ボルツマンソルバーによる
超新星爆発シミュレーションと
ニュートリノ集団振動への応用

山田　章一

早稲田大学

The Evolution of Massive Stars and Formation of Compact Stars: from the Cradle to the Grave

2020/2/26~28@Waseda Univ.

Topics

- Formations of massive stars
- Multi-dimensional evolutions of massive stars
- Evolutions of massive stars in binaries and multiple systems
- Mechanism of core-collapse supernovae
- Neutrino and nuclear physics of relevance for the formation and evolution of compact objects

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http://www.heap.phys.waseda.ac.jp/Conference_waseda_2020/index.html
for registration.

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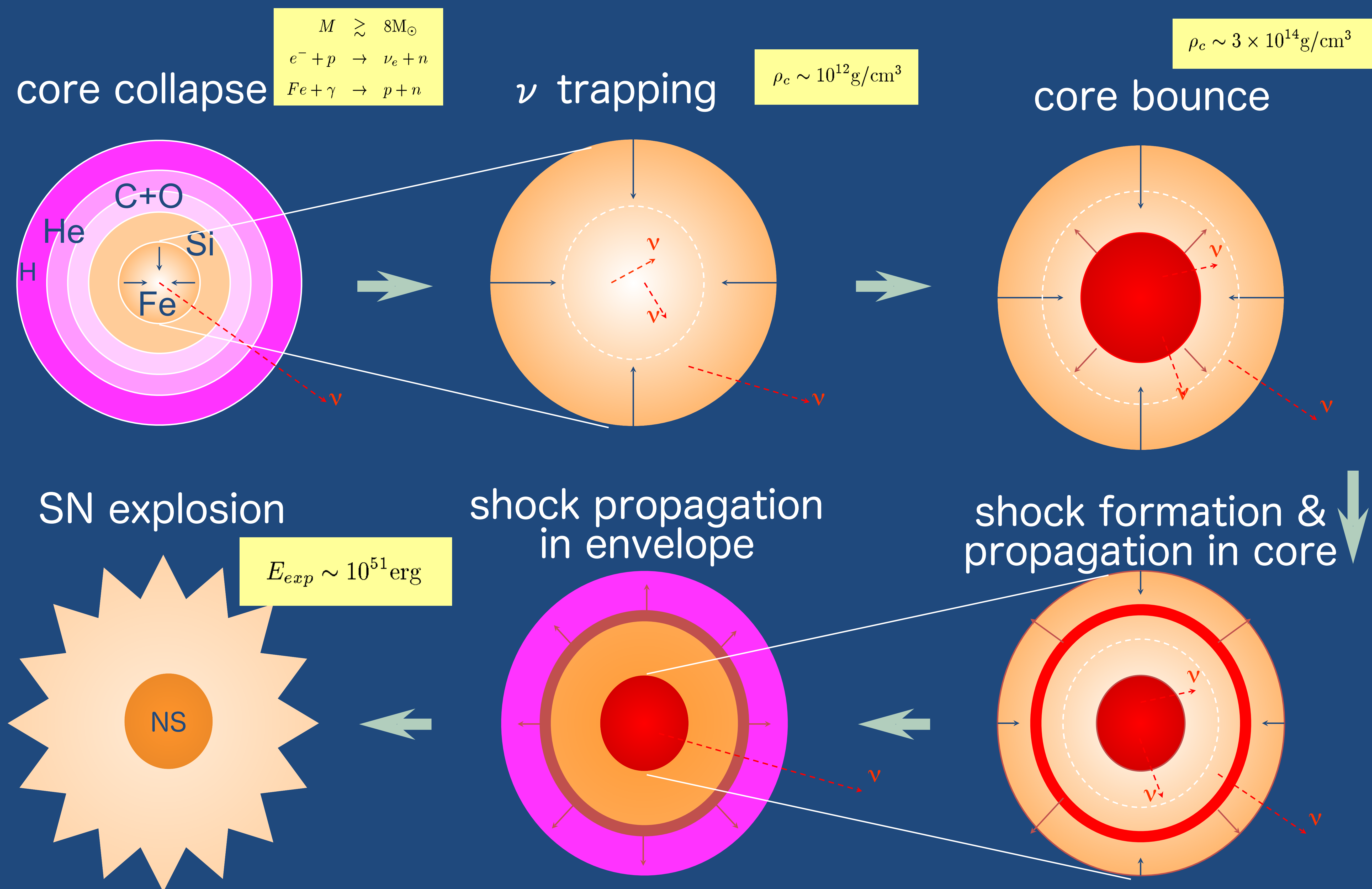
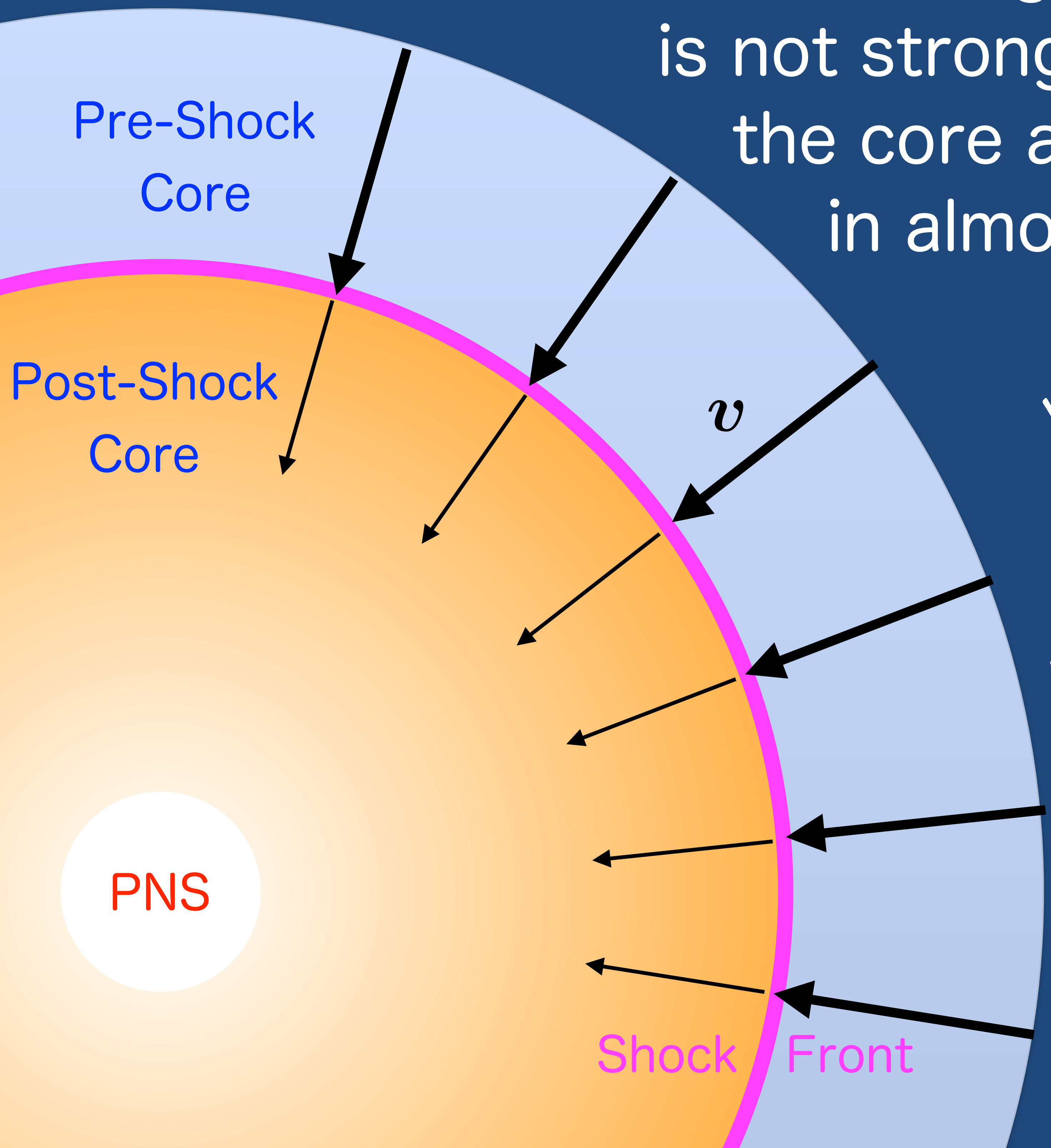
*: to be confirmed

Core-Collapse Supernovae



Neutrino Heating Mechanism



- ✓ The shock generated at core bounce is not strong enough to push through the core and stagnates in the core in almost all cases.
- ✓ The stalled shock need to be revived somehow.
- ✓ Neutrino heating is supposed to be the most promising.

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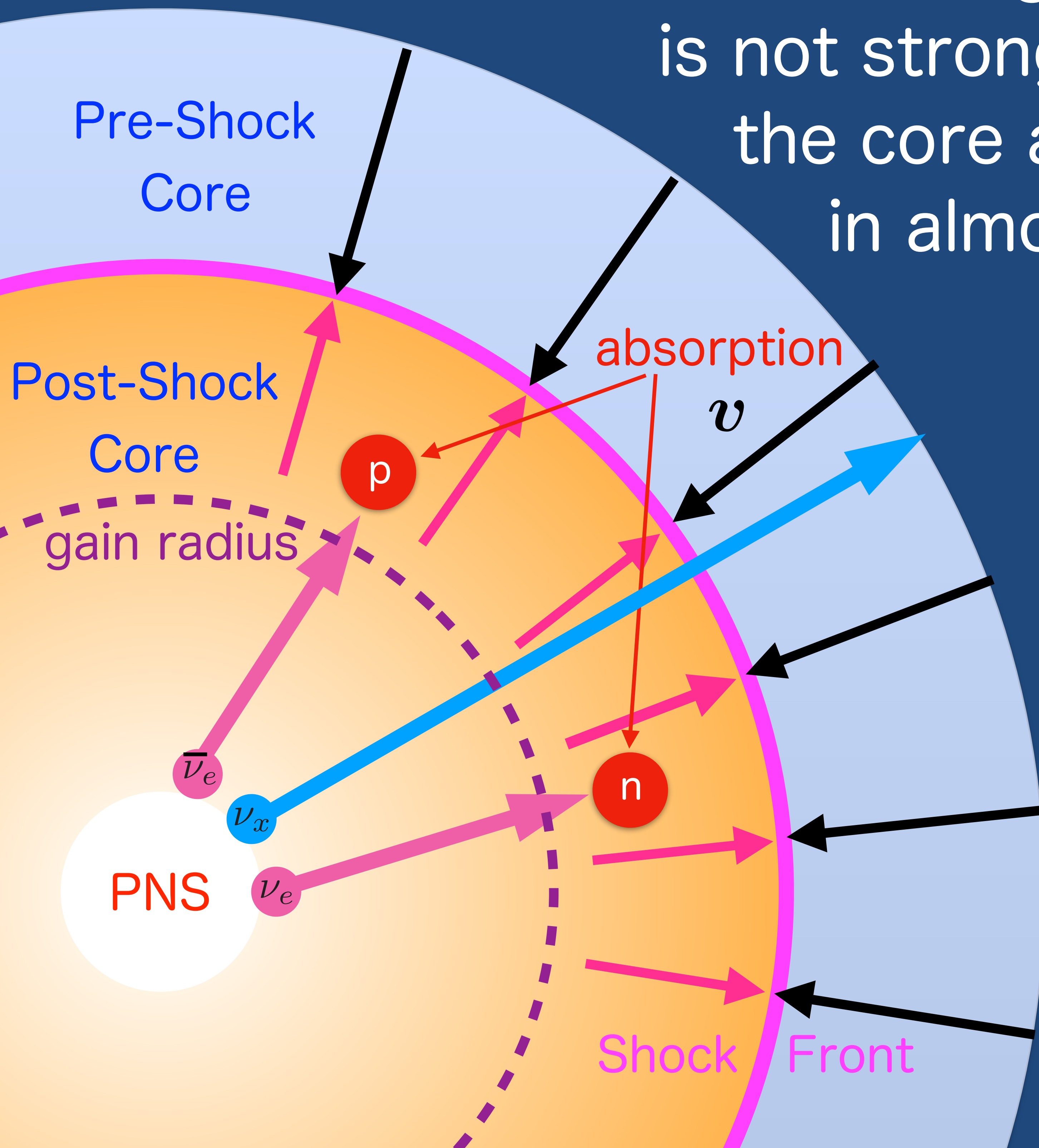


