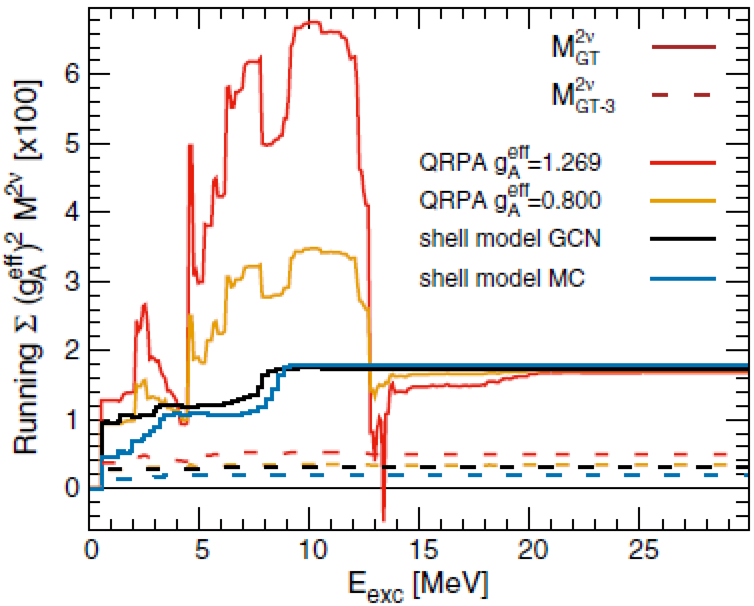


FIG. 4. Running sum of the ^{136}Xe $M^{2\nu}_{GT}$ (solid lines) and $M^{2\nu}_{GT-3}$ (dashed) $2\nu\beta\beta$ NMEs, as a function of the excitation energy of the 1^+ states in ^{136}Cs. Nuclear shell model results with the GCN (MC) interaction, indicated by black (blue) lines, are compared to the QRPA Argonne running sum with $g_A^{eff} = 1.269$ ($g_A^{eff} = 0.80$), shown by red (orange) lines.

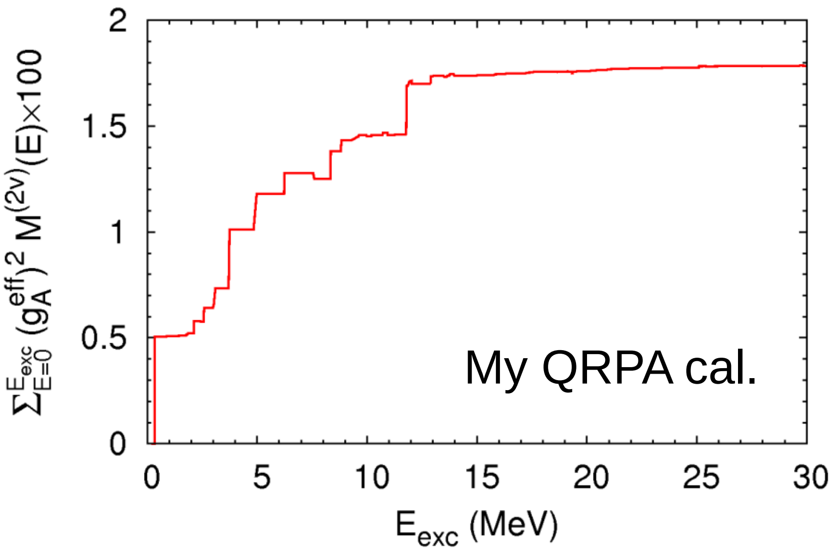
A. Gando et al., PRL **122**, 192501 (2019)

QRPA ← Šimkovic et al.
SM ← Menéndez et al.

Problem:
Significant difference in the running sum of two calculations



A. Gando et al., PRL **122**, 192501 (2019)



$g_A^{\text{eff}} = 0.49 \leftarrow$ Exp. half-life

No adjustment of interaction

$n(0^+)$	$n(1^+)$	$M^{2\nu}$	Ikeda
0	0	0.062	52
0	1	0.091	84
1	1	0.037	84
1	2	0.020	84

M. Horoi and B.A. Brown, PRL **110**, 222502 (2013)

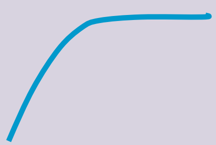
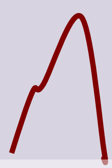
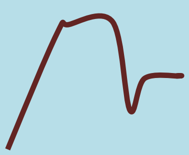
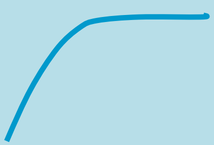
SM, *sd*+*g*7/2+*h*11/2+*g*9/2+*h*9/2

n: number of particles excited from *g*9/2 or to *h*9/2

$n(0^+)$: *n* for the initial and final 0^+ states

$n(1^+)$: *n* for the intermediate 1^+ states

Differences of the calculations of running sum of 2νββ NME of $^{136}Xe \rightarrow {}^{136}Ba$

