原子核殻模型による
キセノン原子核の核行列要素の理論計算

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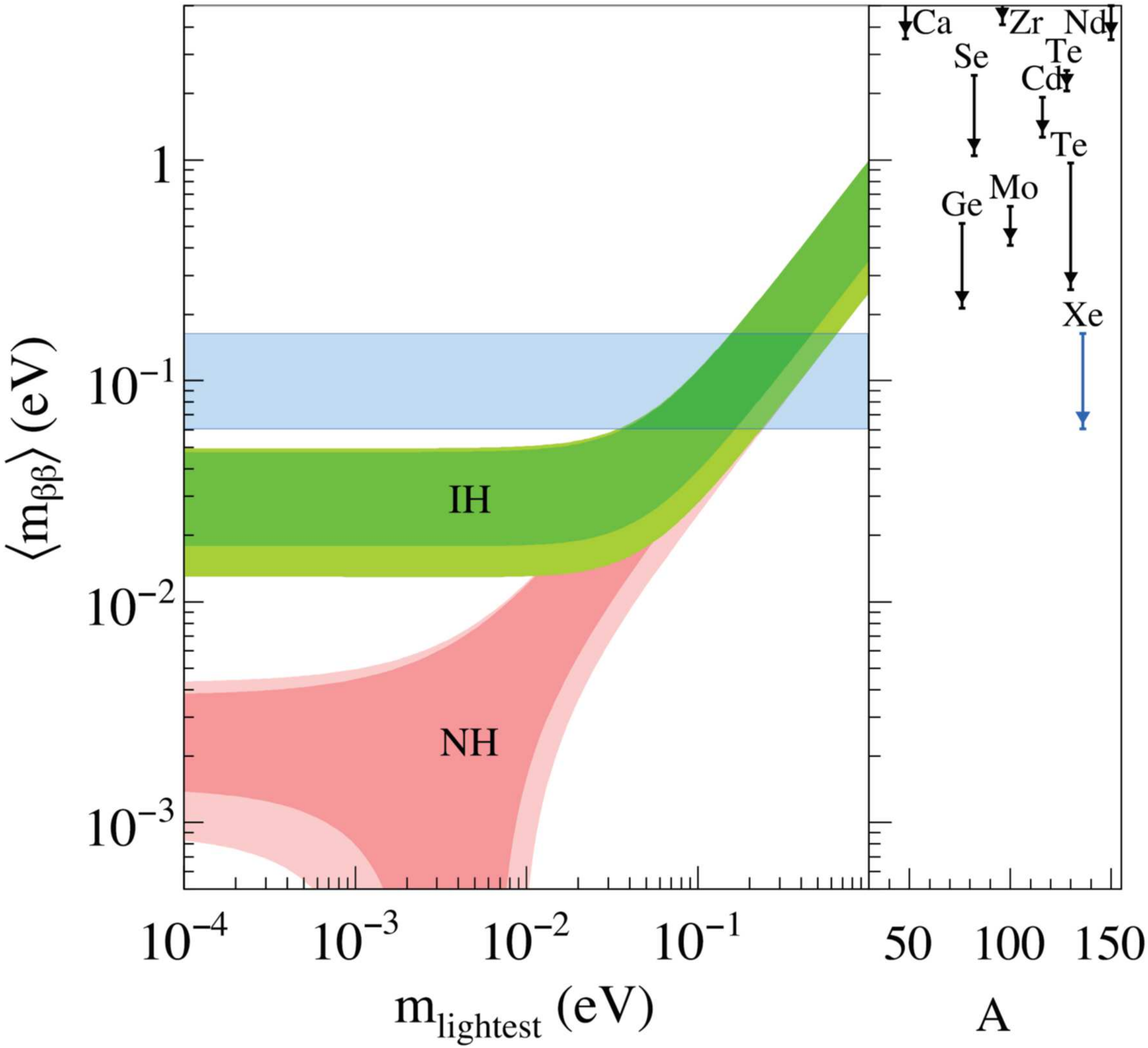
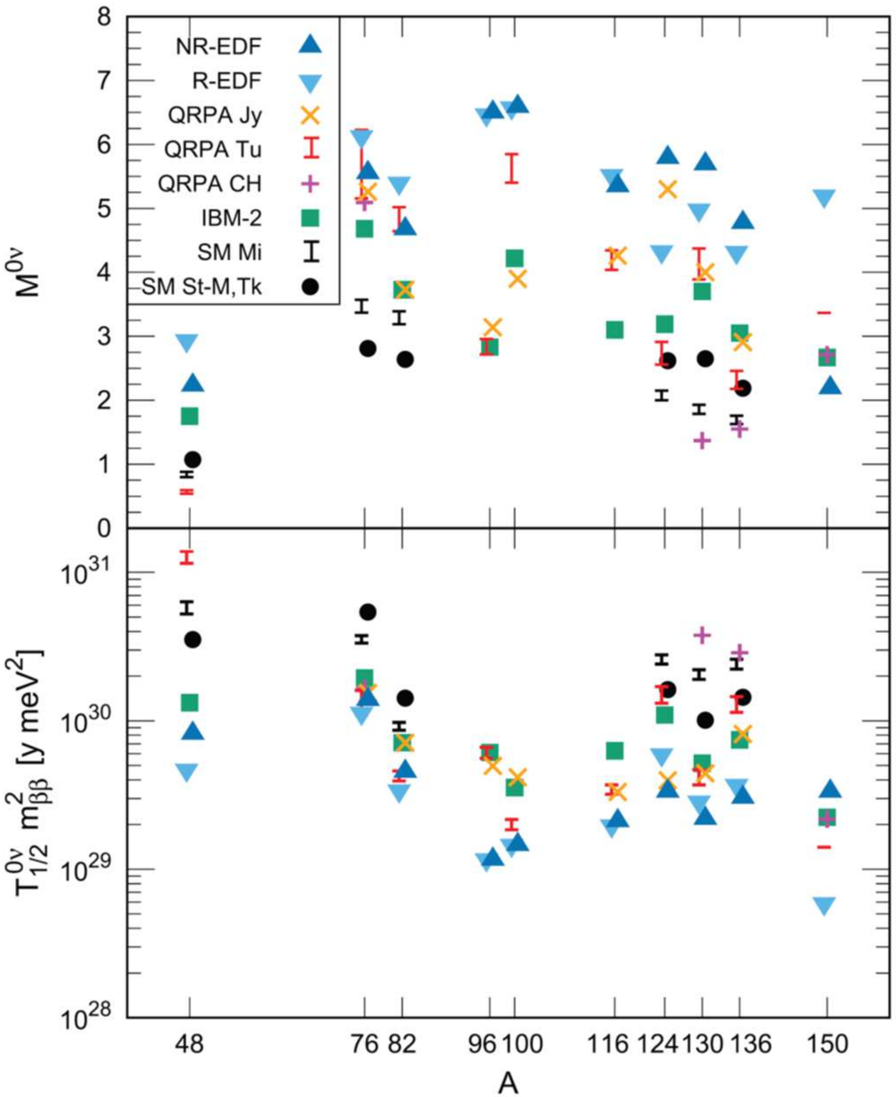