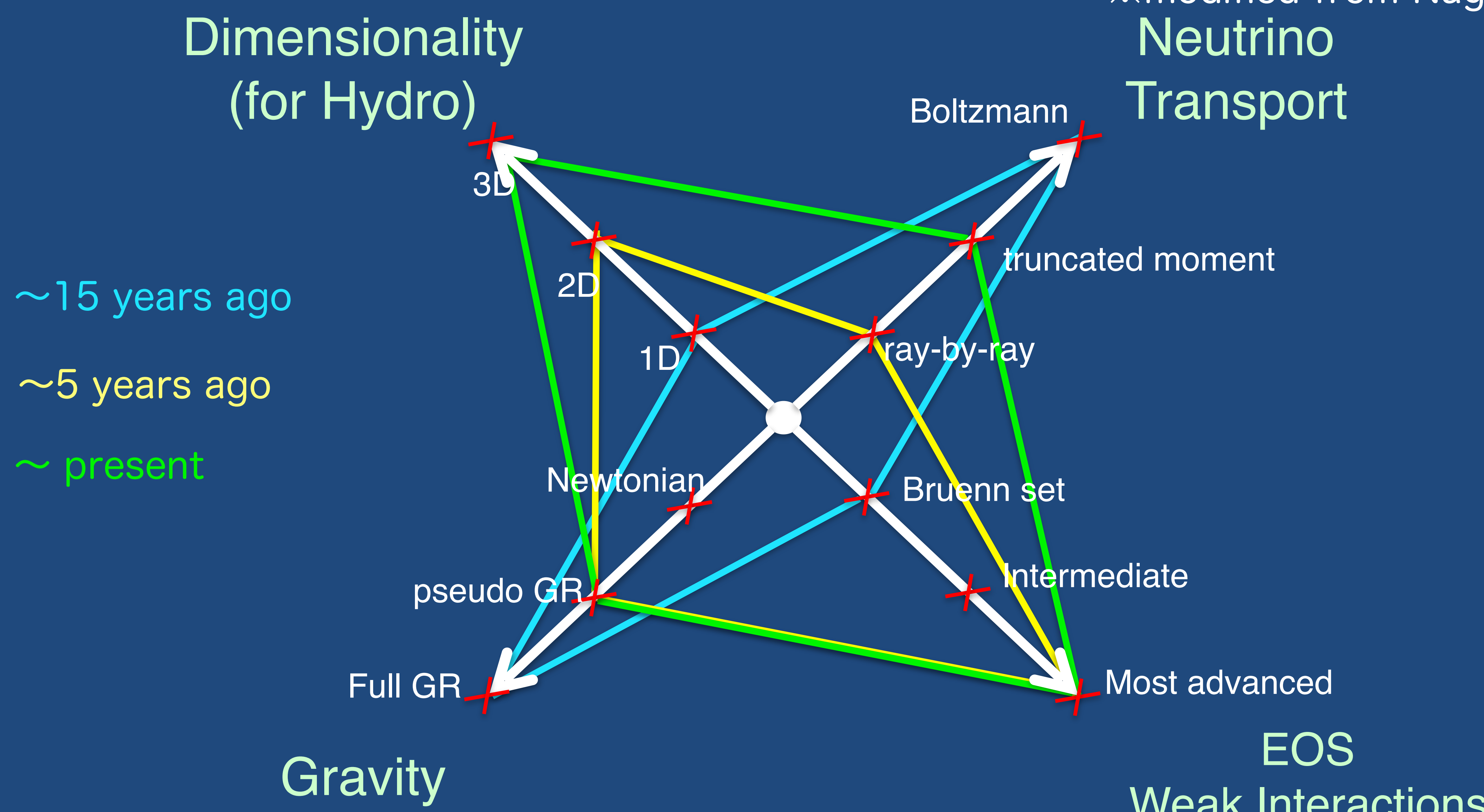
- ✓ The stalled shock need to be revived somehow.
- ✓ Neutrino heating is supposed to be the most promising.

Numerical Investigations of CCSNe

- ✓ Nearby supernovae, that allow us to probe deep inside the core directly in real time via ν and/or GW, are rare (~1 in 100 yrs).
- ✓ We have to rely on theoretical studies, in particular numerical simulations.
- ✓ In so doing, both macro- and microphysics are important.
  - multi-dimensional (magneto)hydrodynamics
  - (general relativistic) self-gravity
  - multi-energy & multi-angle neutrino transport
  - nuclear EOS and weak interactions

Progress in CCSN Modeling

※modified from Nagakura's slide

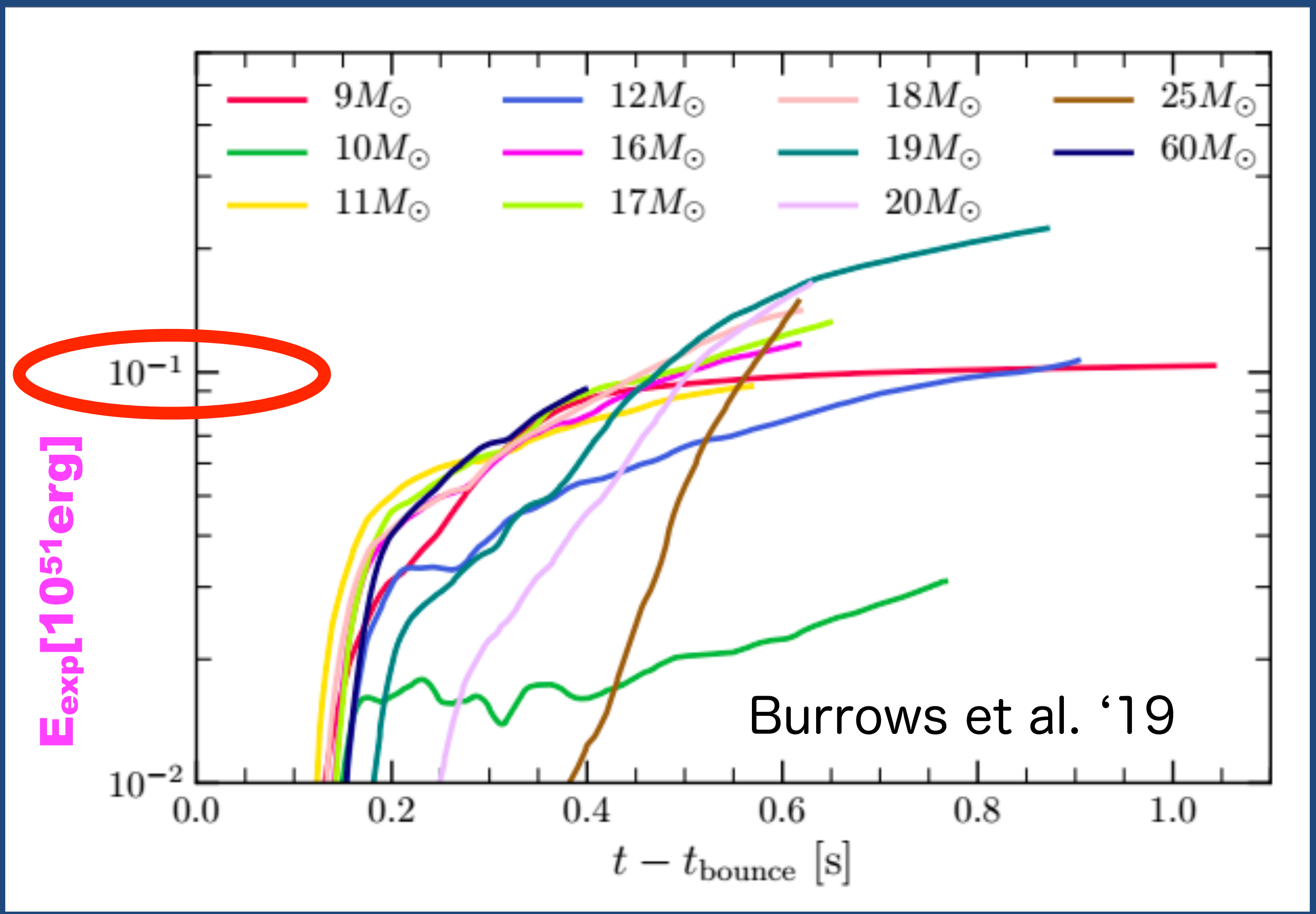


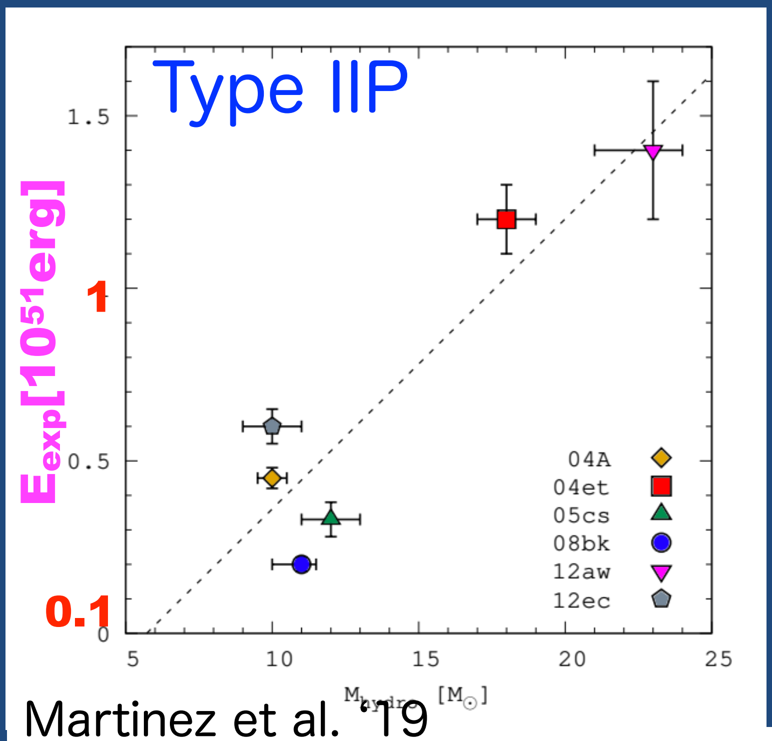
✓ In the last decade, most of the world major groups modeling core-collapse supernovae (CCSNe) numerically have proceeded to 3D in space.

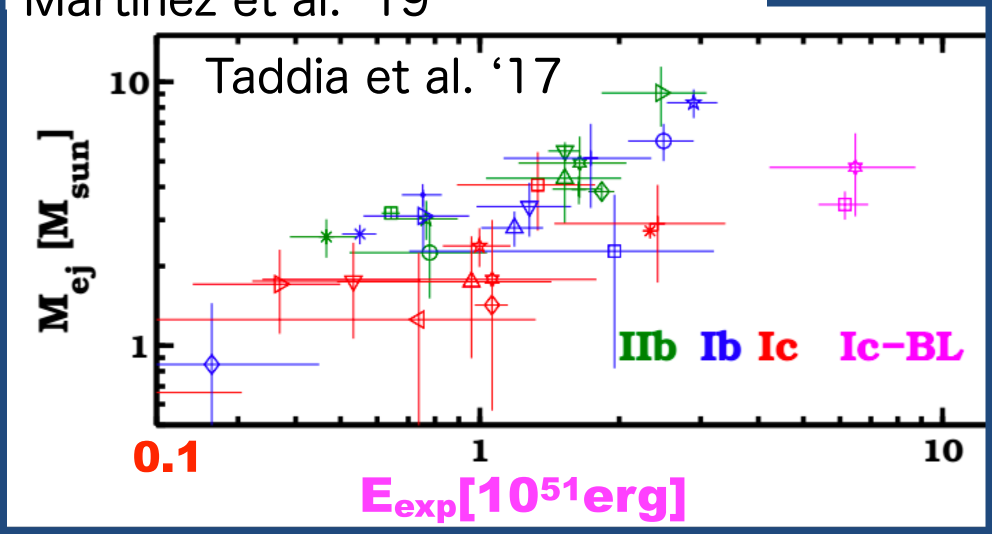
- neutrino transport with the truncated moment method with some closure relation imposed by hand

Recent 3D Simulations

- 3 spatial dimensions
- truncated-moment approximation to ν transfer
- monopole GR correction to self-gravity