Cal.	SM (Menéndez)	SM (Horoi)	QRPA (Šimkovic)	QRPA (Terasaki)
S.p. space	$2s1d+0g_{7/2}$ $+0h_{11/2}$	$2s1d+0g_{7/2}$ $+0h_{11/2}$	$6\hbar\omega+0h_{13/2}$ $+0h_{11/2}$	3800-4500 states (M scheme)
Interaction (particle-hole)	GCN, MC	N3LO potential+ …	G matrixes of CD-Bonn and Argonne V18	Skyrme, SkM*
Running sum				

My speculation based on their result

- The cause is not SM or QRPA. Something else.

Do experiments solve this problem?

$$M_{\mathrm{GT}}^{2\nu} = \sum_j \frac{1}{E_j - (E_i + E_f)/2} \langle 0_f^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 1_j^+ \rangle \cdot \langle 1_j^+ | \sum_l (\boldsymbol{\sigma}\tau^-)_l | 0_i^+ \rangle.$$

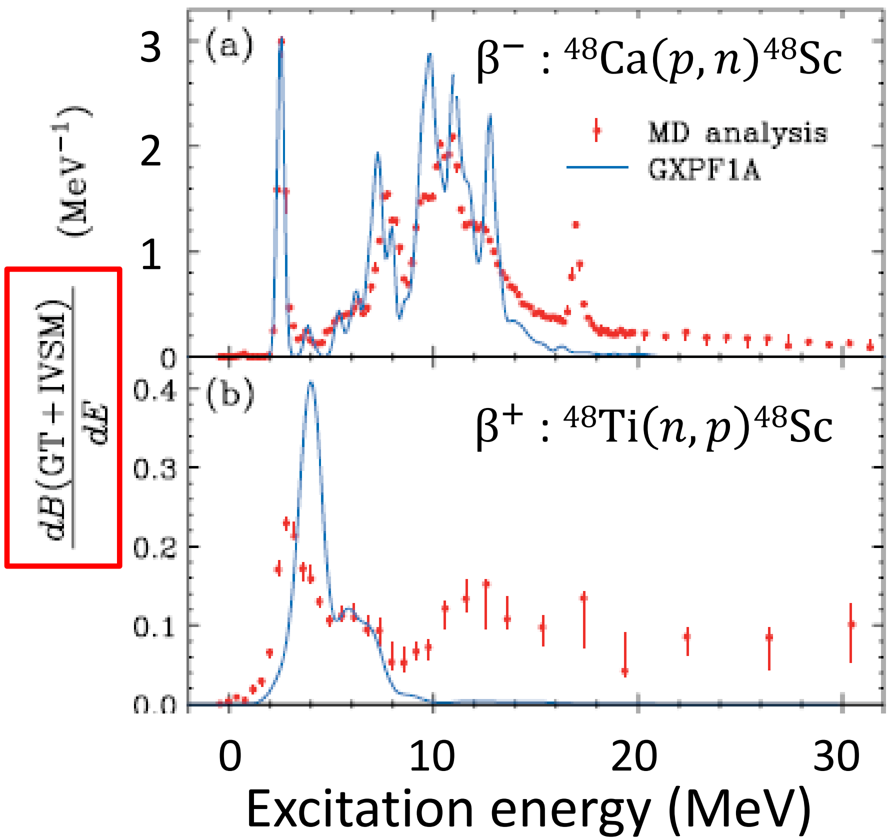
E_j: intermediate-state energy; E_f: final-state energy,
E_i: initial-state energy

If exp. GT strength functions of ^{136}Xe→^{136}Cs and ^{136}Ba→^{136}Cs are available up to 15 MeV, it is possible to judge which behavior of running sum is closer to the truth.

In my opinion, solution by exp. is not possible because it is not possible to remove the contribution of

$$r^n \boldsymbol{\sigma} \tau^-, \qquad n \geq 1,$$

from the exp. strength function.