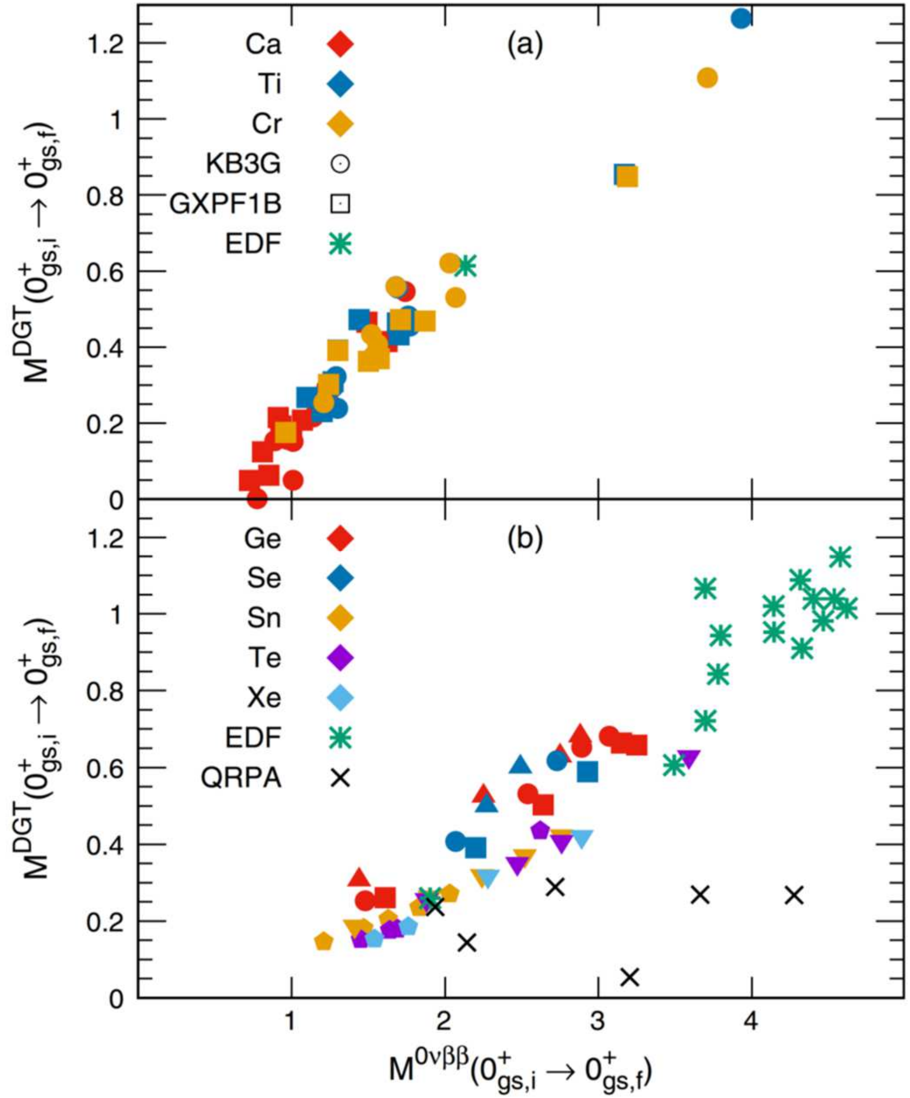
Ca
Zr
Nd
Se
Te
Cd
Te
Ge
Mo
Xe
IH
NH
1
10^{-1}
10^{-2}
10^{-3}
$\langle m_{\beta\beta} \rangle$ (eV)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$m_{lightest}$ (eV)
50
100
150
A

原子核行列要素の理論計算



J. Engel, et al., RPP 80, 046301 (2017).

二重ガモフテラー遷移



N. Shimizu, et al., PRL 120, 142502 (2018).

Nuclear matrix element

0νββ 半減期と原子核行列要素

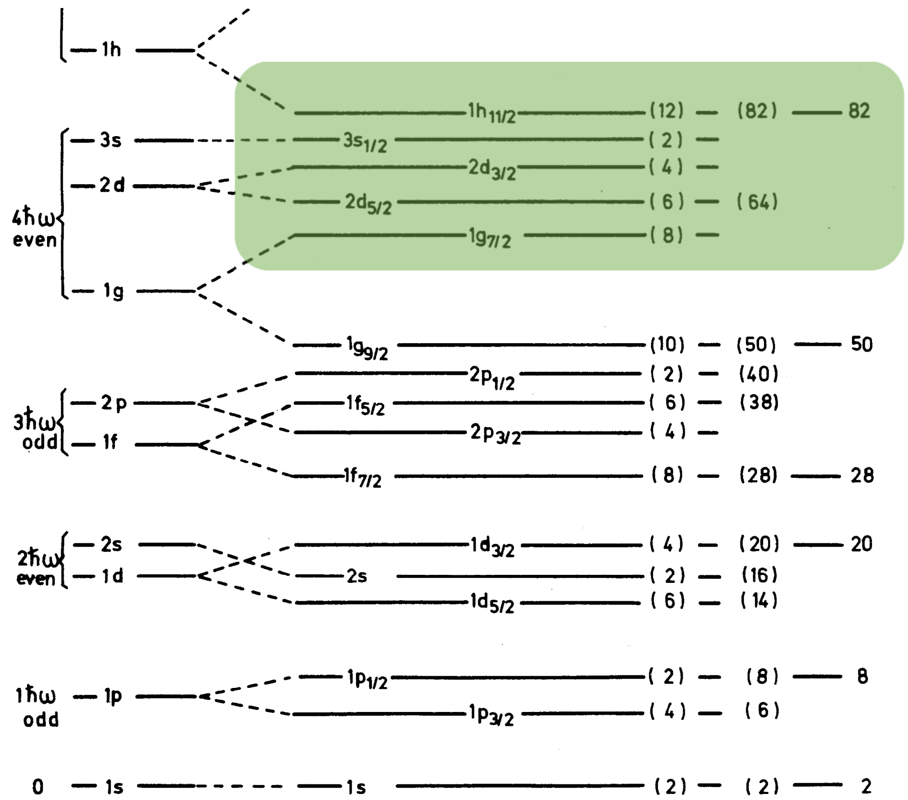
$$[T_{1/2}^{(0\nu)}]^{-1} = G^{(0\nu)}(g_A)^4 |M^{(0\nu)}|^2 \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2$$

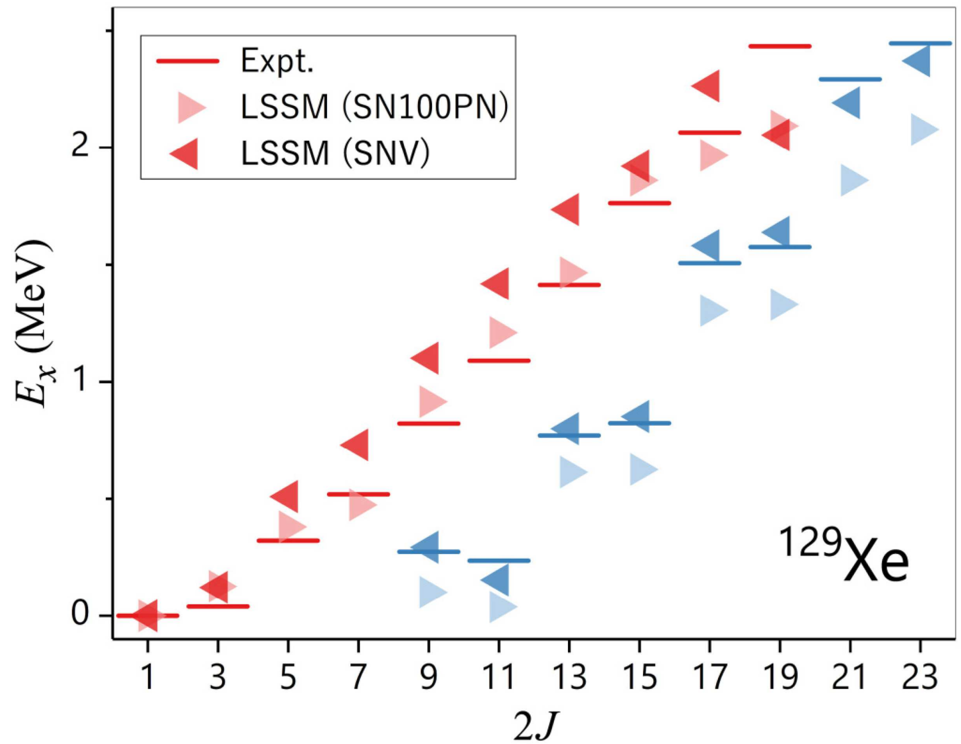
$$M^{(0\nu)} = \sum_K M_K^{(0\nu)} = M_{\mathrm{F}}^{(0\nu)} + M_{\mathrm{GT}}^{(0\nu)} + M_{\mathrm{T}}^{(0\nu)}$$

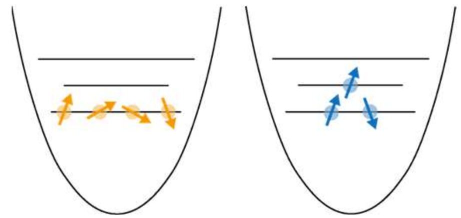
$$M_K^{(0\nu)} = \langle \Psi_{\mathrm{fin}}(0^+_{\mathrm{g.s.}}) |\ \hat{V}_K | \Psi_{\mathrm{ini}}(0^+_{\mathrm{g.s.}}) \rangle$$

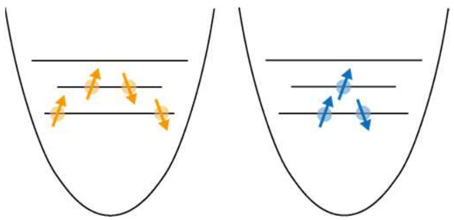
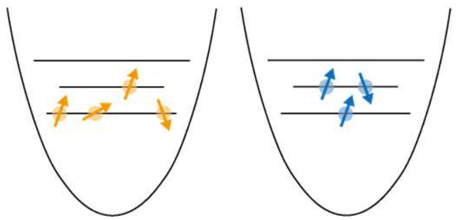
$$\hat{V}_K = A_{s_1 s_2}^{(\lambda)} \sqrt{\frac{\pi}{2\lambda + 1}} \sum_{ij} \tau_i^+ \tau_j^+ H_{\lambda,K}(r_{ij}) \times (Y^{(\lambda)}(\theta_{ij},\ \varphi_{ij}) \cdot [\Sigma_i^{(s_1)} \otimes \Sigma_j^{(s_2)}]^{(\lambda)})$$