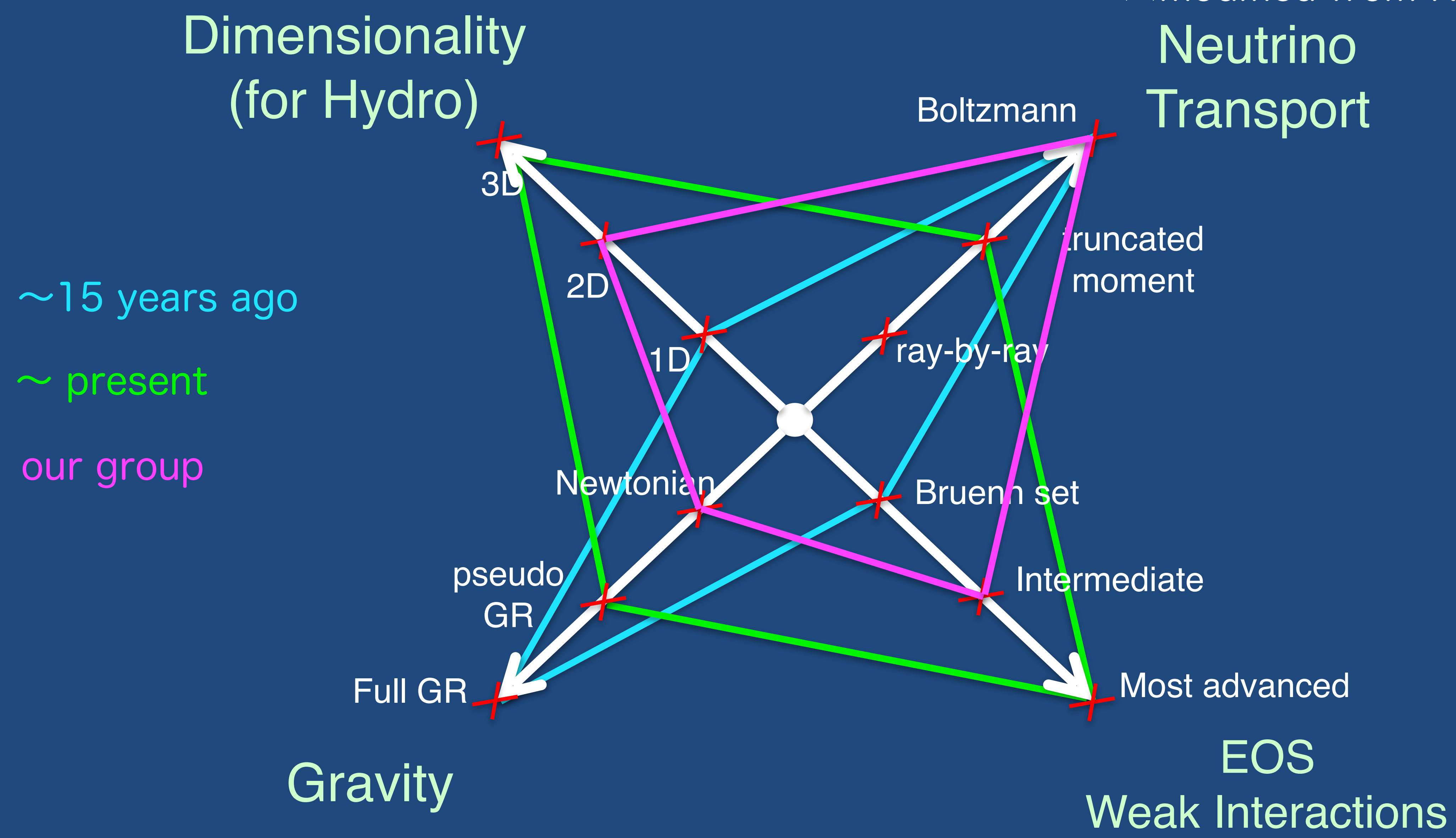




The latest 3D simulations are still unable to reproduce observed explosion energies.

Progress in CCSN Modeling

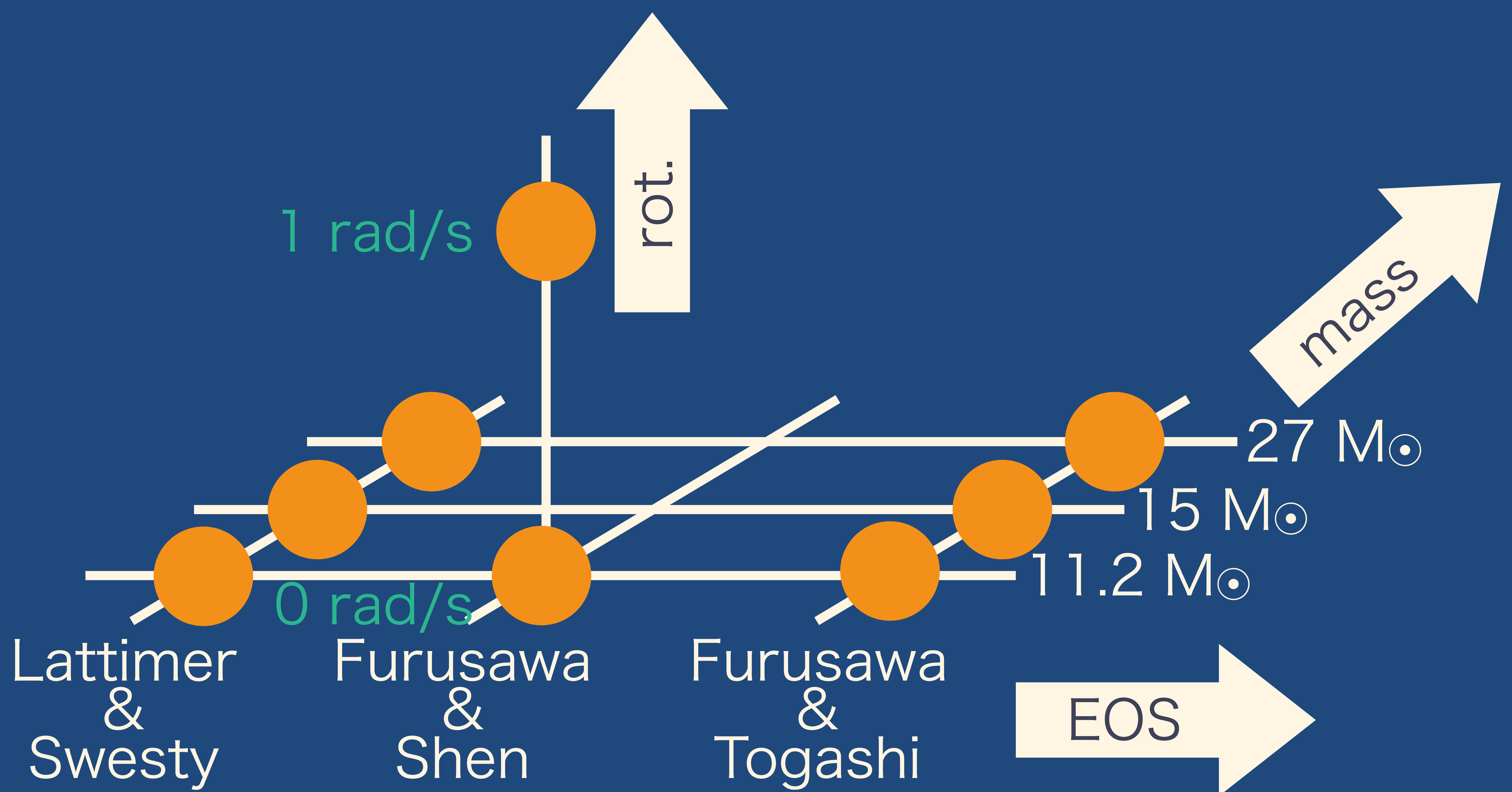
※modified from Nagakura's slide



✓ We have stuck to the Boltzmann solver for ν transport.

- 2 spatial dimensions under axisymmetry

Numerical Models Run So Far



Basic Equations

- ✓ Relativistic Boltzmann eqs. are solved with Newtonian gravity and hydrodynamics.

Neutrino distribution function

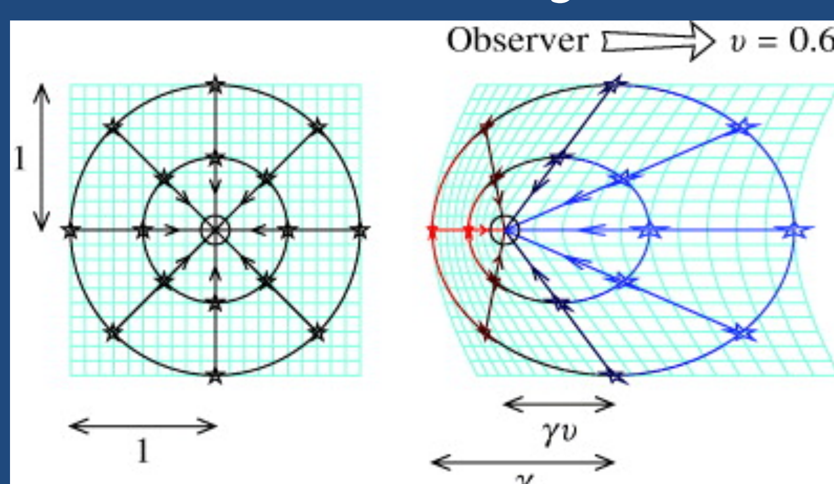
$$f(t, r, \theta, \phi; \varepsilon_\nu, \mu_\nu, \phi_\nu)$$

Boltzmann Equation

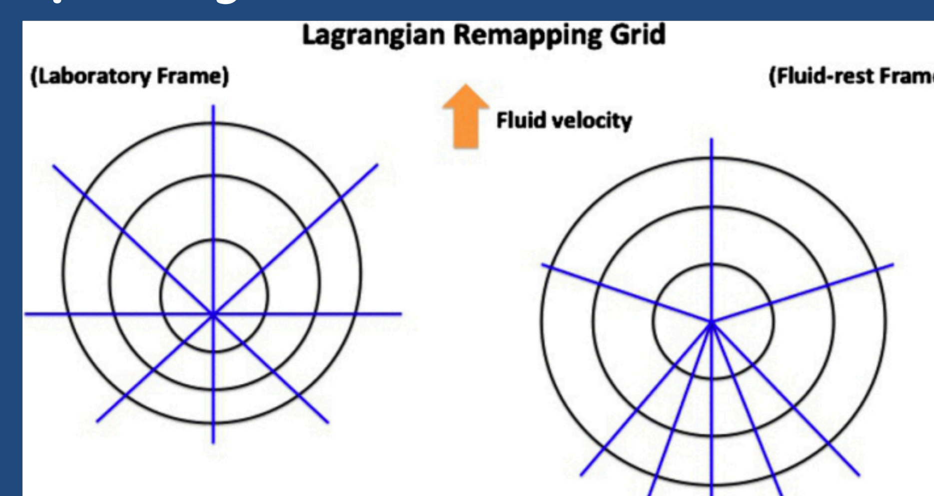
$$\frac{dx^\mu}{d\lambda}\frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\lambda}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \lambda}\right)_{\mathrm{collision}}$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\bigg|_{q_i}\left[\left(e^{\alpha}_{(0)} + \sum_{i=1}^{3}\ell_i e_i^{\alpha}\right)\sqrt{-g}f\right] - \frac{1}{\nu^2}\frac{\partial}{\partial \nu}(\nu^3 f \omega_{(0)}) + \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}(\sin\bar{\theta} f \omega_{(\bar{\theta})}) + \frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\phi}}(f\omega_{(\bar{\phi})}) = S_{\mathrm{rad}},$$

- ✓ Lorentz transformations between the fluid-rest frame and laboratory frame are properly taken into account.