Isolated points with error bars: exp. data by K. Yako et al. PRL **103**, 012503 (2009)

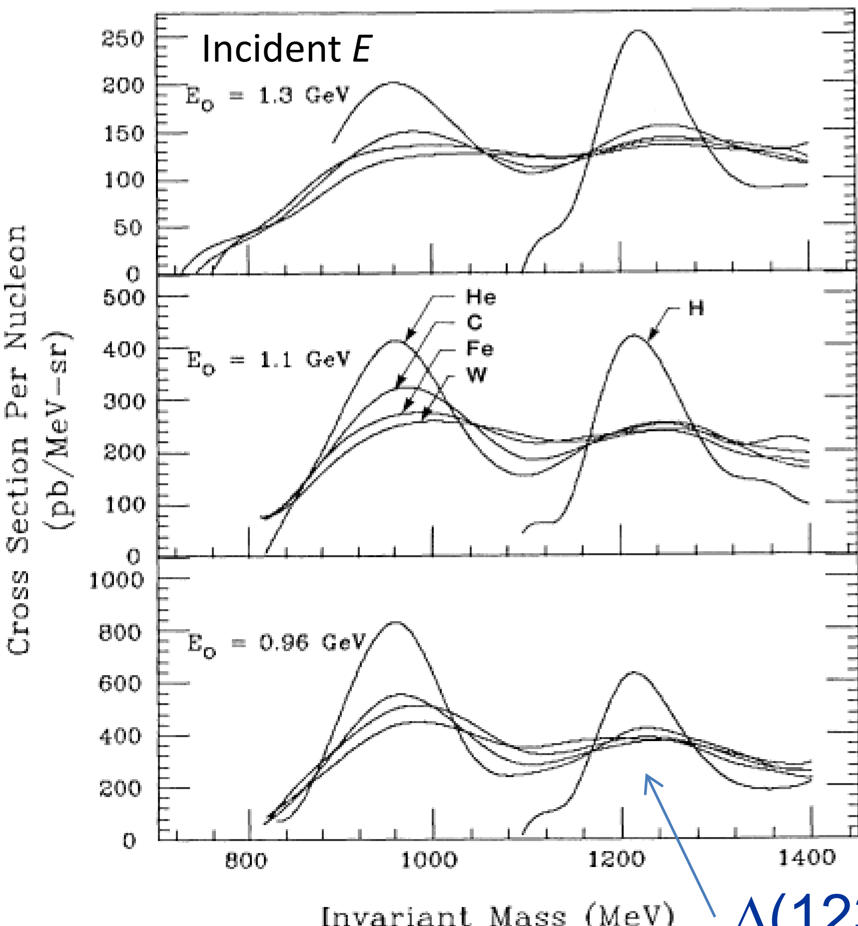
Solid line: shell-model calculation by Horoi et al.

IVSM: isovector spin monopole, *n*=2

5. Contribution of Δ resonance to ββ-decay NME

Cross section measured in inclusive *e* scattering

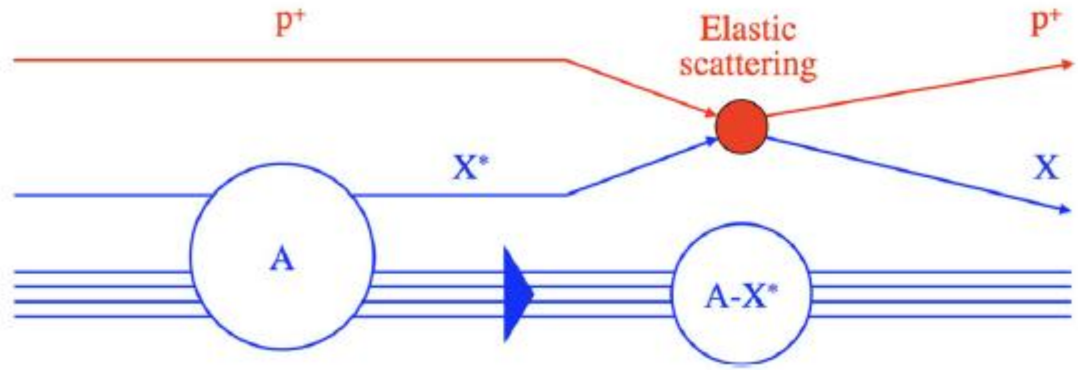
R.M. Sealock et al., PRL **62**, 1289 (1989)



Δ(1232) resonance

"*A* dependence of the cross section per nucleon for the Δ region is slight."

" Because the contributing reaction mechanisms are essentially quasifree, the cross sections roughly scale with *A*."



Schematic illustration of quasifree scattering
V. Panin et al., Eur. Phys. J. A, **57**, 103 (2021)

The final state is noncollective.

Transition strength to the g.s. is anticipated to be small.