Nuclear shell model





Ground state (^{129}Xe) = C_1 

$+C_2$  **$+C_3$**  **+···**

Residual interaction (P+QQ)

現象論的な有効相互作用（Pairing + QQ 相互作用）

$$H = H_\nu + H_\pi + H_{\nu\pi}$$

一粒子エネルギー　　二体相互作用の強さ

$$H_\tau = \sum_{jm} \epsilon_{j\tau} c^\dagger_{jm\tau} c_{jm\tau} - G_{0\tau} P^{\dagger(0)}_\tau P^{(0)}_\tau - \kappa_\tau : Q_\tau \cdot Q_\tau :$$

$$- \sum_{L=2,4,6,8,10} G_{L\tau} P^{\dagger(L)}_\tau \cdot \tilde{P}^{(L)}_\tau ,$$

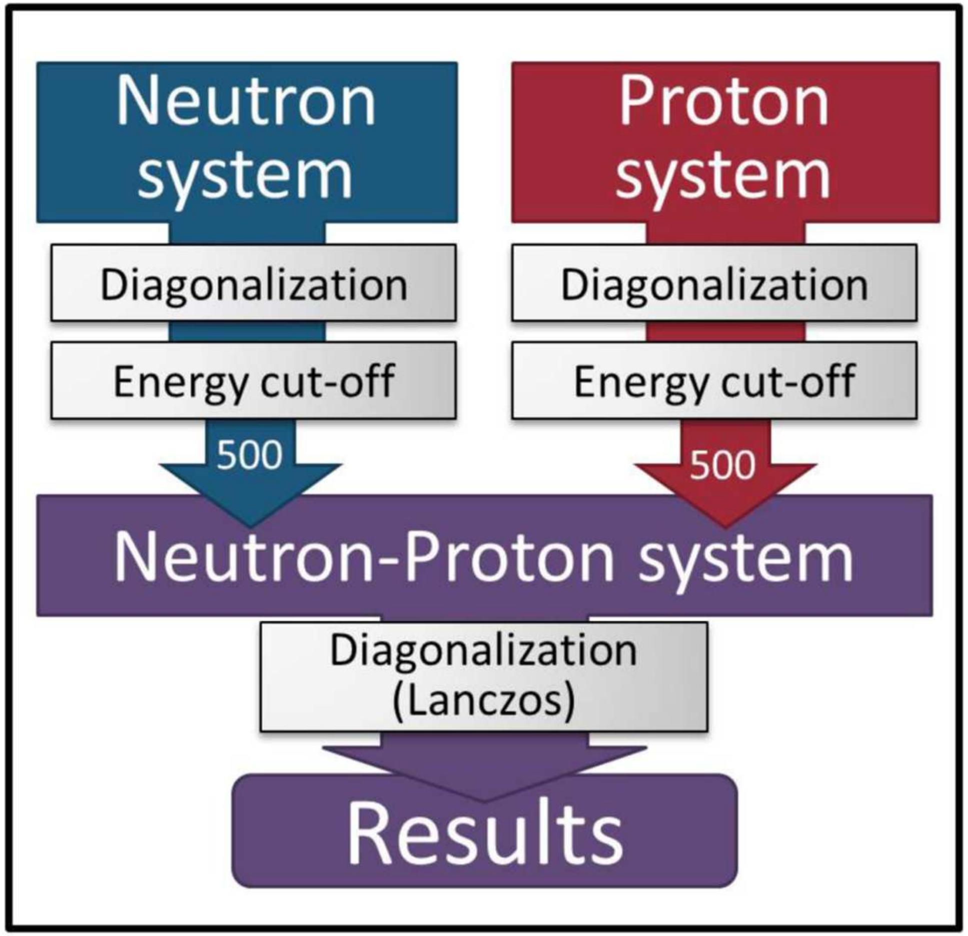
$$H_{\nu\pi} = -\kappa_{\nu\pi} Q_\nu \cdot Q_\pi .$$

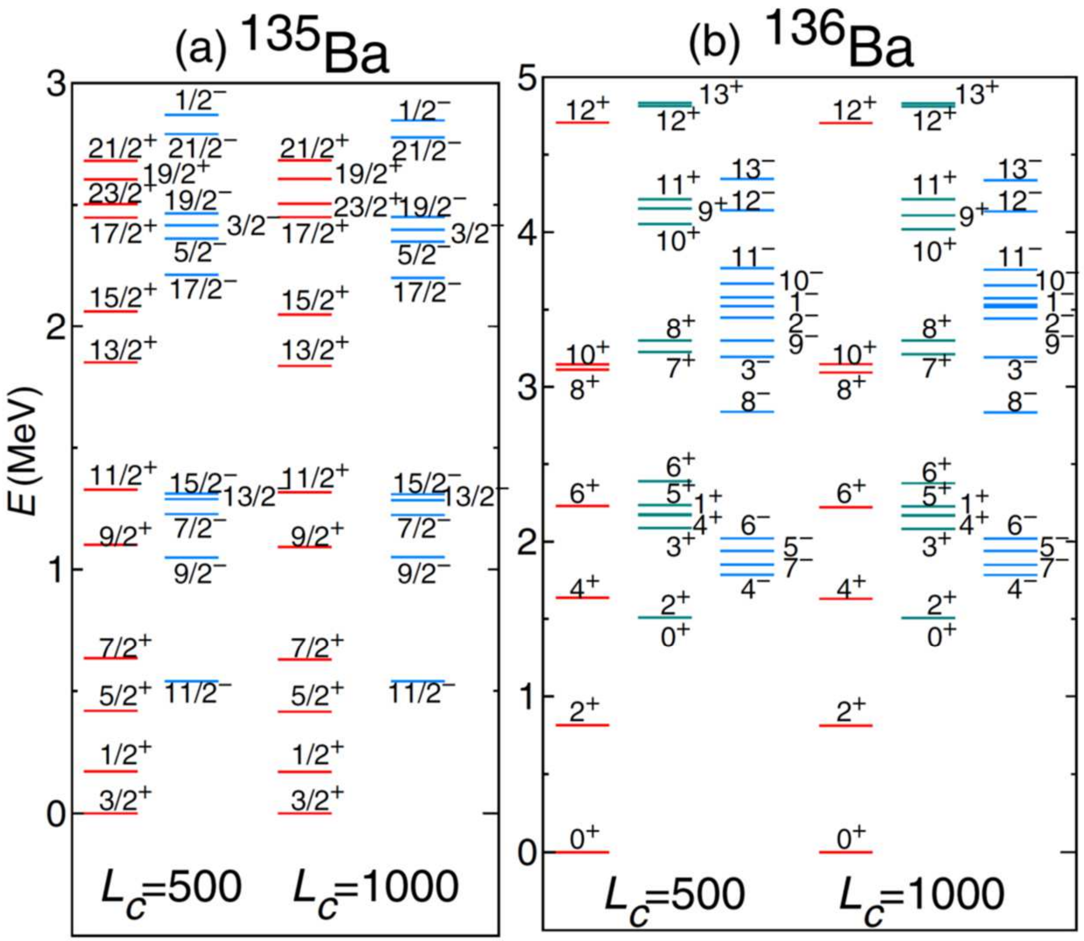
$$\hat{P}^{\dagger(0)}_\tau = \sum_j \frac{\sqrt{2j+1}}{2} A^{\dagger(0)}_{0\tau}(jj)$$

$$A^{\dagger(J)}_M(j_1 j_2) = \sum_{m_1 m_2} (j_1 m_1 j_2 m_2 | J M) c^\dagger_{j_1 m_1} c^\dagger_{j_2 m_2}$$

$$= [c^\dagger_{j_1} c^\dagger_{j_2}]^{(J)}_M ,$$

Residual interaction (P+QQ)