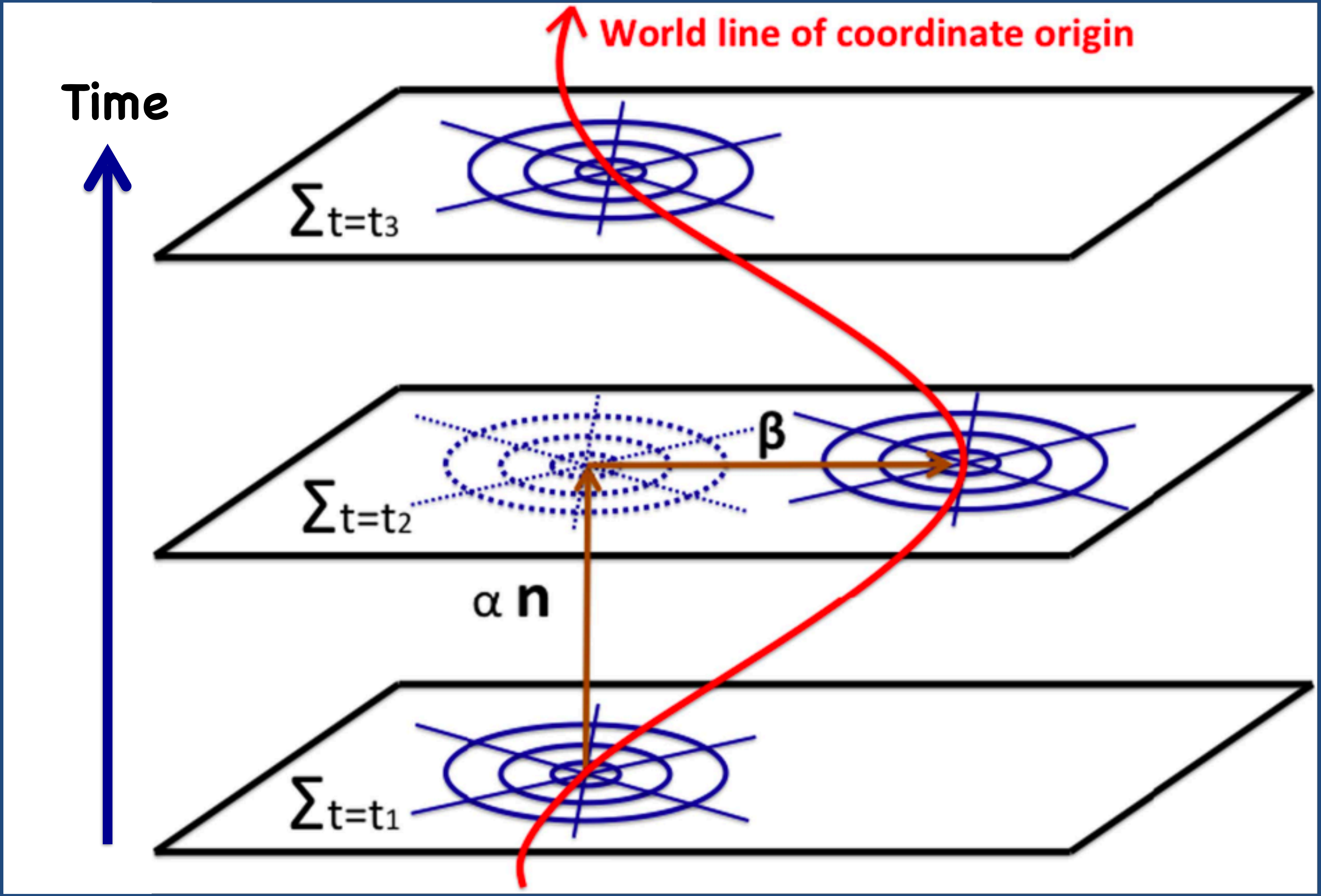


Hamilton et al. '10



Nagakura et al. '14

Tracking of Proper Motion of PNS



Progress in CCSN Modeling

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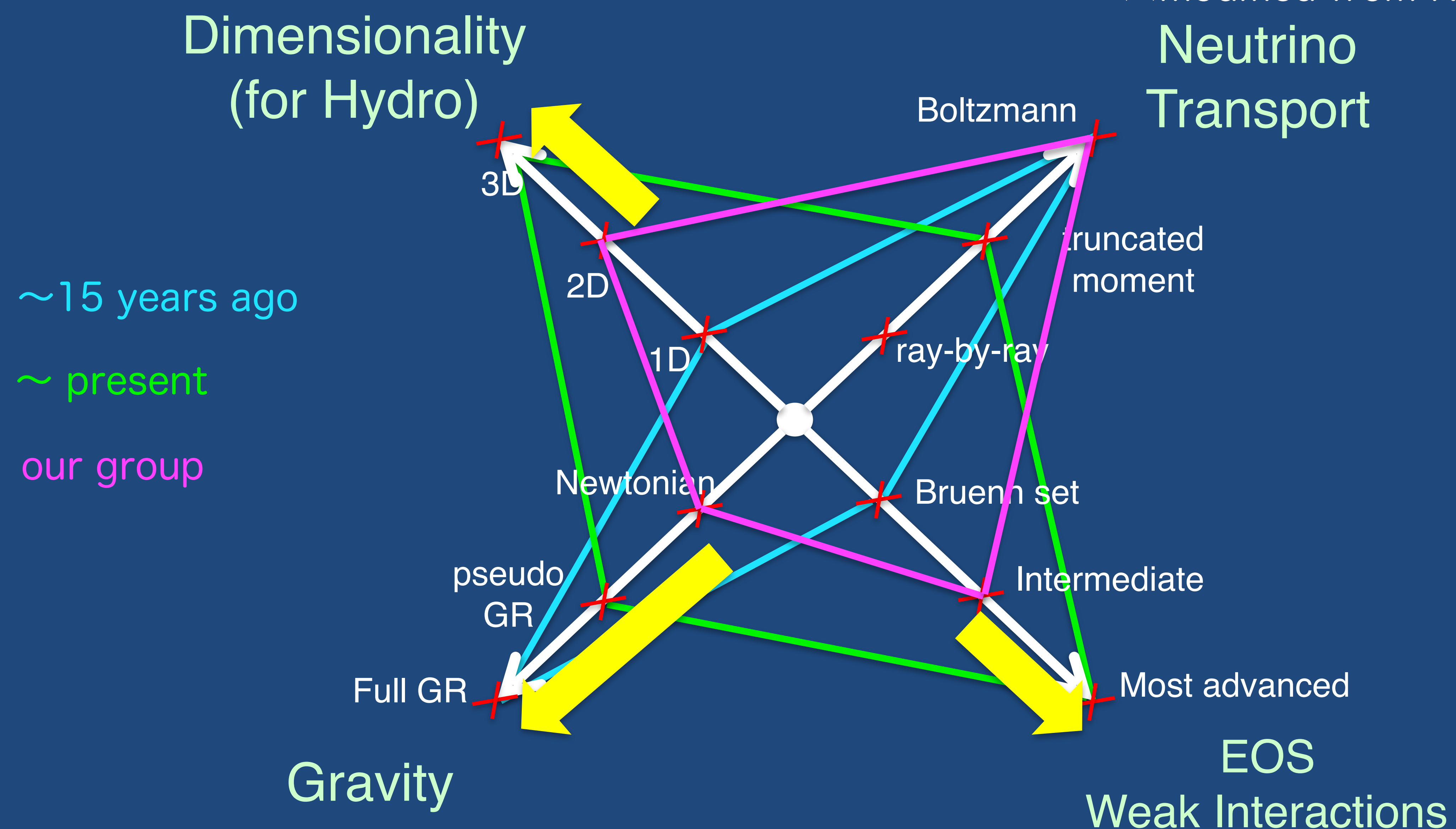
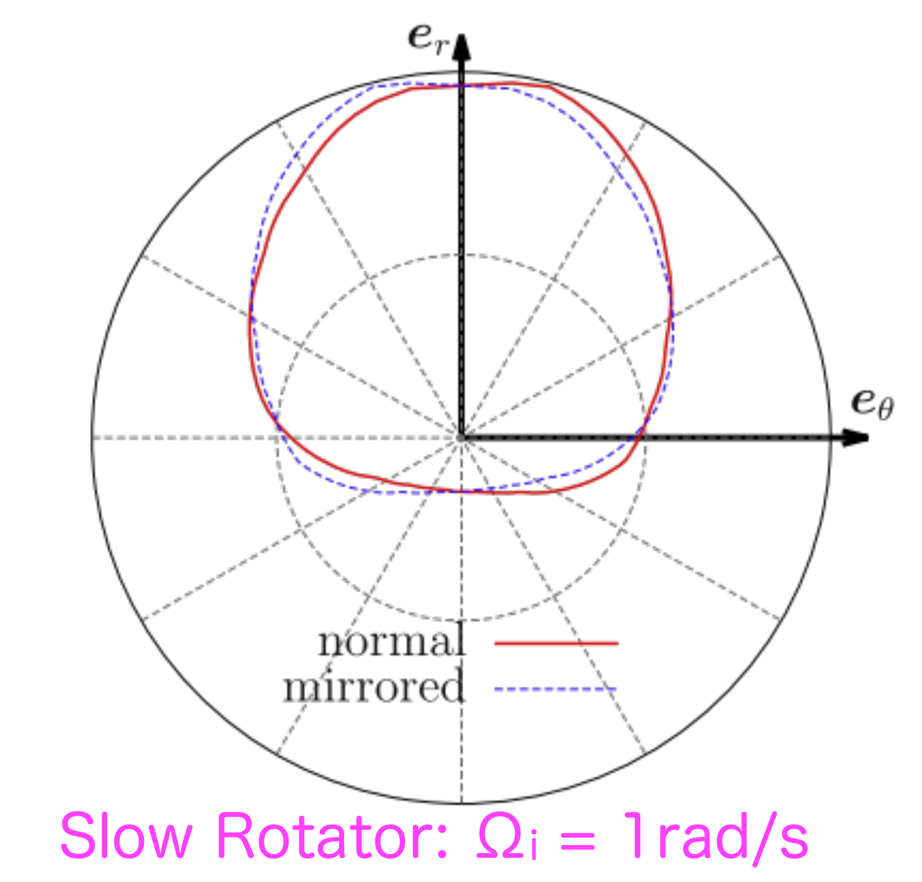


✓ We have stuck to the Boltzmann solver for ν transport.

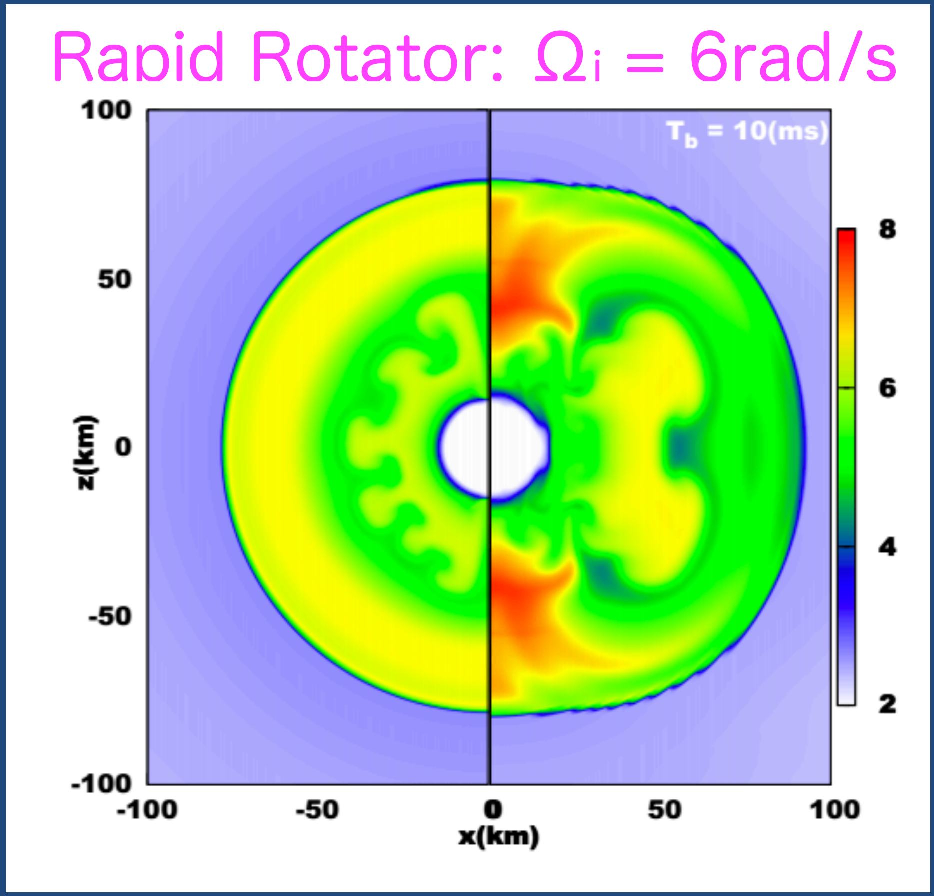
- 2 spatial dimensions under axisymmetry
- currently pushing for full GR and 3D

Probing Momentum Space

✓ Neutrino distributions in momentum space are non-axisymmetric in general even in 2D in space.



Harada et al. '19



Nagakura et al. '19

✓ The Boltzmann solver enables us to study neutrino distributions in momentum space in detail.

✓ It will help us calibrate the closure relation.

Probing Momentum Space

✓ In the truncated moment method, some closure relation is imposed by hand on the Eddington tensor.

$$E_\nu = \int d\Omega_\nu \, \varepsilon_\nu \, f_\nu(\varepsilon_\nu, \Omega_\nu)$$

$$\boldsymbol{F}_\nu = \int d\Omega_\nu \, \varepsilon_\nu \, \boldsymbol{n}_\nu \, f_\nu(\varepsilon_\nu, \Omega_\nu)$$

$$\boldsymbol{P}_\nu = \int d\Omega_\nu \, \varepsilon_\nu \, \boldsymbol{n}_\nu \boldsymbol{n}_\nu \, f_\nu(\varepsilon_\nu, \Omega_\nu)$$

flux factor : $f = \dfrac{|\boldsymbol{F}_\nu|}{E_\nu}$

Eddington tensor:

$$\boldsymbol{D} = \frac{\boldsymbol{P}_\nu}{E_\nu}$$

M1 closure

$$\boldsymbol{D} = \frac{1-\chi}{2}\boldsymbol{1} + \frac{3\chi - 1}{2}\boldsymbol{n}_F \boldsymbol{n}_F$$