Order estimation of cross section

$$\frac{d^2\sigma}{dEd\Omega}\frac{1}{A}(1250\text{MeV}) \times \text{width} \times 4\pi \times 184 \approx 0.00013\ \text{b}$$

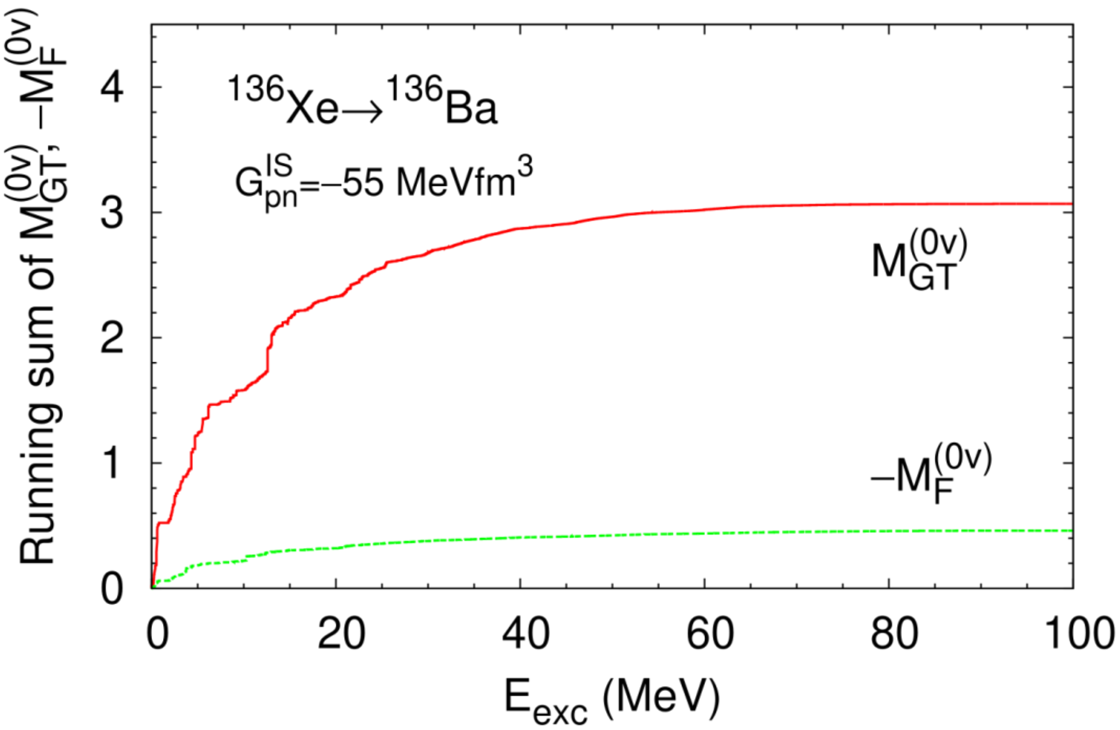
Comparison

Photoabsorption cross sec. of GR of ^{184}W ($\sim$14 MeV) $= 0.35$ b

G.M. Gurevich et al., NPA **351**, 257 (1981)

Suppose that the cross section is strongly correlated with the transition strength, and

$$\mathrm{tr.\,str.\,(GR, low}-E) \gg \mathrm{tr.\,str.\,(\Delta\,res.)}$$



Contribution of GR
≈ 0.4

The contribution of the Δ resonance to the ββ NME is evaluated to be very small.

5. Summary

List of possible impacts of experiments to theory

- Charge change strength function of GT + higher order up to 30 MeV $r^n \nearrow$

  ⇒ Phenomenological modification of SM and QRPA values of the 0νββ NME.

- Spectrum of the intermediate nucleus

  ⇒ Check of interaction

- $M^{2\nu}_{\mathrm{GT}\text{-}3}$ (higher-order term of 2νββ NME)

  ⇒ Can this be reproduced with the g_{A} for $M^{2\nu}_{\mathrm{GT}}$?.

- *Pure* GT strength function up to 15 MeV

  ⇒ Solution of the discrepancy of the running sum of 2νββ NME

- Cross section of Δ resonance

  ⇒ Anticipated contribution to ββ NME is very small.

Appendix

1. Introduction — physical implication of 0νββ decay

1. *Leptogenesis*

A scenario tries to explain the matter-antimatter asymmetry in the universe with the right-handed neutrino, which causes the breaking of the lepton number conservation.

M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986)

2. *See-saw model*

This theory assumes the right-handed neutrino to explain why the neutrino is so light ($m_{\nu e}$< 0.8 eV)

T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980)

The 0νββ decay proves that the neutrino is a Majorana neutrino, which has a right-handed component.

Neutrino mass-scale parameters

- Average of three eigen masses ⇐ Astrophysical approach and partially ν oscillation
- Expectation value of ν_e mass ⇐ KATRIN
- Effective ν mass (Majorana mass) ⇐ $0\nu\beta\beta$ decay

Average of three eigen masses

$$0.020 \text{ eV} \leq \bar{m}_\nu \leq 0.04 \text{ eV}$$

Neutrino oscillation data with $m_1 = 0$, normal ordering,
P.F. de Salas *et al.*, arXiv: 2006.11237 (2021)

Astrophysical result,
N. Aghanim *et al.*, (Planck 2018 collaboration), Astron. Astrophys. **641**, A6 (2020).