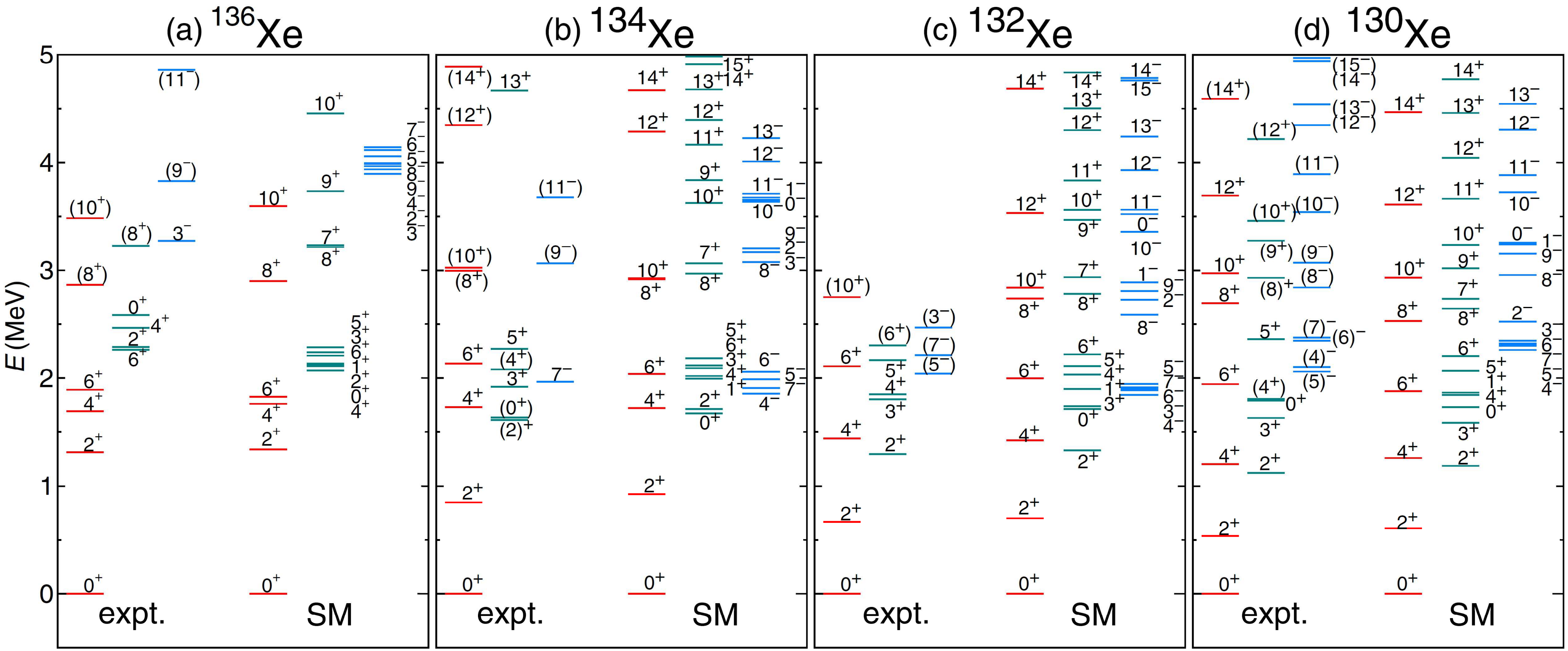




Energy spectra

エネルギースペクトルを系統的に再現するように

二体相互作用の強さを決める



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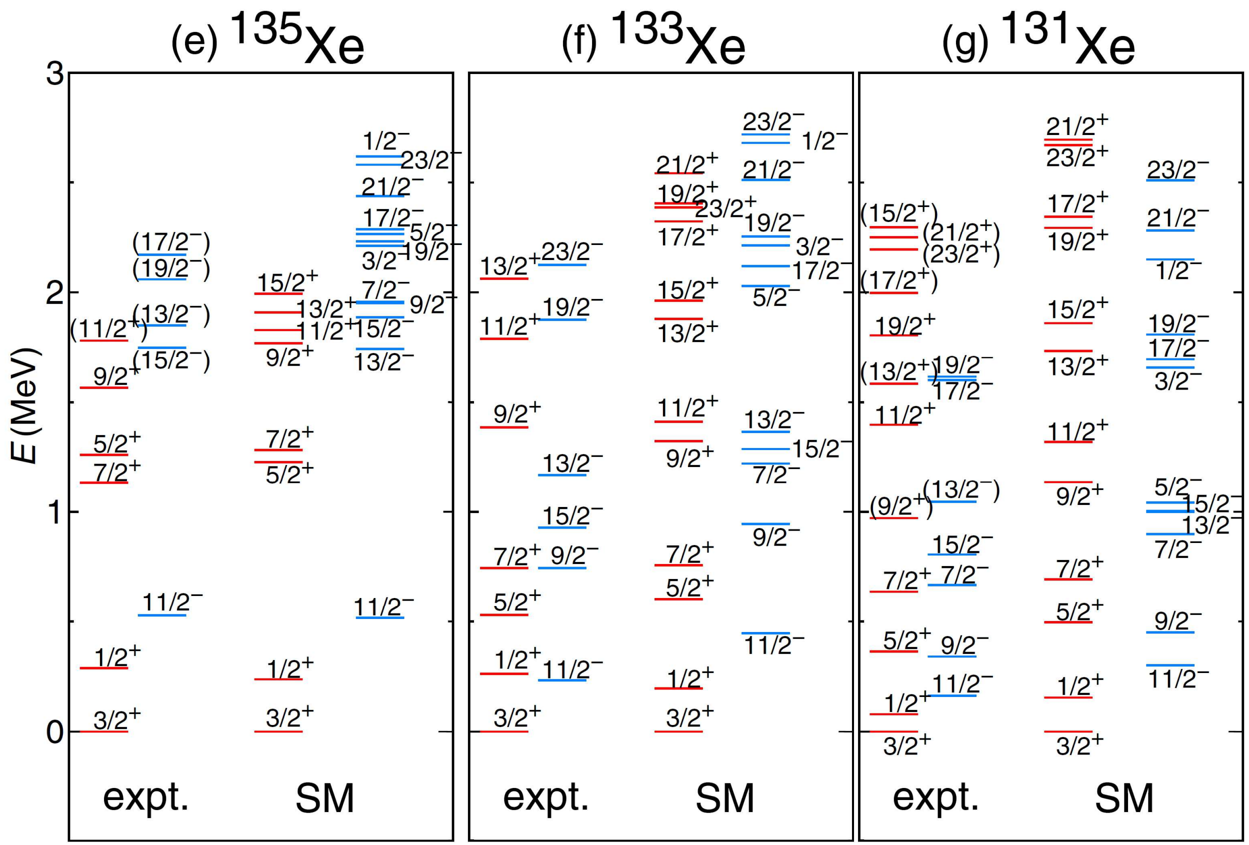