$$\boldsymbol{n}_F = \boldsymbol{F}_\nu / |\boldsymbol{F}_\nu|$$

$$\chi = \frac{3 + 4f^2}{5 + 2\sqrt{4 - 3f^2}}$$

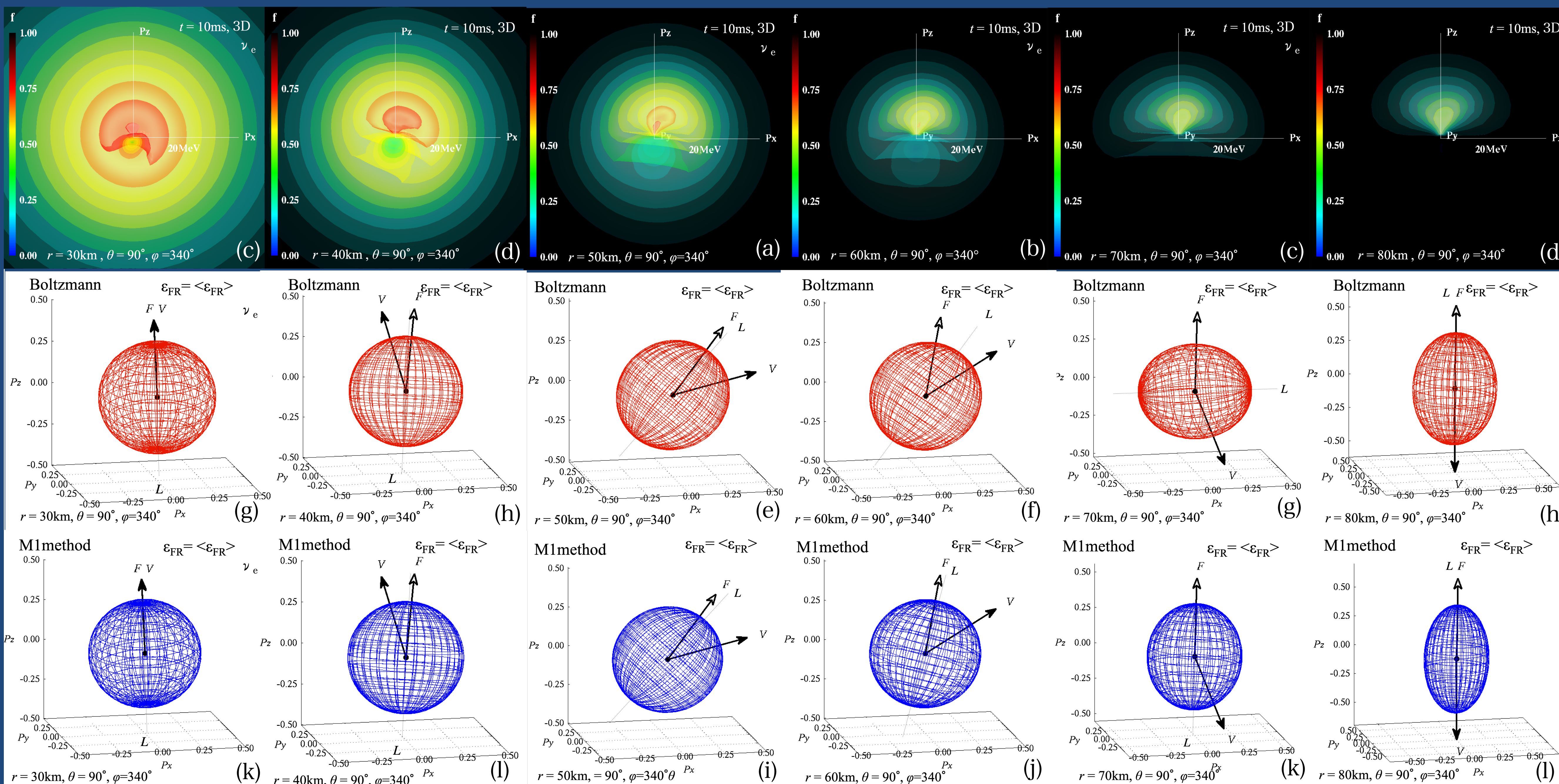
✓ Since the Eddington tensor is a second order symmetric tensor, it may be characterized as ellipsoid.

Probing Momentum Space

Iwakami et al. ’19

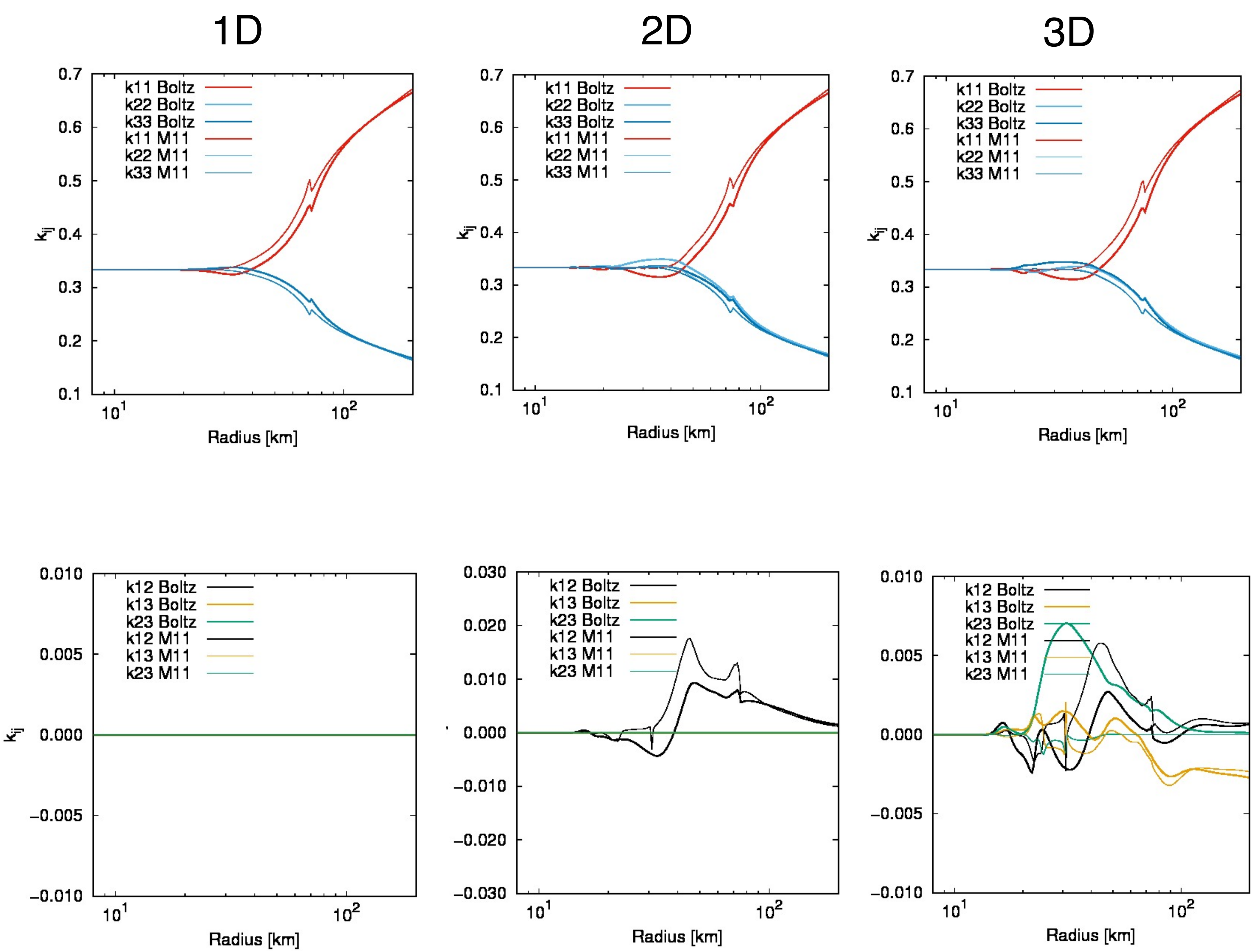
$t_{\rm pb} = 10{\rm ms},\ \theta \approx 90^\circ,\ \phi \approx 340^\circ$

r = 30km r = 40km r = 50km r = 60km r = 70km r = 80km



Comparison of Eddington-Tensor between the M1 method and the Boltzmann transport

Iwakami et al. ’19



対角成分

1Dよりも2Dと3Dの方がより内部の領域からM1とBoltzmannの結果にズレが見られる

非対角成分

3Dのk^{23}成分についてBoltzmannよりもM1の方が小さい値になる傾向がある?

Energy @ Fluid Rest-Frame 3.3MeV

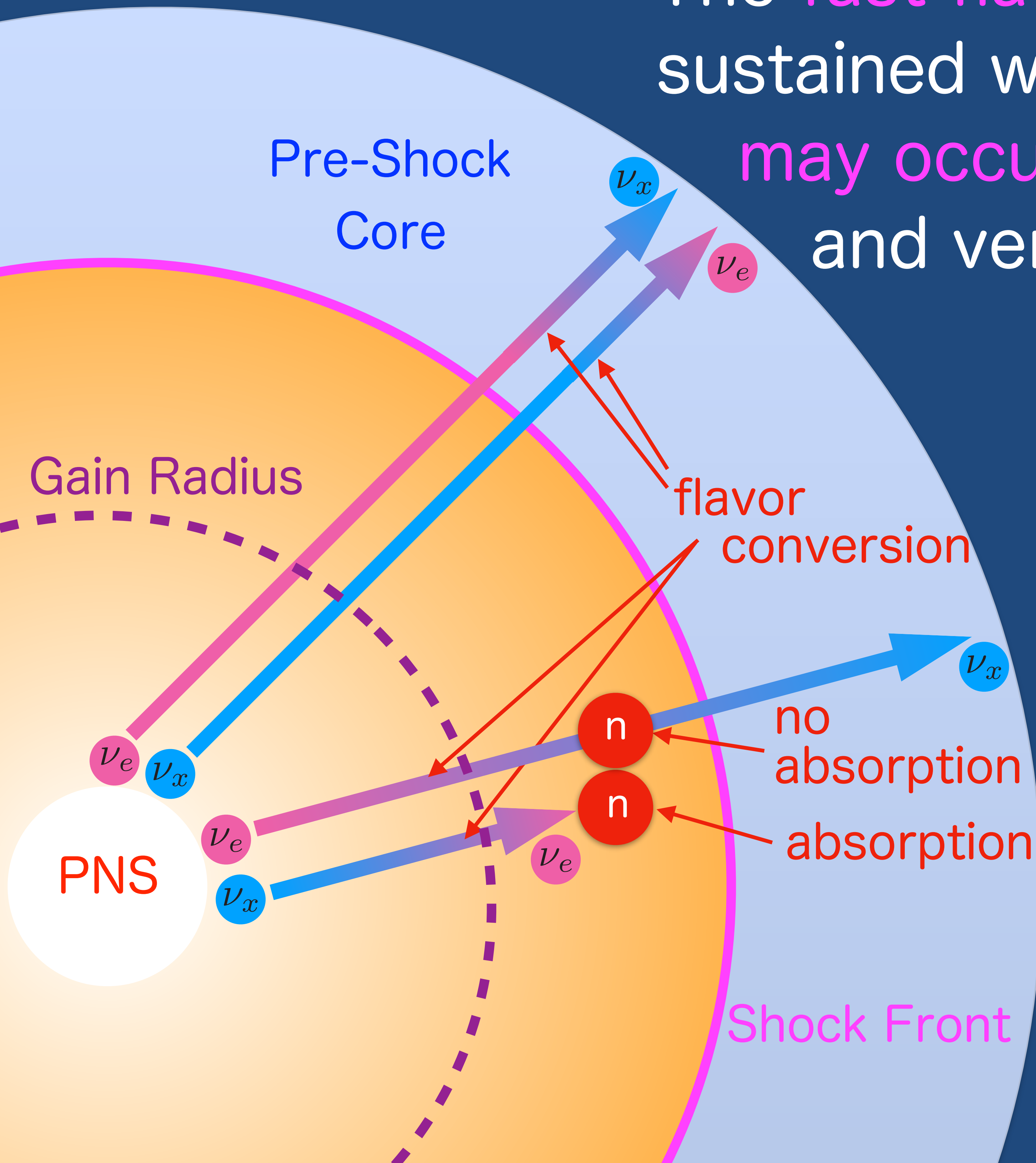
Probing Momentum Space

- ✓ The principal axes are not aligned with coordinates.
- ✓ M1 closure does a good job but is not perfect.

The Boltzmann solver puts us also in a unique position in the study of neutrino oscillations in CCSNe.

- The fast conversion mode feeds on the angular distribution difference between ν_e and $\bar{\nu}_e$.

Fast Neutrino-Flavor Conversion



- ✓ The fast flavor conversion is self-sustained without the mass term and may occur near the neutrino sphere and very rapidly. Sawyer '05

  ν-potential: $\mu \sim \sqrt{2}\, G_{\mathrm{F}}\, n_\nu$

- ✓ If true, it may affect the supernova explosion, nucleosynthesis and the observation of ν.

- ✓ It feeds on the difference in angular distributions of ν_e and $\bar{\nu}_e$.

Crossing in ν Angular Distributions

✓ The fast flavor conversion is a nonlinear phenomenon but its onset can be studied linearly.