Neutrino mass-scale parameters

- Average of three eigen masses ⇐ Astrophysical approach and partially ν oscillation
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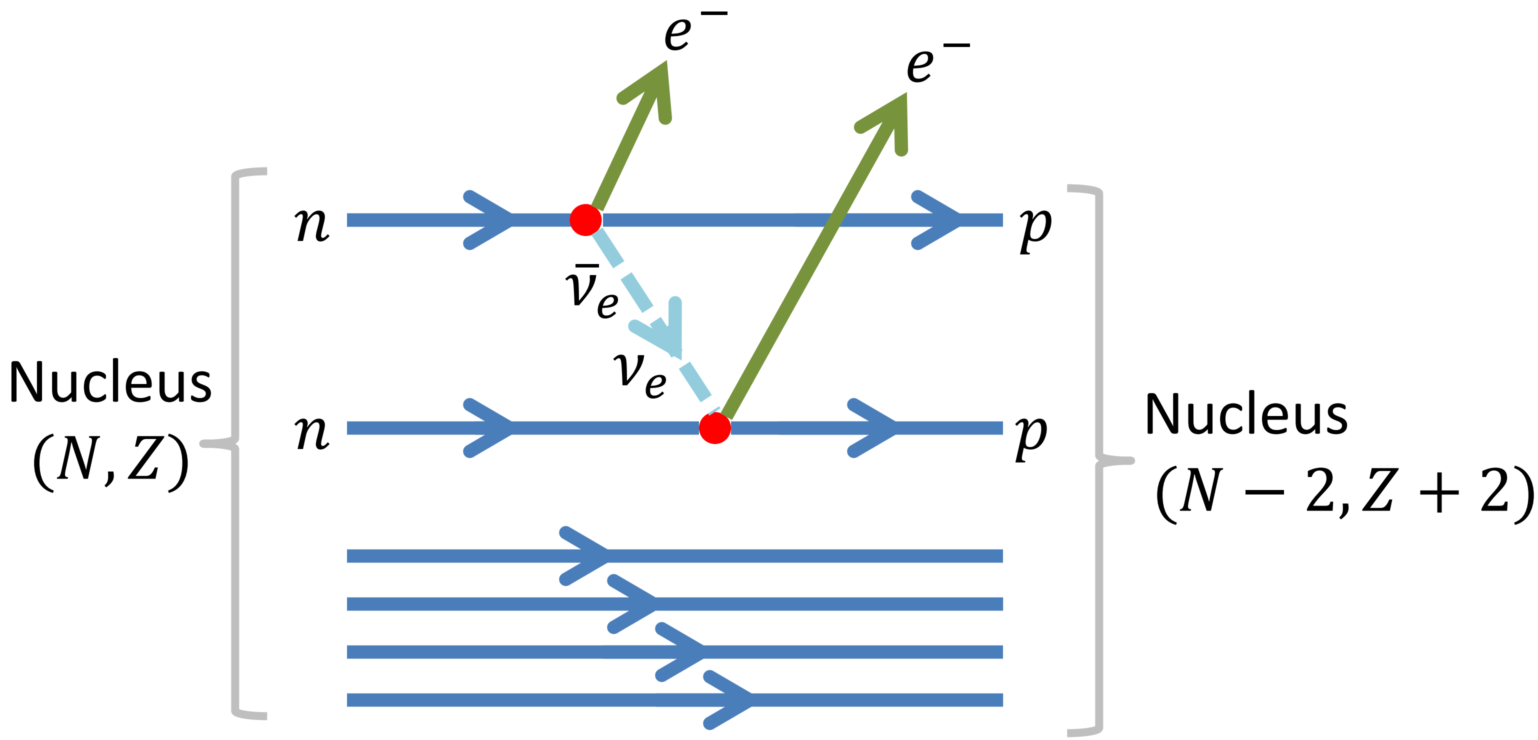
- Effective ν mass (Majorana mass) ⇐ $0\nu\beta\beta$ decay

The differences of these parameters are important.
⇒ *PMNS matrix elements*

The effective neutrino mass gives a constraint to the Majorana phase included in the PMNS matrix.

Neutrinoless double-β decay



If the neutrino is a Majorana particle ($\nu_e = \bar{\nu}_e$), this decay occurs.