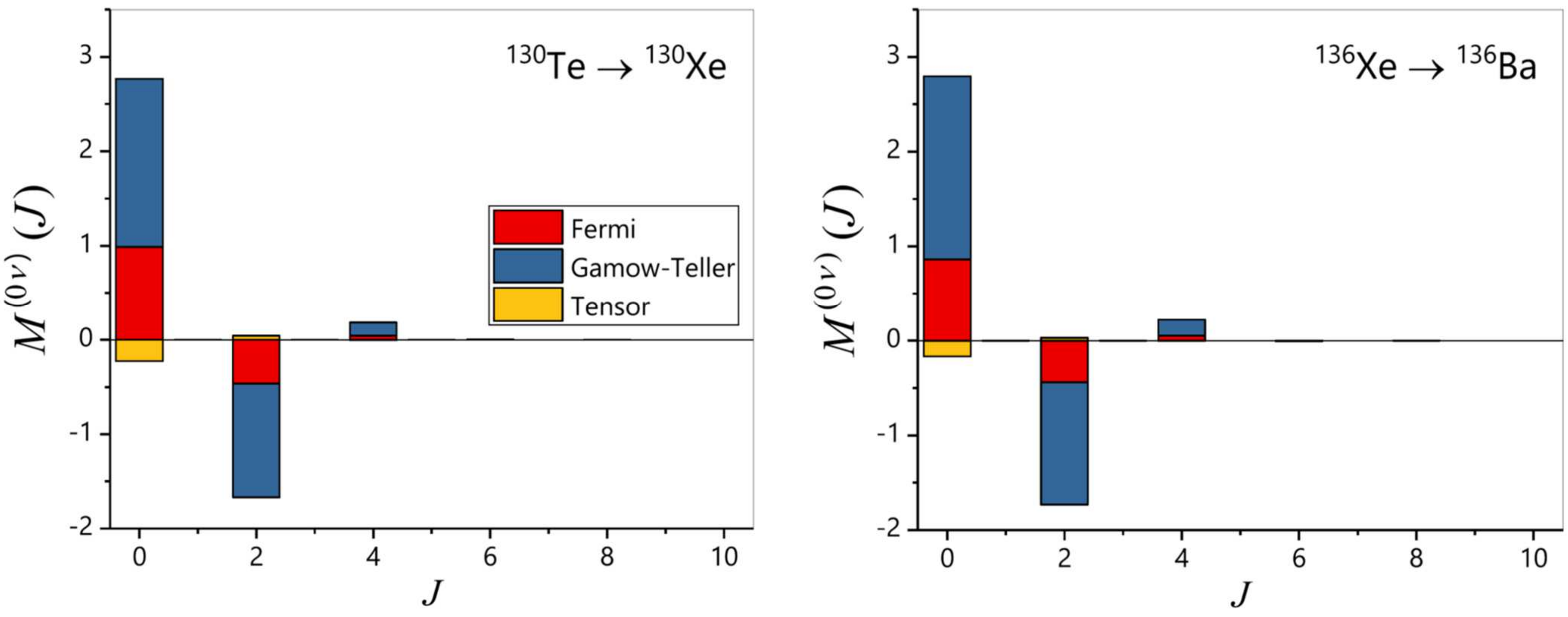
Nuclear matrix element

Model (Interaction)	g_A	$M_{\mathrm{F}}^{(0\nu)}$	$\widetilde{M}_{\mathrm{F}}^{(0\nu)}$	$M_{\mathrm{GT}}^{(0\nu)}$	$M_{\mathrm{T}}^{(0\nu)}$	$M^{(0\nu)}$	$T_{1/2}^{(0\nu)}\langle m_\nu\rangle^2$
SM (optimized)	1.25	0.478	−0.747	0.803	−0.135	1.145	5.57
SM ($1.3G_0$)	1.25	0.712	−1.113	1.235	−0.185	1.763	2.35
PTSM-*SDGH*	1.25	0.630	−0.984	0.932	−0.169	1.394	3.76
PTSM-*SDG*	1.25	0.626	−0.978	0.818	−0.177	1.267	4.55
PTSM-*SD*	1.25	0.715	−1.117	1.252	−0.180	1.787	2.29
SM (SVD) [35]	1.254		−0.40	1.50	−0.01	1.76	
SM (GCN50.82) [31]	1.25					1.76	1.78
GCM [60]	1.254		−0.32	2.17	−0.02	2.35	
QRPA (Argonne V18) [47]	1.27		−0.806	1.959	−0.282	2.177	
QRPA (CD-Bonn) [47]	1.27		−0.858	2.181	−0.254	2.460	
IBM [54]	1.269		−0.52	2.83	−0.10	3.05	0.74

J=0 contribution

$$M_K^{(0\nu)}(J) = \langle \Psi_{\rm fin}(0^+_{\rm g.s.})|\ \hat{V}_K^{(J)} |\Psi_{\rm ini}(0^+_{\rm g.s.})\rangle$$

$$\hat{V}_K^{(J)} = \sum_{j_1 j_2 j_3 j_4} V^{(\lambda)}_{s_1,s_2}(j_1 j_2 j_3 j_4\,;\,J)([c^\dagger_{\pi j_1}\, c^\dagger_{\pi j_2}]^{(J)} \cdot [\tilde{c}_{\nu j_3}\tilde{c}_{\nu j_4}]^{(J)})$$



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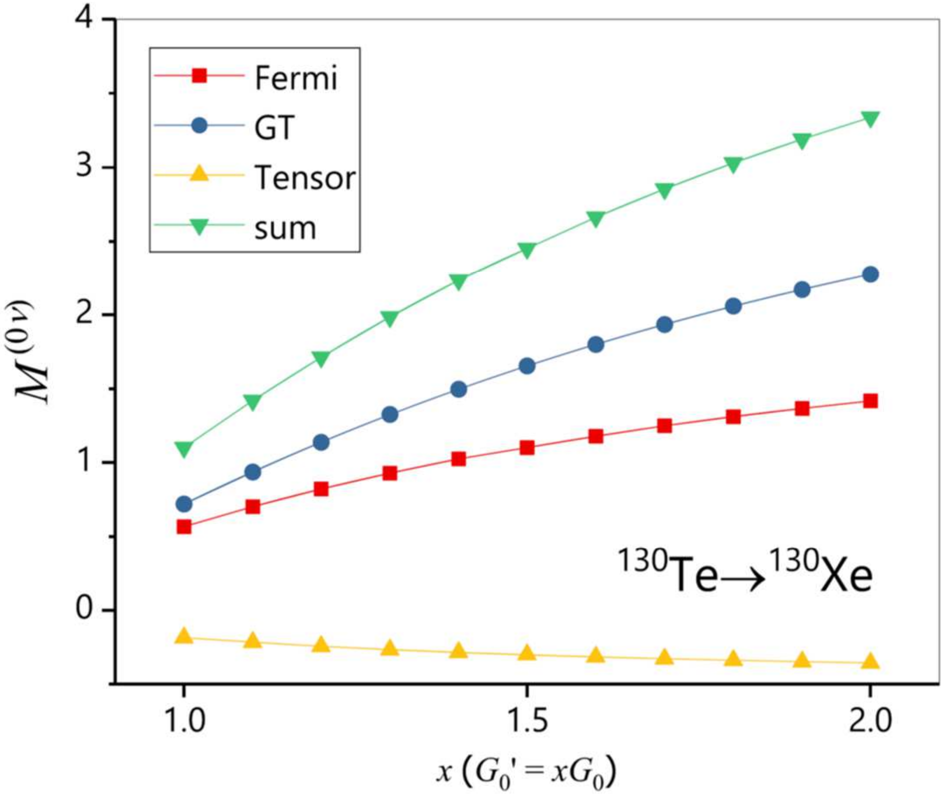
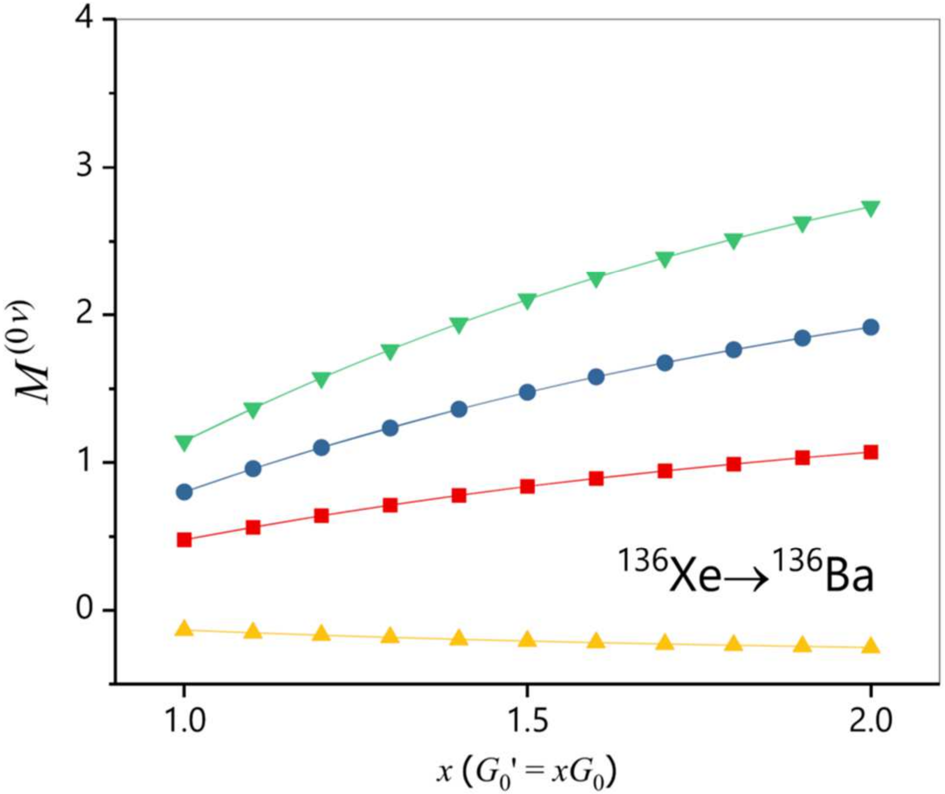