Density Matrix (in 2-flavor approx.)

$$\rho = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s_{\mathbf{p}} & S_{\mathbf{p}} \\ S^*_{\mathbf{p}} & -s_{\mathbf{p}} \end{pmatrix}$$

EOM for the small off-diagonal component

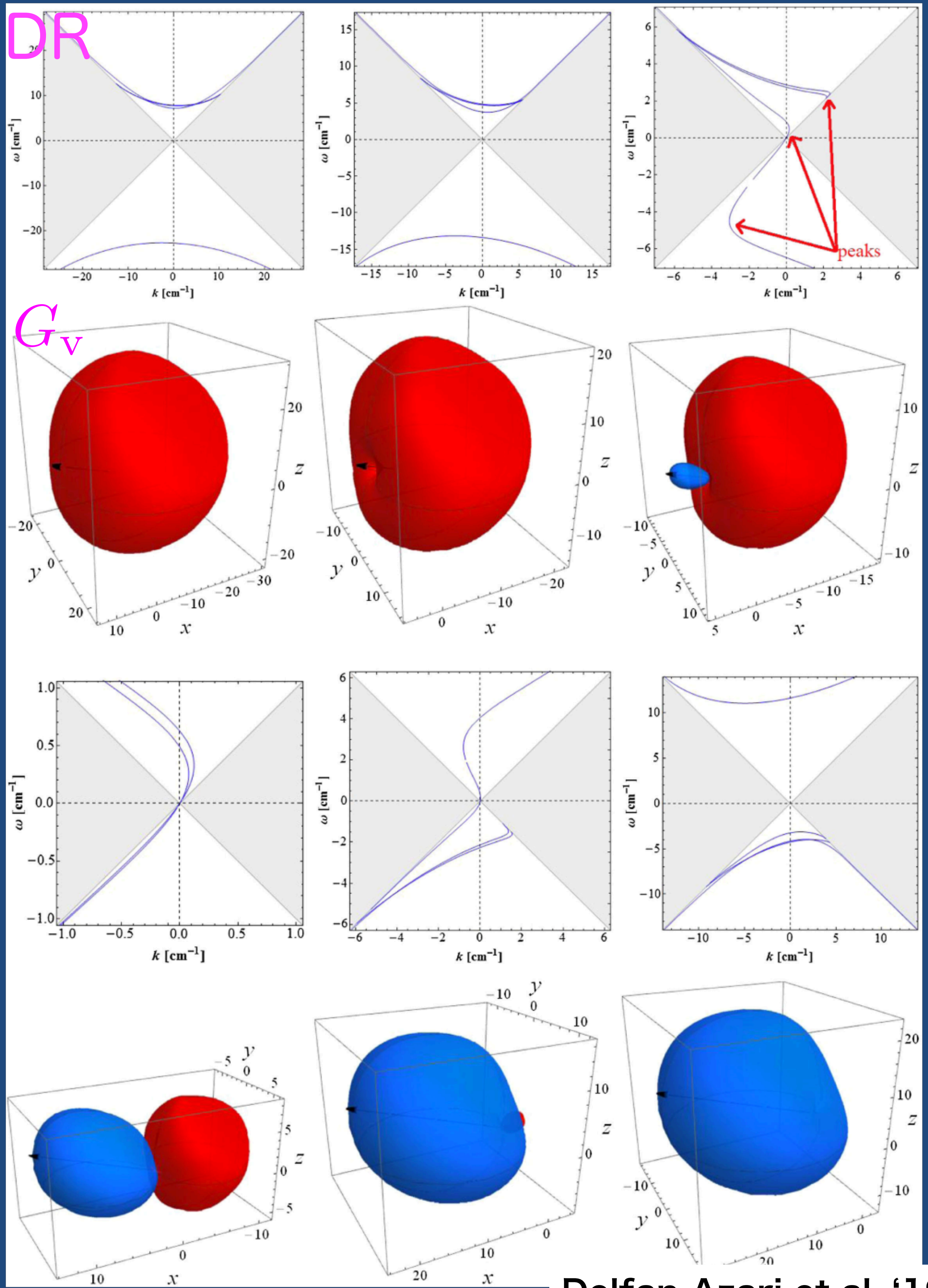
$$i(\partial_t + \mathbf{v}\cdot\nabla_{\mathbf{r}})S_{\mathbf{v}} = v^\mu(\Lambda_\mu + \Phi_\mu)S_{\mathbf{v}} - \int d\Upsilon' v^\mu v'_\mu G_{\mathbf{v}'} S_{\mathbf{v}'}$$

Dispersion Relation (DR)

$$D(\omega, \boldsymbol{k}) = 0$$

✓ The angular distributions are the important ingredient for the fast flavor conversion.

$$G_{\mathbf{v}} = \sqrt{2}G_F \int_0^\infty \frac{dE E^2}{2\pi^2}[f_{\nu_e}(E,\mathbf{v}) - f_{\bar{\nu}_e}(E,\mathbf{v})]$$



Delfan Azari et al. ‘19

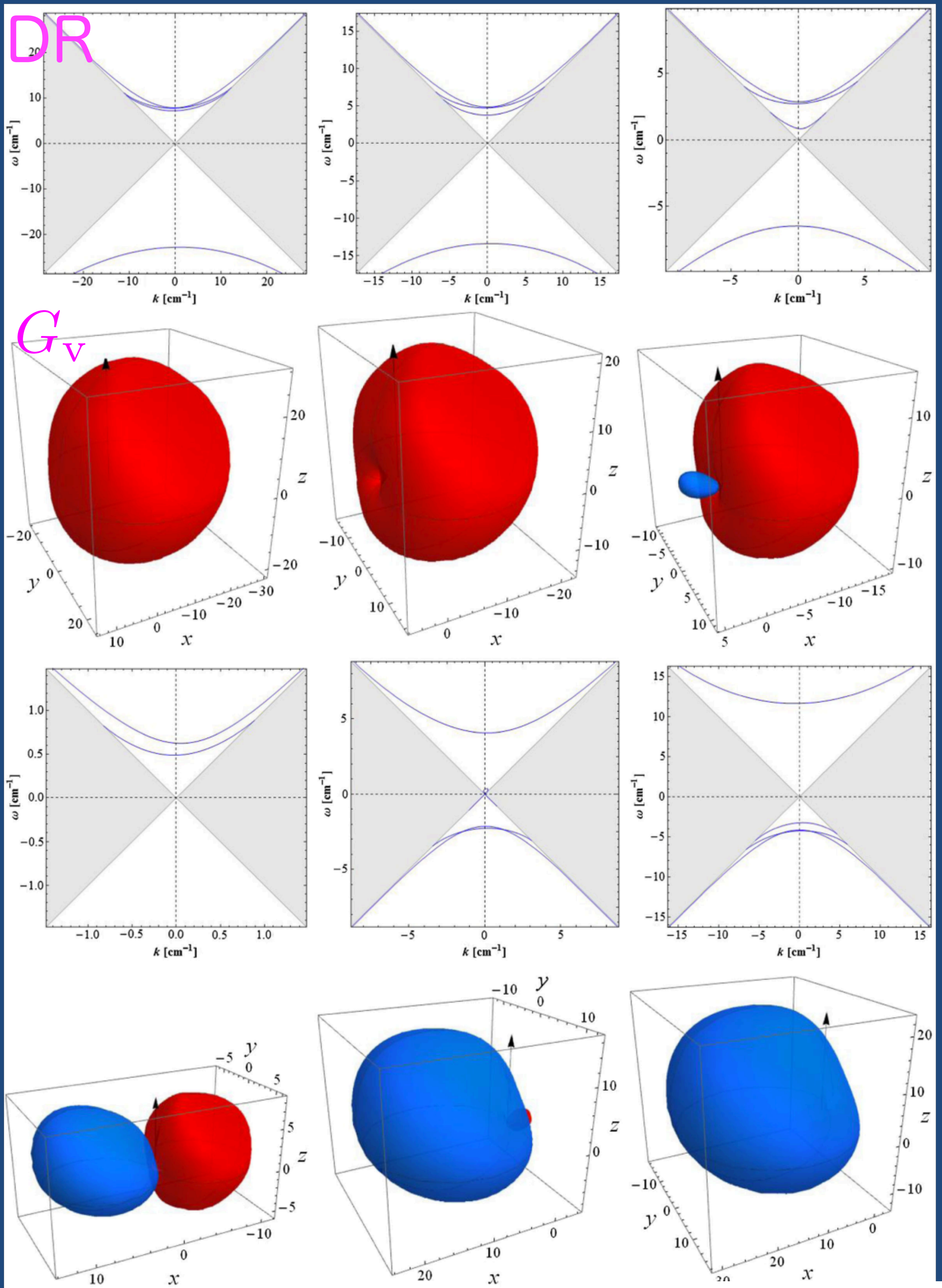
Crossing in ν Angular Distributions

DR depends on the direction of k.

✓ The sign change of G_v, or the crossing in angular distributions, seems indeed to be the indication of the fast flavor conversion.

$$G_{\mathbf{v}} = \sqrt{2} G_F \int_0^\infty \frac{dE E^2}{2\pi^2} [f_{\nu_e}(E, \mathbf{v}) - f_{\bar{\nu}_e}(E, \mathbf{v})]$$

We will hence look for the crossing first.

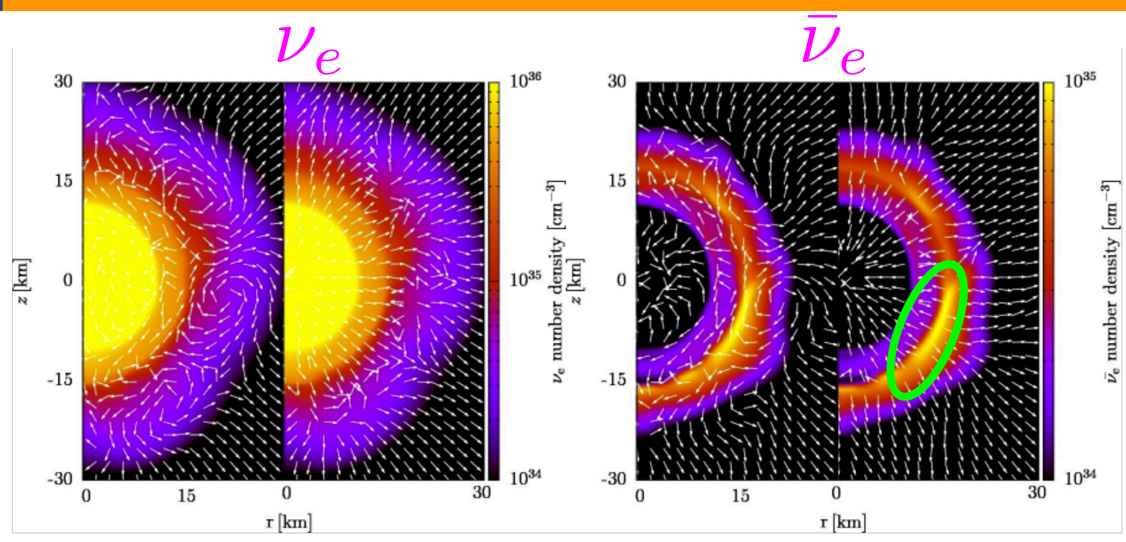


Delfan Azari et al. '19

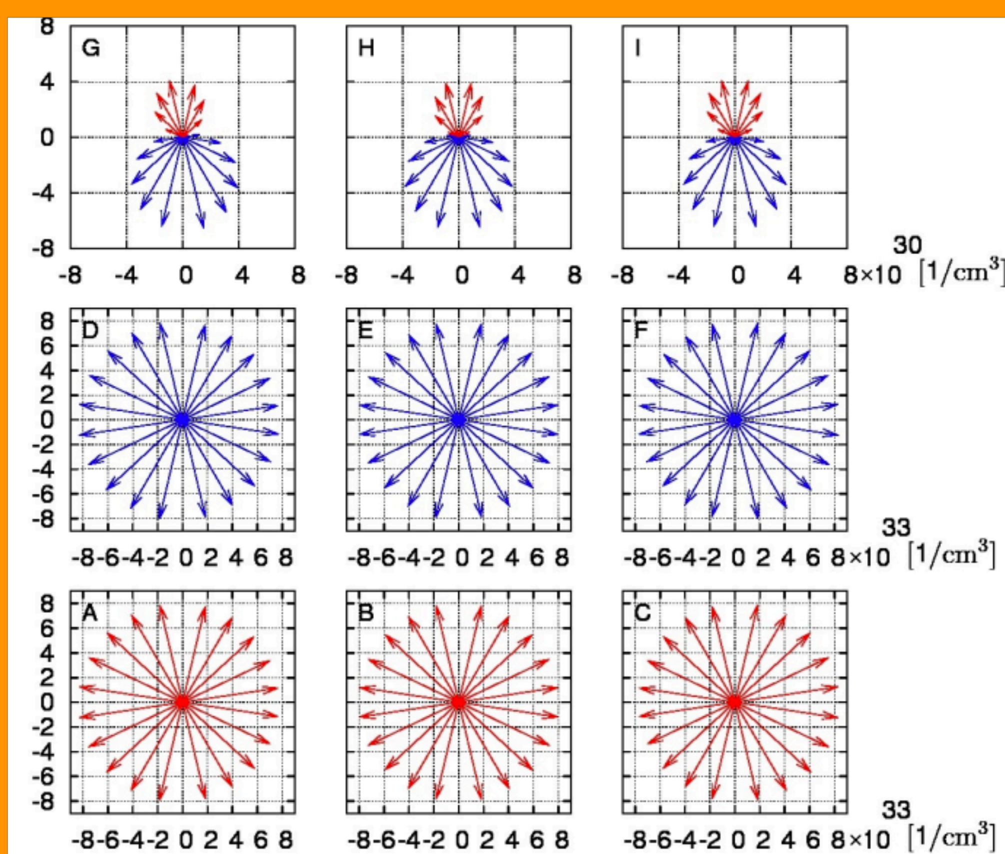
We have spotted three possible cases.