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

$$1/T_{0\nu}(\mathrm{g.s.} \rightarrow \mathrm{g.s.}) = g_A^4 \left| M^{(0\nu)} \right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2$$

Half-life

Nuclear matrix element

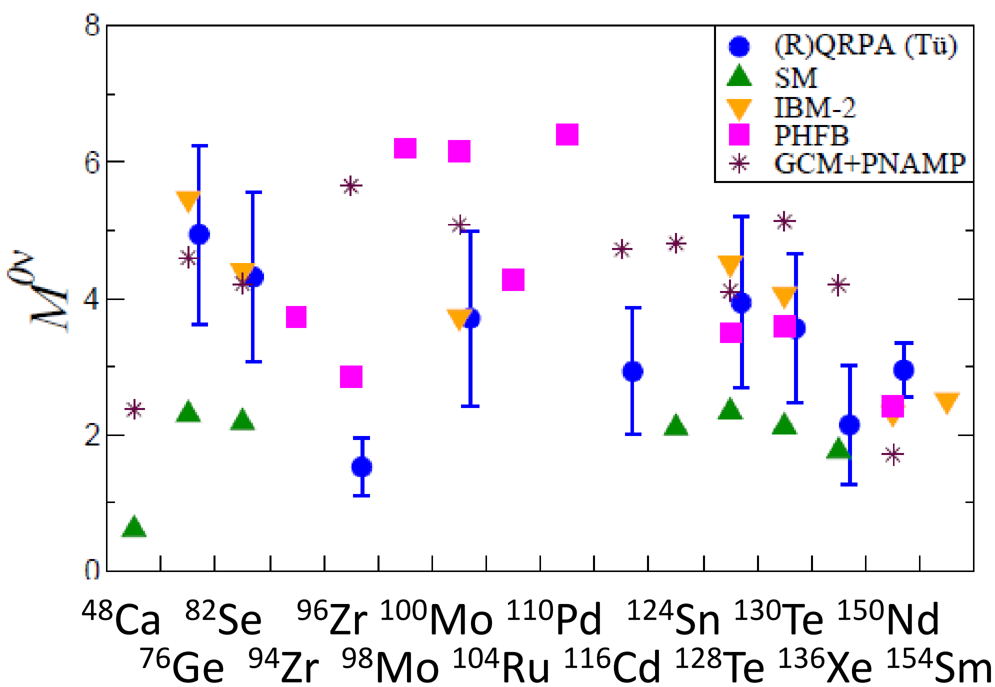
Phase -space factor

Effective ν mass

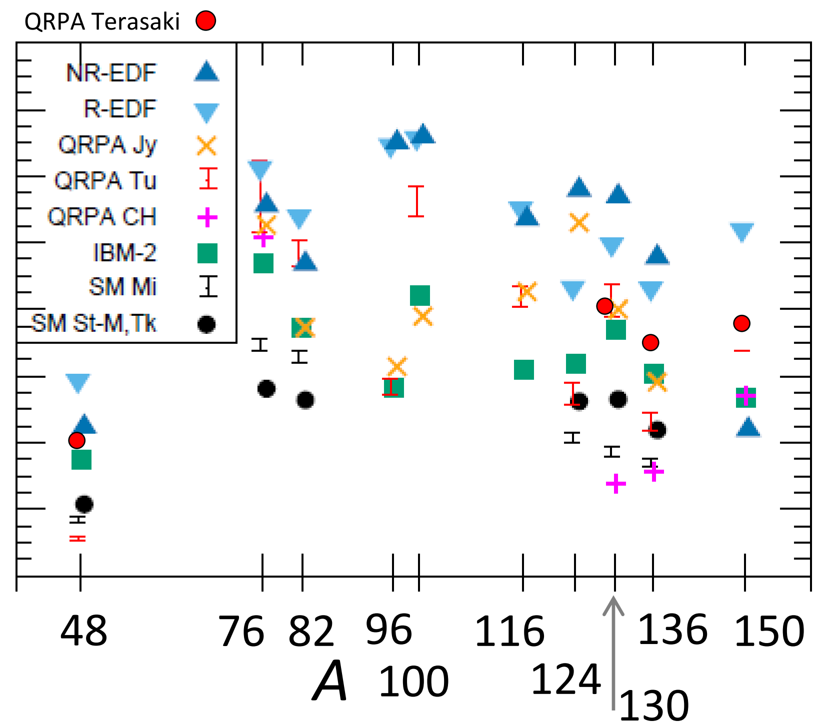
Experimental measurement

Theoretical calculation

Status of 0νββ nuclear matrix element $M^{0\nu}$



A. Feassler, J. Phys.: Conf. Ser. **337**, 012065 (2012)



J. Engel and J. Menéndez, Rep. Prog. Phys. 80 (2017) 046301
+ my cal. (g_A=1.254)

This large uncertainty of $M^{0\nu}$ is the most serious problem of the 0νββ study.