Nucleon-pair truncation

核子対 (J=0,2,4) で構成される配位に制限した殻模型計算 (PTSM)

$$S^\dagger = \sum_j \alpha_j A_0^{\dagger(0)}(jj)$$

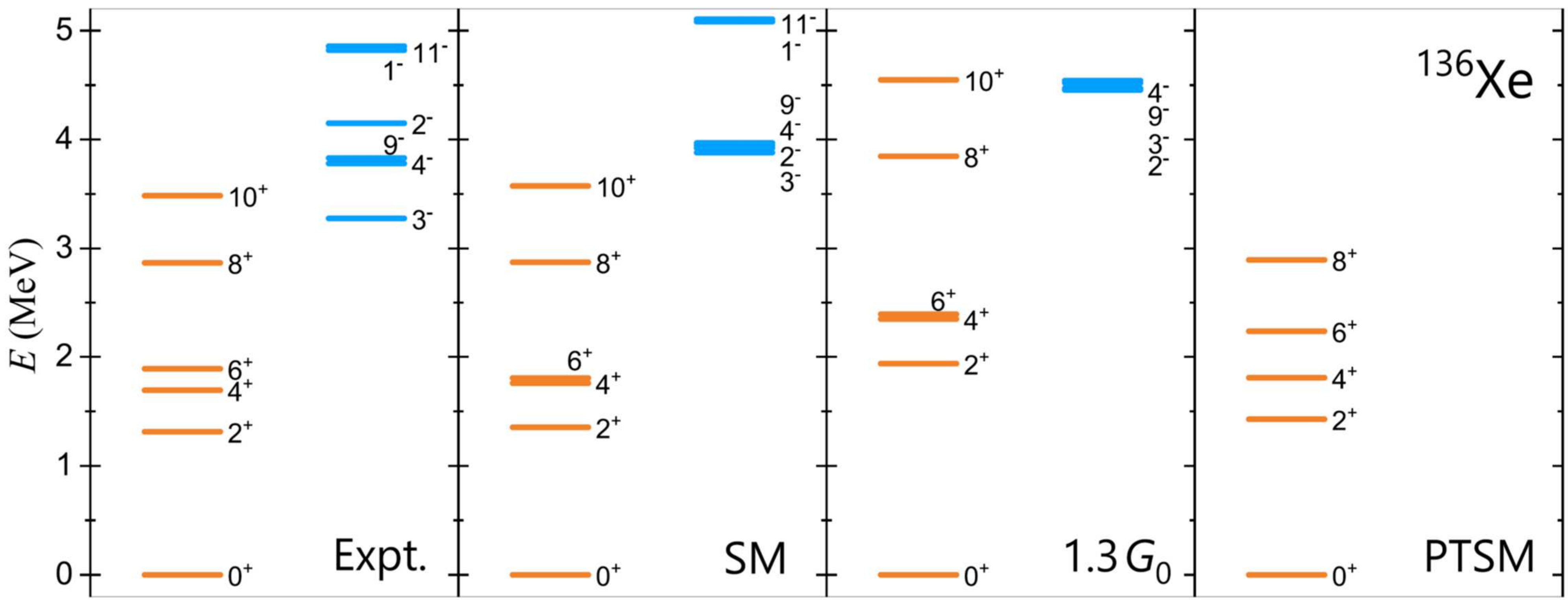
$$D_M^\dagger = \sum_{j_1 j_2} \beta_{j_1 j_2} A_M^{\dagger(2)}(j_1 j_2)$$

$$G_M^\dagger = \sum_{j_1 j_2} \gamma_{j_1 j_2} A_M^{\dagger(4)}(j_1 j_2)$$

$$A_M^{\dagger(J)}(j_1 j_2) = \sum_{m_1 m_2} (j_1 m_1 j_2 m_2 | JM) c^\dagger_{j_1 m_1} c^\dagger_{j_2 m_2} = [c^\dagger_{j_1} c^\dagger_{j_2}]_M^{(J)}$$

$$|S^{n_s} D^{n_d} G^{n_g} H^{n_h} \eta I\rangle = (S^\dagger)^{n_s} (D^\dagger)^{n_d} (G^\dagger)^{n_g} (H^\dagger)^{n_h} | - \rangle$$

Energy spectra



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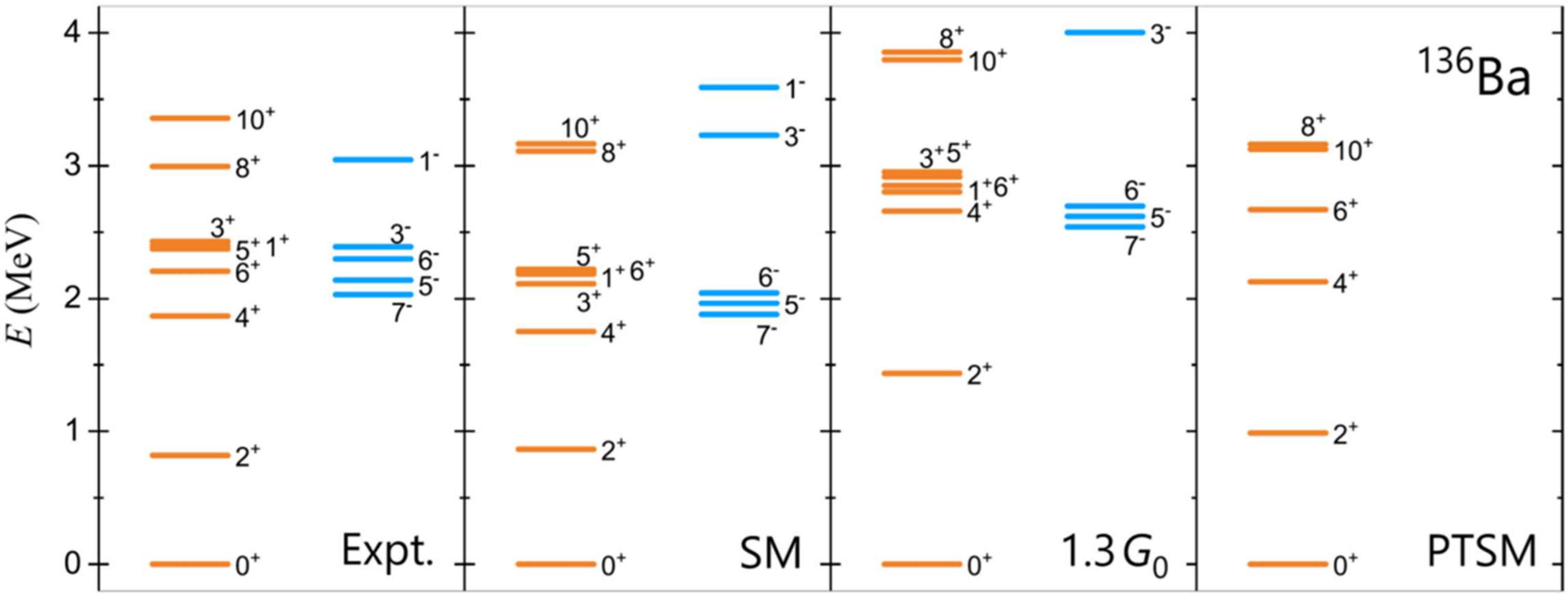
Energy spectra