✓ Inside ν sphere

Delfan Azari et al. '19

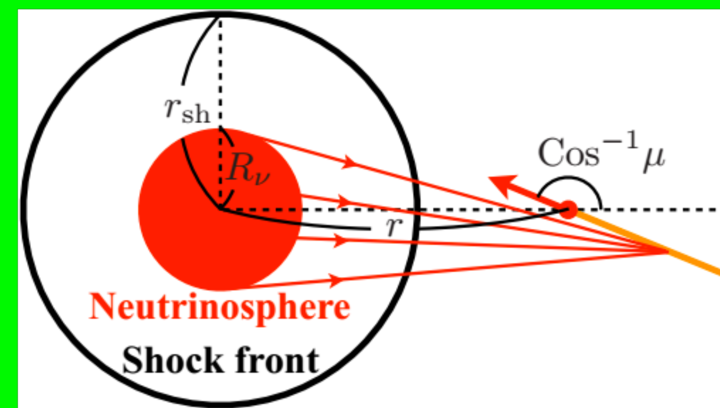


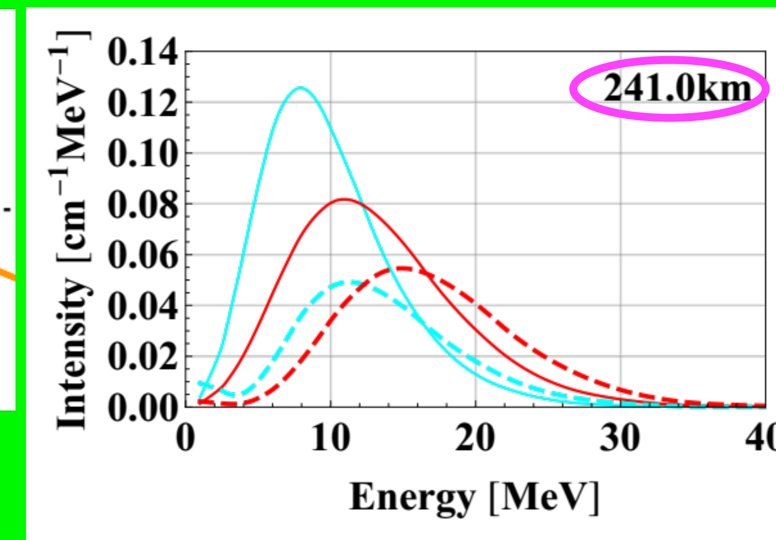
Angular Distributions of ν_eと$\bar{\nu}_e$



✓ Pre-Shock Region

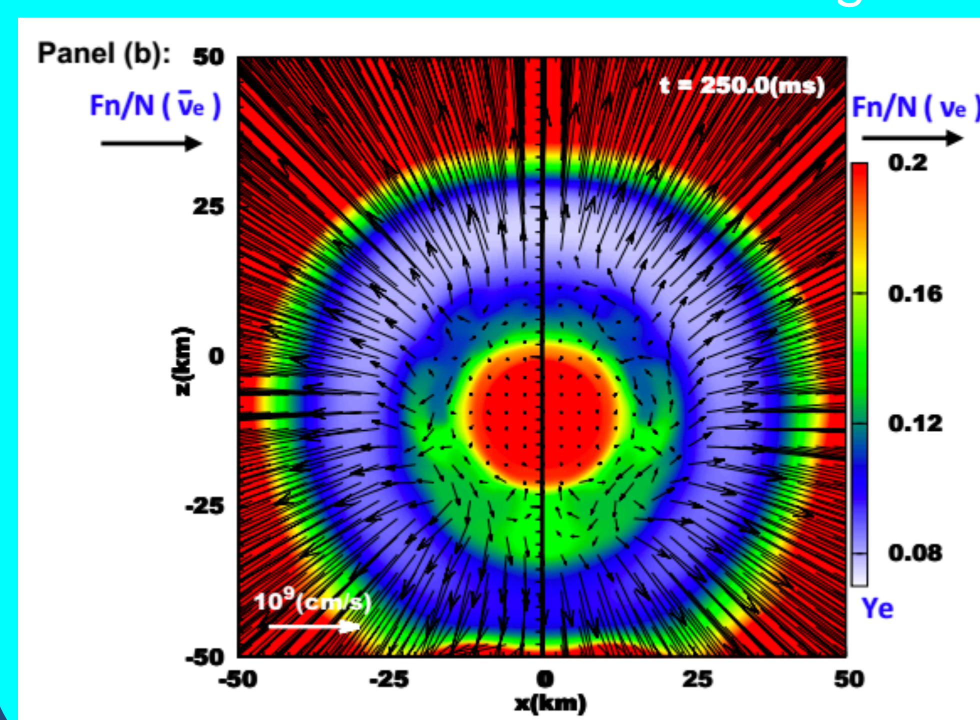
Morinaga et al. '19

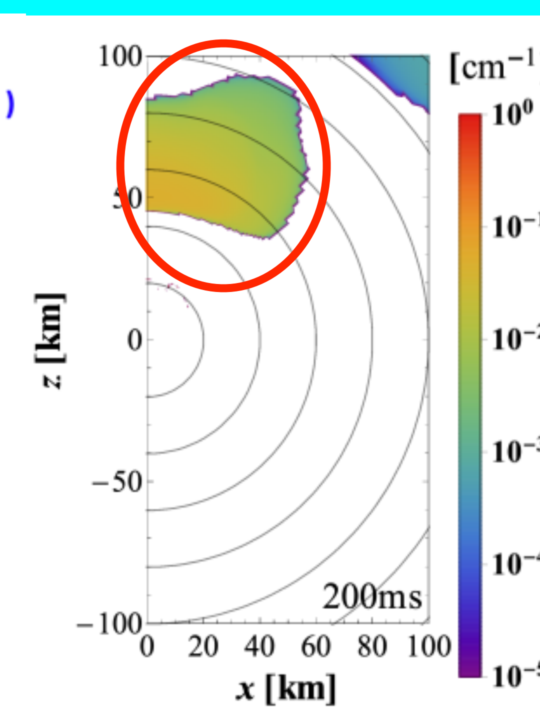




✓ Post-Shock Region

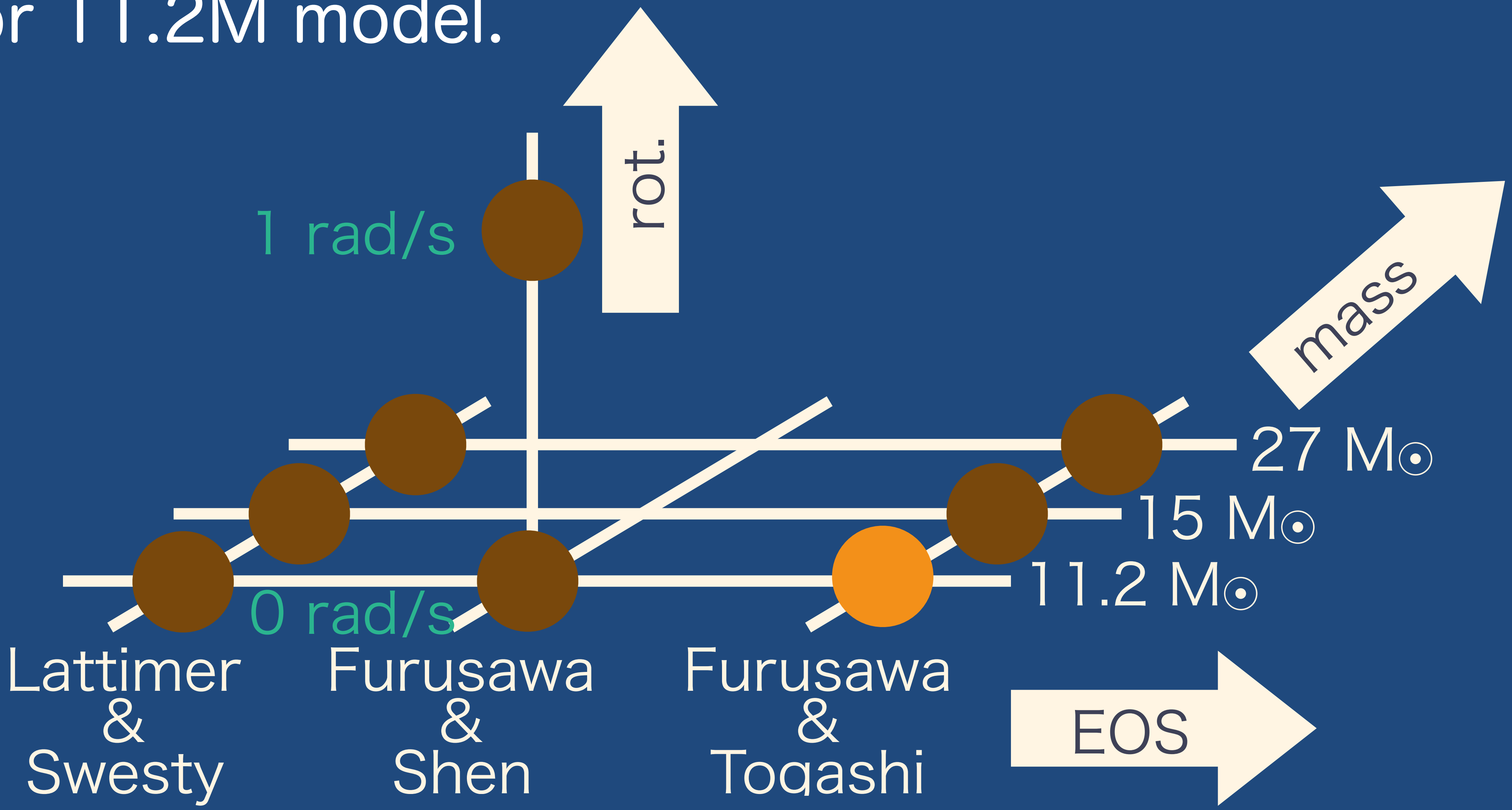
Nagakura et al. '19





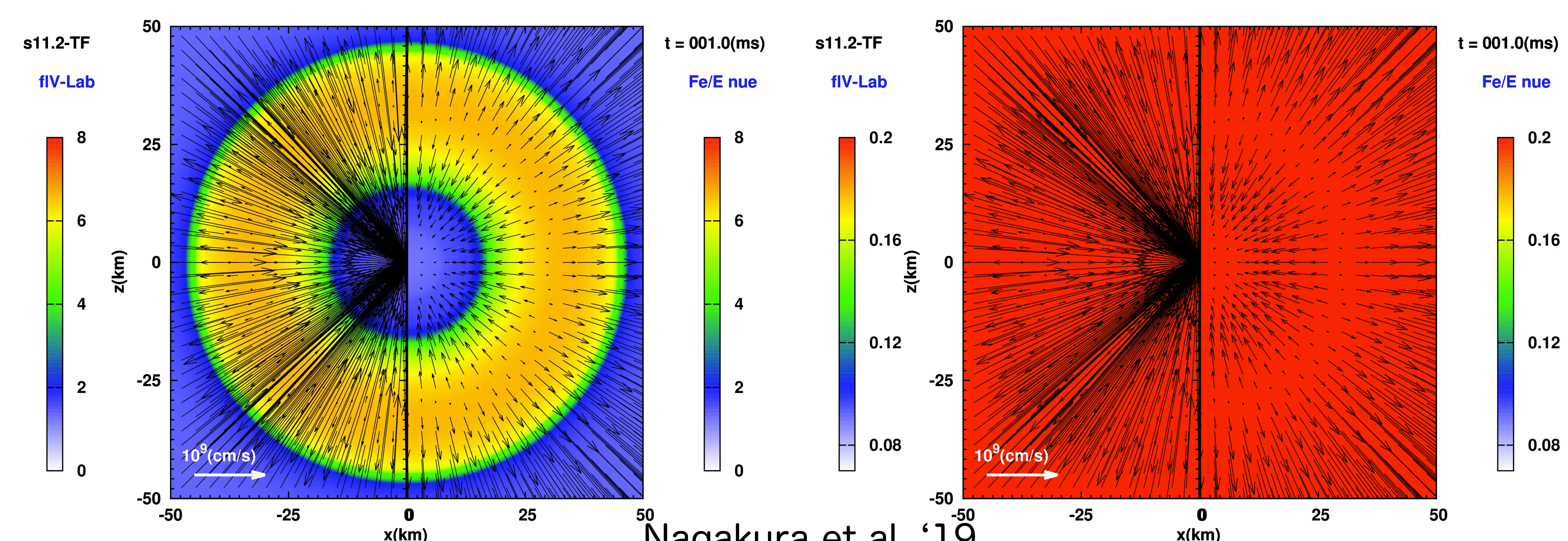
Fast Flavor Conversion in the Post-Shock Region

- ✓ We found the ELN crossing inside the shock wave at $t_{pb} \gtrsim 200$ms in one of our latest 2D simulations for 11.2M model.