b. Modification of the QRPA result

Basic idea

Phenomenologically in a low-*E* region

Quenching factor for GT str. fun (SM) ≡ q(SM)

quenching factor for GT str. fn. (QRPA) in low-*E* region ≡ q(QRPA) = factor due to insufficient mpmh correlations x factor necessary, if exact nuclear wave fn. is available

$$R_q = \sqrt{\frac{q(\text{QRPA})}{q(\text{SM})}}$$

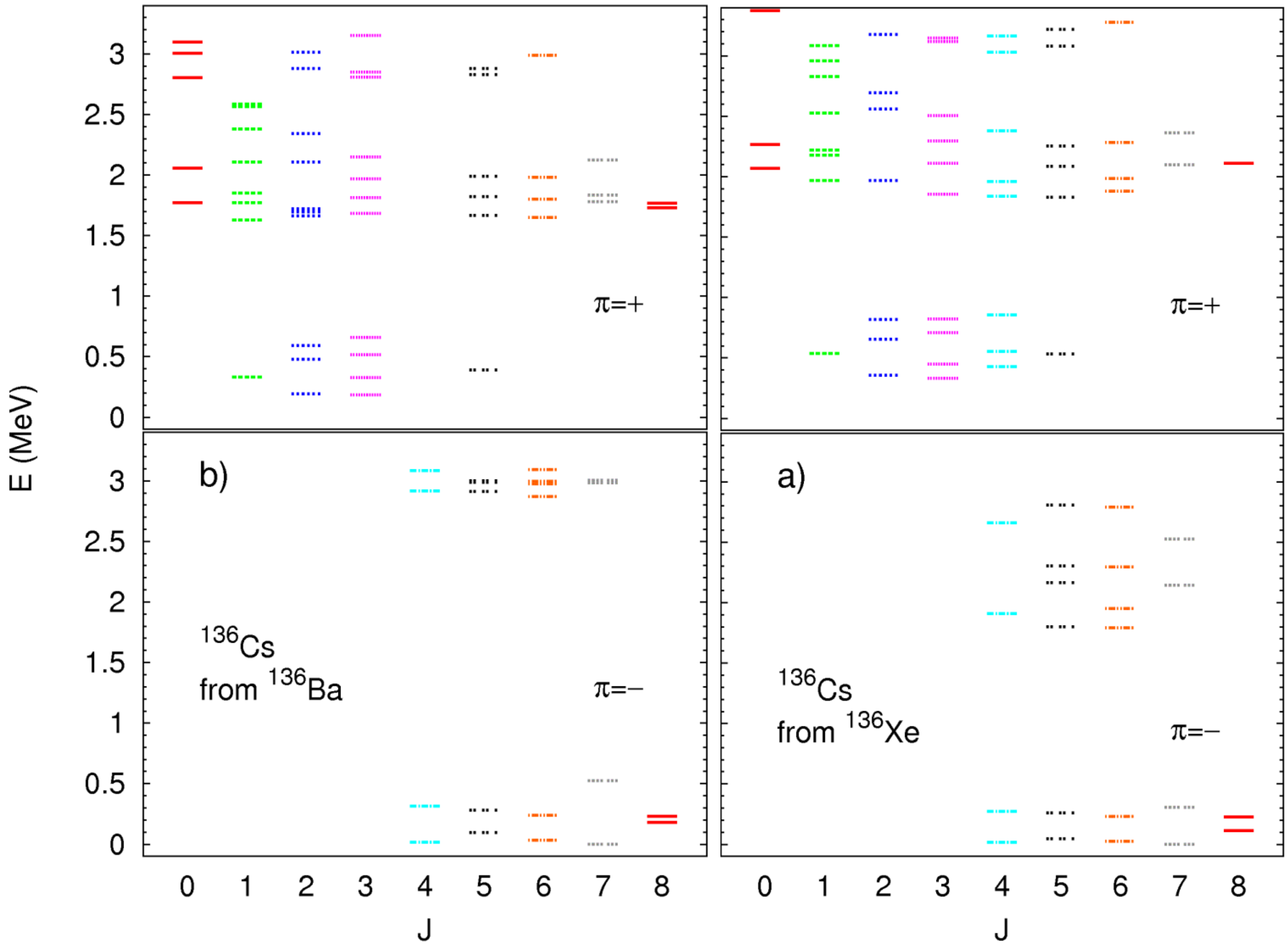
is the modification factor used perturbatively for the components of $M_{\mathrm{GT}}^{(0\nu)}$ of QRPA in the low-*E* region

Modification factor in the high-E region

This can be estimated because of the sum rule but for the uncertainty of the sign.

2. Spectrum of intermediate nucleus ^{136}Cs

^{136}Xe→^{136}Cs→^{136}Ba



J. T. PRC **100**, 034325 (2019)

0. Introduction

My idea on the methodology to obtain a reliable $0\nu\beta\beta$ NME

An experimental proof of the true NME is not possible.