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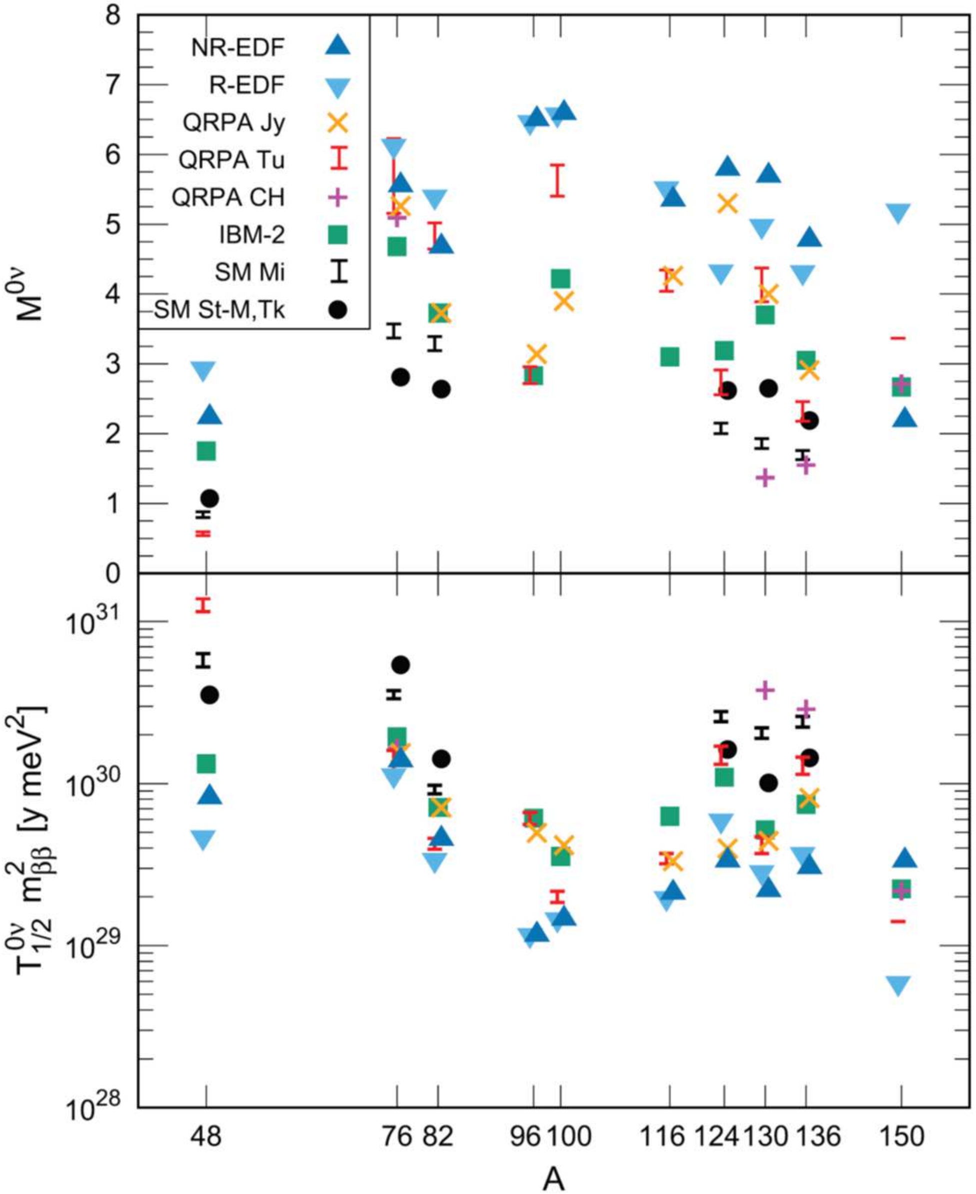
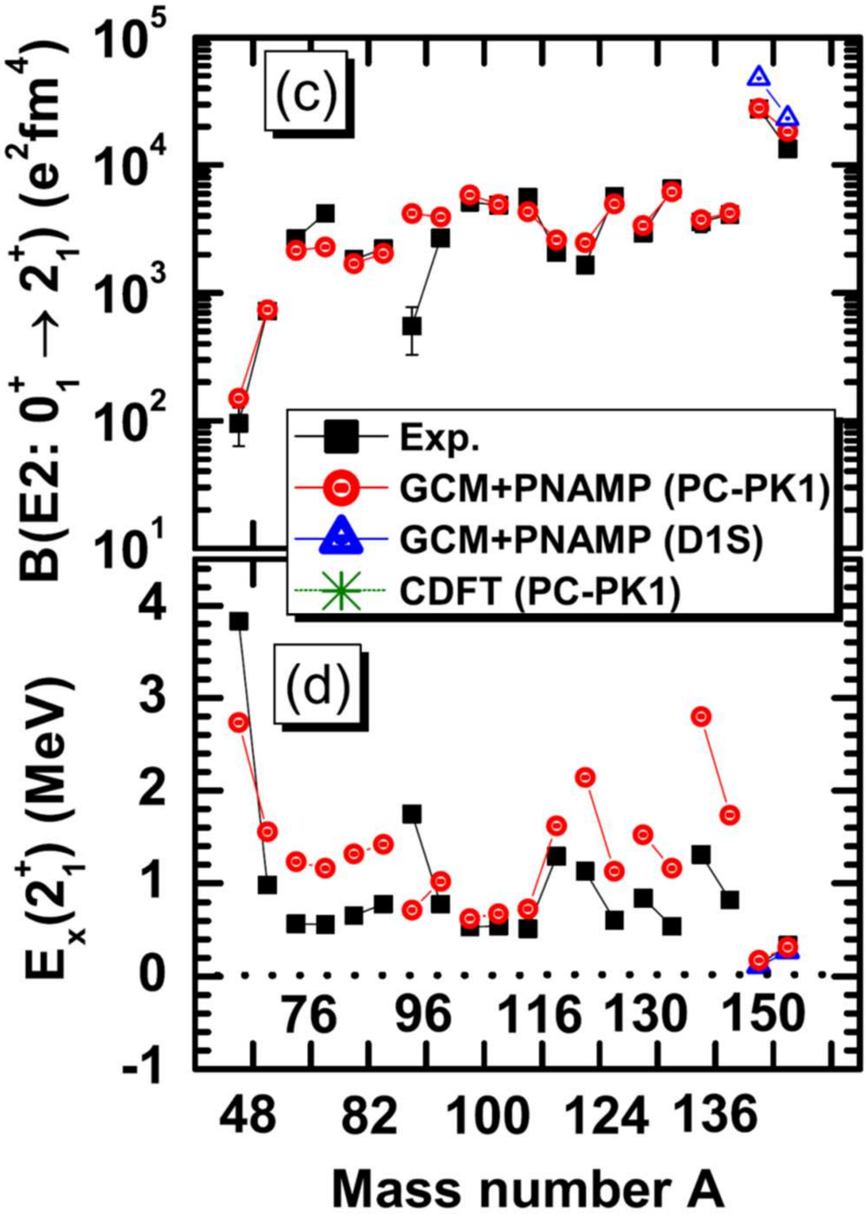




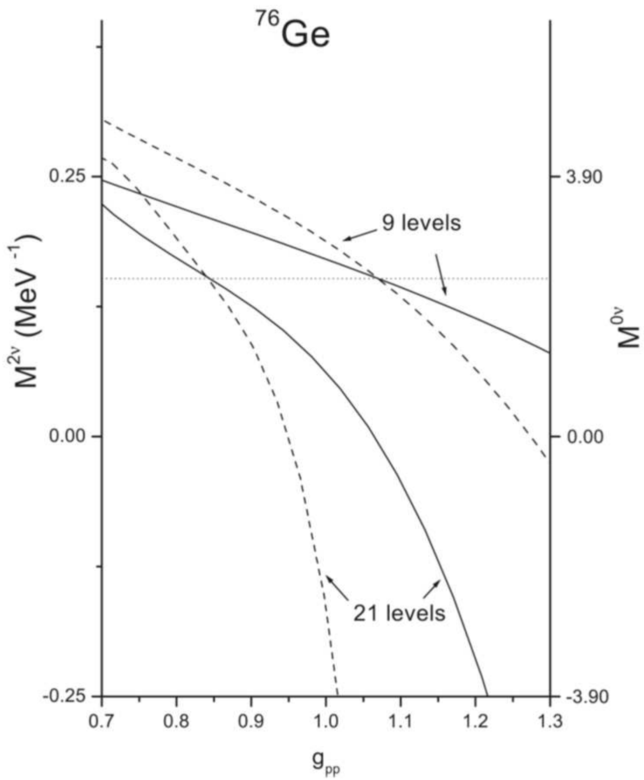
J. M. Yao, et al., PRC 91, 024316 (2015).

Summary

- 殻模型計算で予言される核行列要素は比較的小さい
- 核行列要素はアイソベクトル・ペアリングに敏感
- 励起スペクトル (2+の励起エネルギー) との相関

Perspectives

- アイソスカラー・ペアリング
- 準粒子真空殻模型
- 二重ガモフテラー遷移



Residual interaction (P+QQ)

現象論的な有効相互作用である Pairing + QQ 相互作用を用いる。

^{130}Xe	Expt.	SM
$2_1^+ \to 0_1^+$	38(5)	41.0
$4_1^+ \to 2_1^+$	44.5(20)	59.6
$6_1^+ \to 4_1^+$	>0.033	62.3
$8_1^+ \to 6_1^+$	>0.020	50.3
$10_1^+ \to 8_1^+$	1.69(4)	5.99

^{131}Xe	Expt.	SM
$1/2_1^+ \to 3/2_1^+$	<37	17.9
$5/2_1^+ \to 1/2_1^+$	7.64(24)	2.30
$5/2_1^+ \to 3/2_1^+$	27.8(9)	33.3
$7/2_1^+ \to 3/2_1^+$	22.2(19)	27.6
$7/2_1^- \to 11/2_1^-$	0.494532(20)	20.8
$7/2_2^- \to 11/2_1^-$		4.73
$9/2_1^- \to 11/2_1^-$	39(10)	42.0

^{132}Xe	Expt.	SM
$2_1^+ \to 0_1^+$	23.1(15)	27.7
$2_2^+ \to 0_1^+$	0.079(11)	0.000150
$2_2^+ \to 2_1^+$	41(4)	37.3
$4_1^+ \to 2_1^+$	28.6(23)	40.4
$6_1^+ \to 4_1^+$		30.5
$8_1^+ \to 6_1^+$		17.4

^{133}Xe	Expt.	SM
$1/2_1^+ \to 3/2_1^+$		15.1
$5/2_1^+ \to 1/2_1^+$		0.235
$5/2_1^+ \to 3/2_1^+$		19.9
$7/2_1^+ \to 3/2_1^+$		17.5
$9/2_1^- \to 11/2_1^-$		15.2
$15/2_1^- \to 11/2_1^-$		13.8

^{134}Xe	Expt.	SM
$2_1^+ \to 0_1^+$	15.3(11)	15.3
$4_1^+ \to 2_1^+$	11.6(8)	18.6
$6_1^+ \to 4_1^+$		3.43
$8_1^+ \to 6_1^+$		4.95
$10_1^+ \to 8_1^+$	160(50),0.64(1)	0.610

^{135}Xe	Expt.	SM
$1/2_1^+ \to 3/2_1^+$		9.07
$7/2_1^+ \to 3/2_1^+$		11.3
$5/2_1^+ \to 3/2_1^+$		8.24
$5/2_1^+ \to 1/2_1^+$		4.26

^{136}Xe	Expt.	SM
$2_1^+ \to 0_1^+$	16.6(24)	8.59
$4_1^+ \to 2_1^+$	1.281(17)	1.53
$6_1^+ \to 4_1^+$	0.0132(4)	0.00212
$8_1^+ \to 6_1^+$		4.08

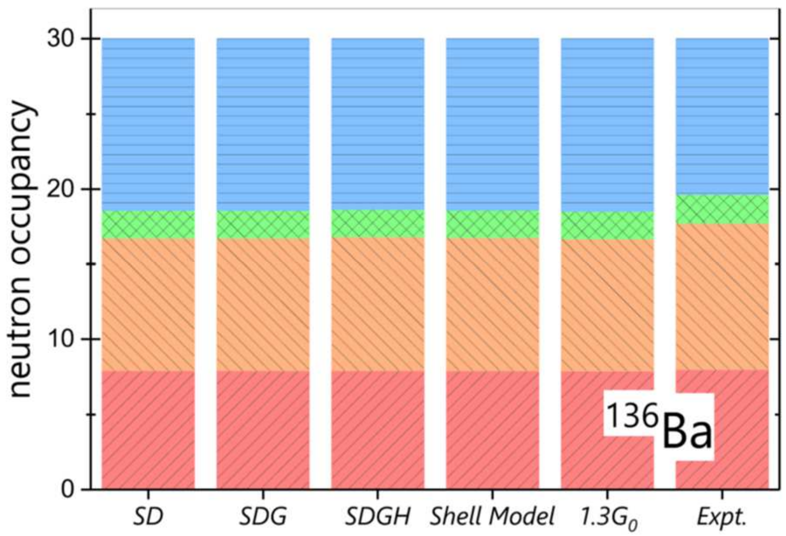
Residual interaction (P+QQ)

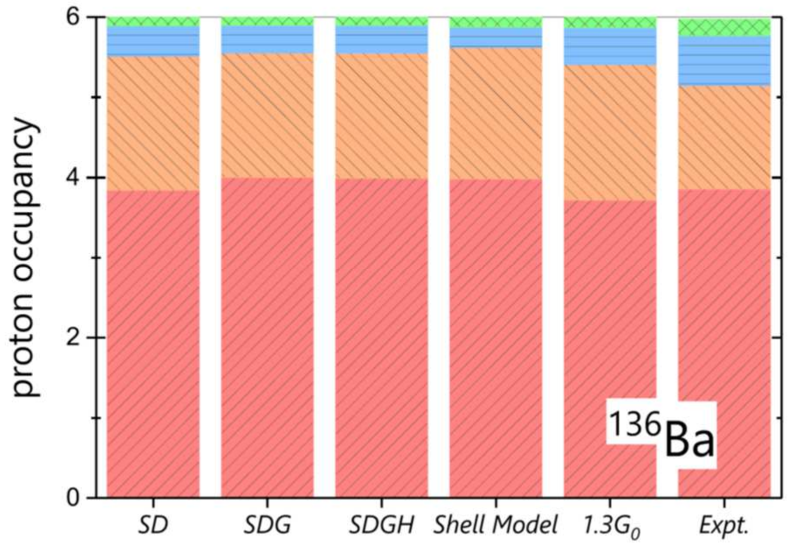
現象論的な有効相互作用である Pairing + QQ 相互作用を用いる。

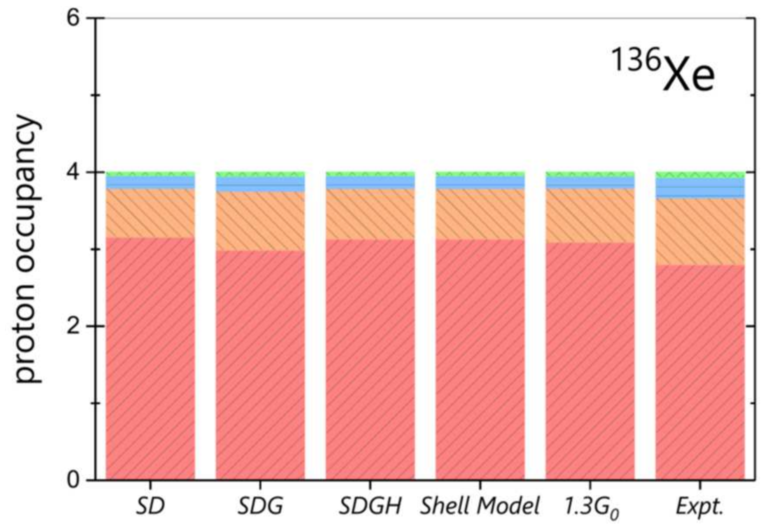
^{130}Xe	μ		Q	
	Expt.	SM	Expt.	SM
2_1^+	+0.76(14)	+0.611		−0.262
4_1^+	+1.68(20)	+1.50		−0.395
6_1^+		+2.86		−0.508
10_1^+	−2.05(14)	−1.86		+0.452
^{131}Xe	Expt.	SM	Expt.	SM
$3/2_1^+$	+0.691862(4)	+0.892	−0.114(1)	+0.303
$5/2_1^+$		+0.649		−0.249
$7/2_1^-$		−1.34		+0.138
$9/2_1^-$		−1.08		+1.05
$11/2_1^-$	−0.994048(6)	−1.16	+0.73(3)	+0.479
^{132}Xe	Expt.	SM	Expt.	SM
2_1^+	+0.651(24)	+0.622		−0.218
2_2^+	+0.2(4)	+0.563		+0.251
4_1^+	+2.4(4)	+1.85		−0.300
6_1^+		+4.74		−0.517
10_1^+	(−)1.95(5)	−2.28		+1.04

^{133}Xe	Expt.	SM	Expt.	SM
$3/2_1^+$	+0.81340(7)	+0.892	+0.142(5)	+0.293
$5/2_1^+$		+0.651		−0.204
$9/2_1^-$		−1.10		+0.782
$11/2_1^-$	−1.08247(15)	−1.25	+0.77(3)	+0.622
^{134}Xe	Expt.	SM	Expt.	SM
2_1^+	+0.708(14)	+0.647		−0.167
4_1^+	+3.2(6)	+2.22		−0.382
6_1^+		+4.14		−0.268
^{135}Xe	Expt.	SM	Expt.	SM
$3/2_1^+$	+0.9032(7)	+0.889	+0.214(7)	+0.212
$5/2_1^+$		+0.770		+0.109
$7/2_1^+$		+2.46		+0.0795
$11/2_1^-$	−1.1036(14)	−1.27	+0.618(21)	+0.575
^{136}Xe	Expt.	SM	expt.	SM
2_1^+	+1.53(9)	+1.45		−0.0967
4_1^+	3.2(6)	+2.77		−0.0378
6_1^+		+4.07		−0.111

Occupation number







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$$H_\tau = \sum_{jm} \epsilon_{j\tau} c^\dagger_{jm\tau} c_{jm\tau} - G_{0\tau} P^{\dagger(0)}_\tau P^{(0)}_\tau - \kappa_\tau : Q_\tau \cdot Q_\tau :$$

$$- \sum_{L=2,4,6,8,10} G_{L\tau} P^{\dagger(L)}_\tau \cdot \tilde{P}^{(L)}_\tau ,$$

$d_{5/2}$	$2s_{1/2}$	$0h_{11/2}$	$1d_{3/2}$
.655	0.332	$\epsilon_\nu(h_{11/2})$	0.000
.962	3.000	2.791	2.440

	G_0	G_2	G_4	G_6	G
ν-ν	0.170	0.018	−0.50	−2.00	−
π-π	0.165	0.007	0.20	0.10	
ν-π					

$$H_{\nu\pi} = -\kappa_{\nu\pi} Q_\nu \cdot Q_\pi .$$