My Hope

Approaching the true value limitlessly
by accumulating circumstantial evidences.

If many people think that value is probably very close to the true one, that is the goal.

Statistical approach from many different calculated values
→ Does this make sense?
→ I would take deductive approaches,
i.e., to make the machinery and input sure.

- (e,e') and (e,$e'p$) cross section up to GeV/c

  ⇒ ◦ Capability of calculation to deal with the singularity in coordinate, or a long tail in q, of v potential.

  ◦ Check of the phenomenological vertex form factor.

Neutrino mass-scale parameters

- Average of three eigen masses ⇐ Astrophysical approach and partially *ν* oscillation
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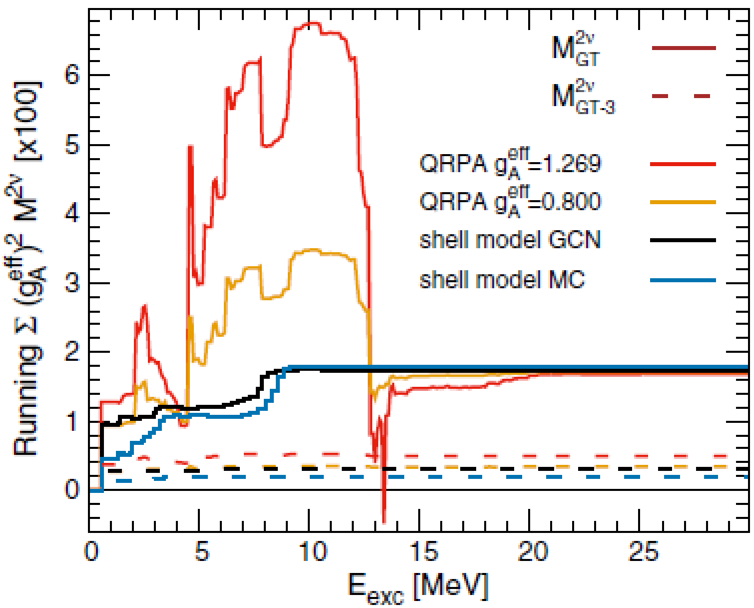
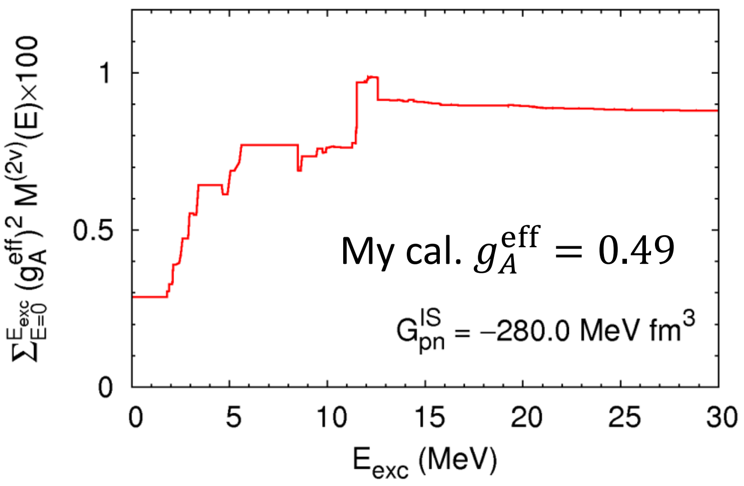
N. Aghanim *et al*., (Planck 2018 collaboration), Astron. Astrophys. **641**, A6 (2020).

Interaction strength



A. Gando et al., PRL **122**, 192501 (2019)



The strength of the isoscalar proton-neutron pairing interaction is unrealistically large.

- Strength of the *pn* isoscalar pairing interaction is not enough for explaining the difference.

Is there an explanation to these puzzles?

0. Introduction — physical implication of 0νββ decay

1. Leptogenesis

梅原、吉田、日本物理学会誌 **77**, 514 (2022)

“(宇宙の)この粒子と反粒子の量の差は，なぜあるのだろうか？この謎の解明のために，何十年も前から科学者は研究を続けてきた．この物質優勢宇宙を作り上げるシナリオはいくつか存在するが，その中で有力なシナリオの一つとされているのが，レプトン数生成（レプトジェネシス）である．このレプトジェネシスでは，物質優勢宇宙の条件の一つである「バリオン数の破れ」を，粒子数の一つである「レプトン数の破れ」で説明するものである．ニュートリノを伴わない二重ベータ（0νββ）崩壊の研究は，このレプトン数保存則の破れを，ニュートリノのマヨラナ粒子性，すなわち粒子（物質）と反粒子（反物質）の転換可能性，で検証する研究である。”

2. See-saw model needs the right-handed neutrino.