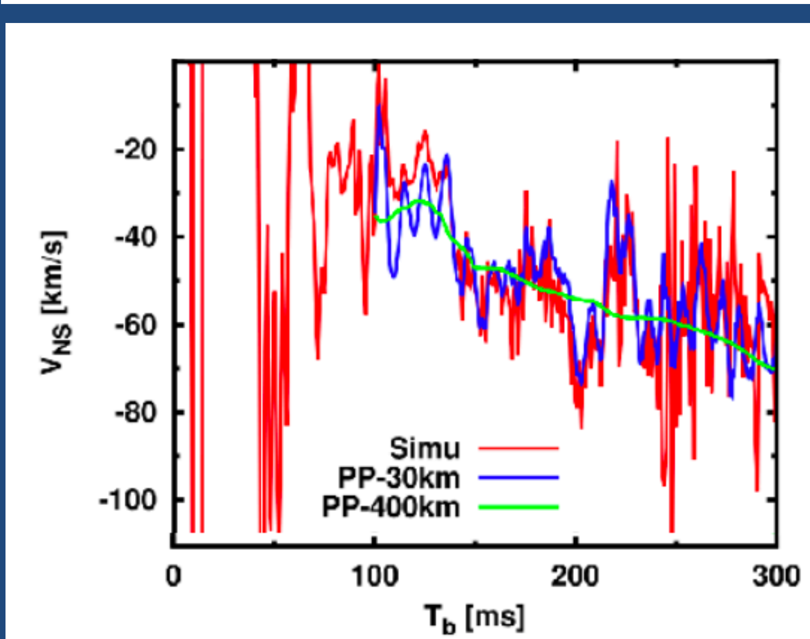


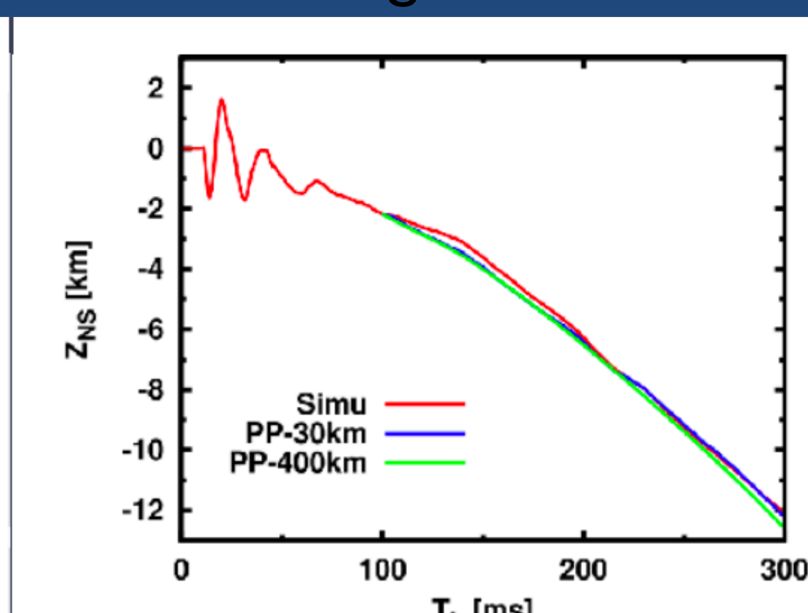
- ✓ In this model, PNS starts to move southward at around the same time.

Asymmetric ν Emissions and Proper Motion of Proto Neutron Star

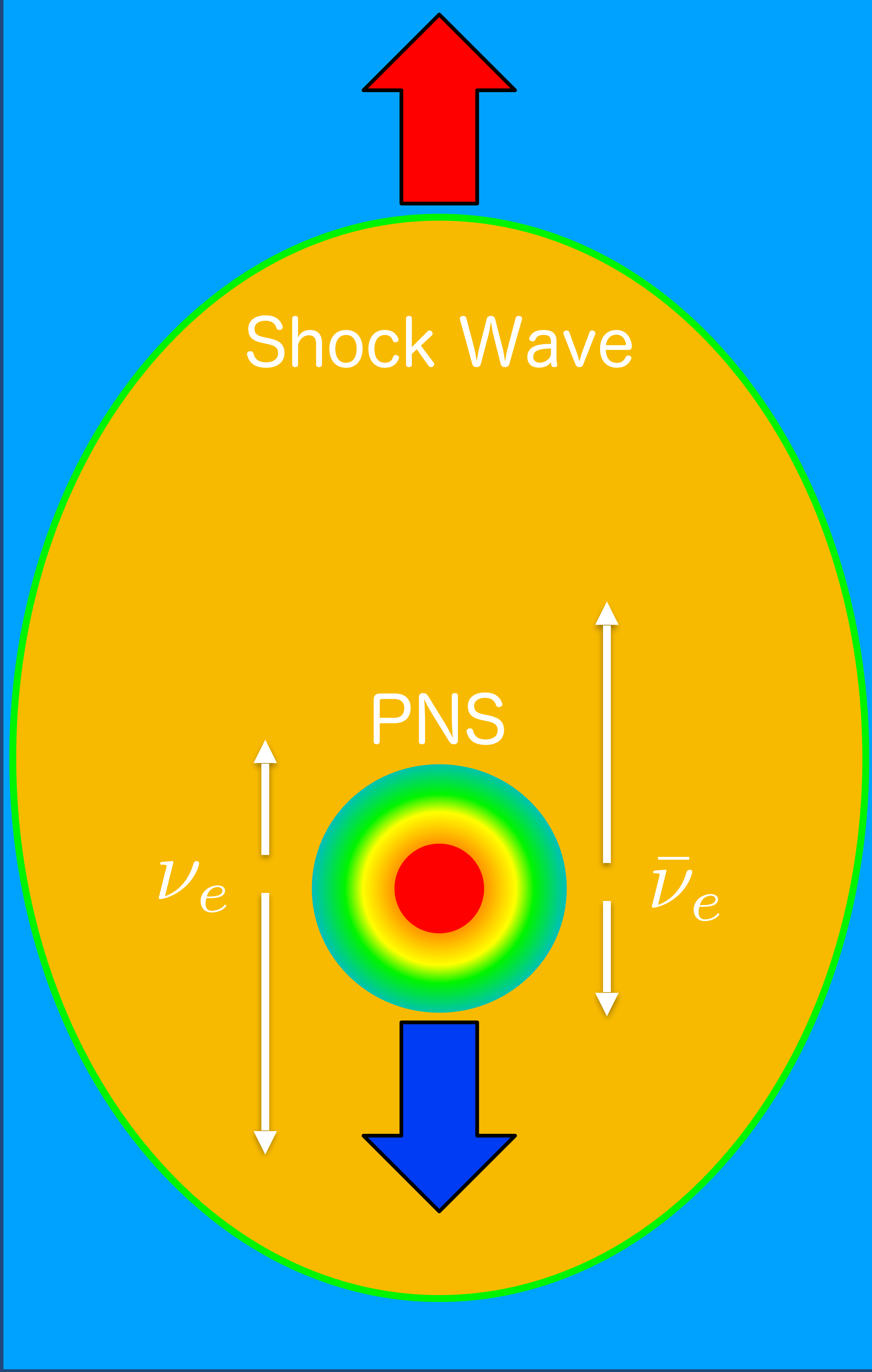


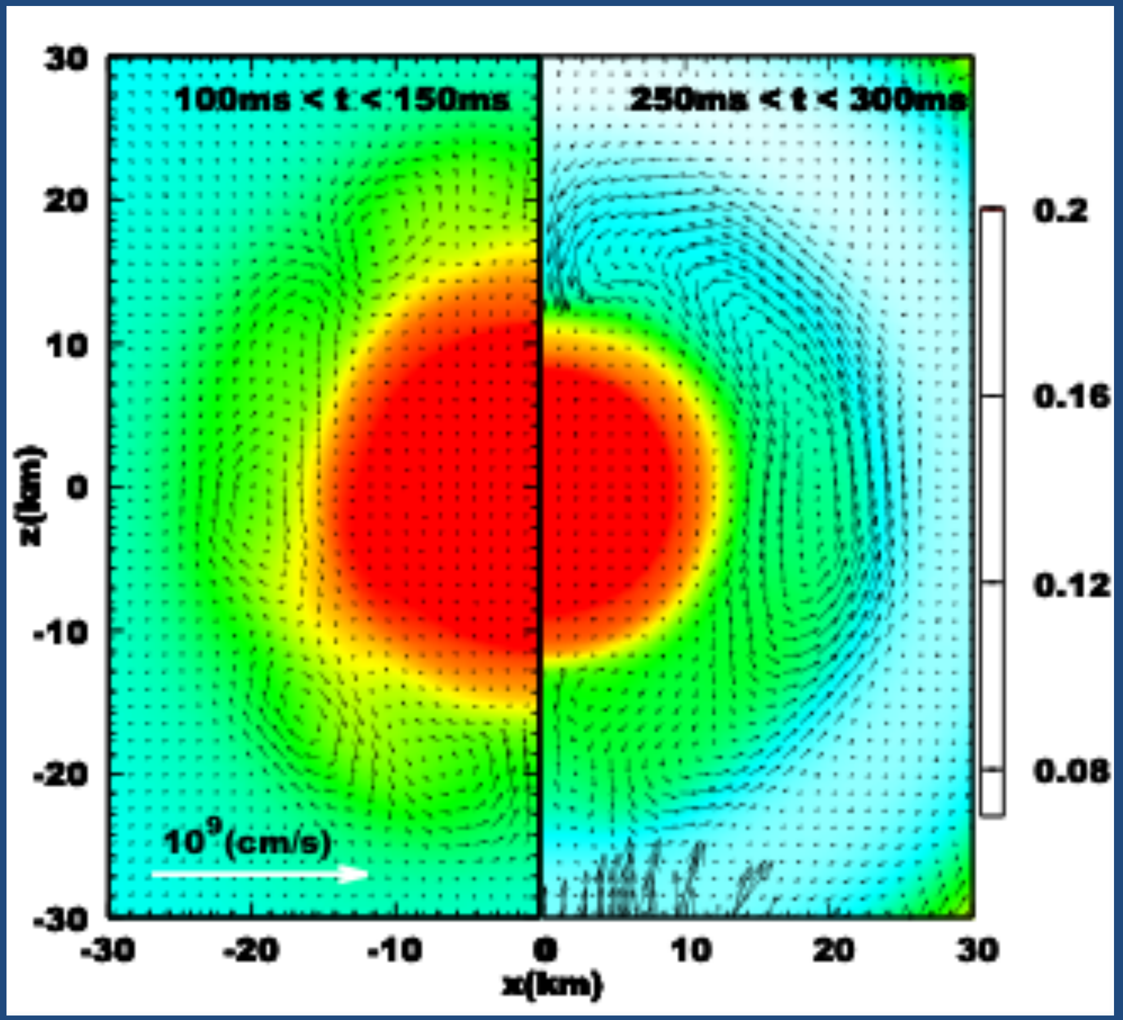
Nagakura et al. '19





In some cases, a strong asymmetry in ν emission is sustained, in association with a proper motion of PNS.

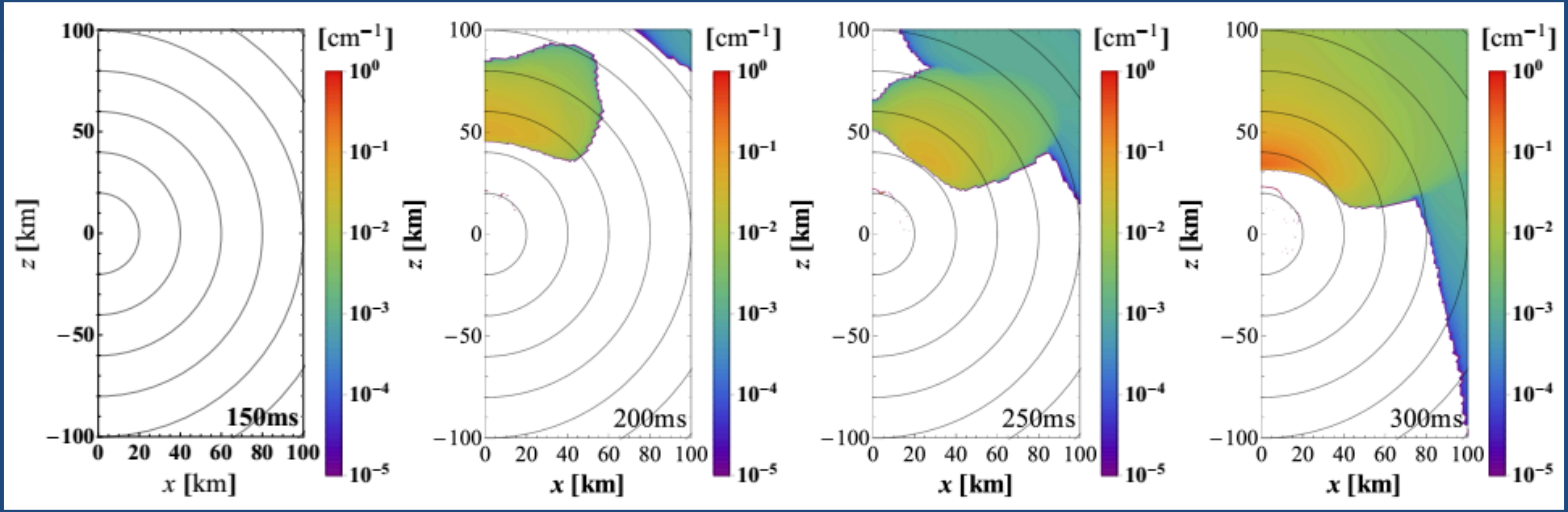




✓ Interplays between PNS motions and asymmetric neutrino emissions in operation

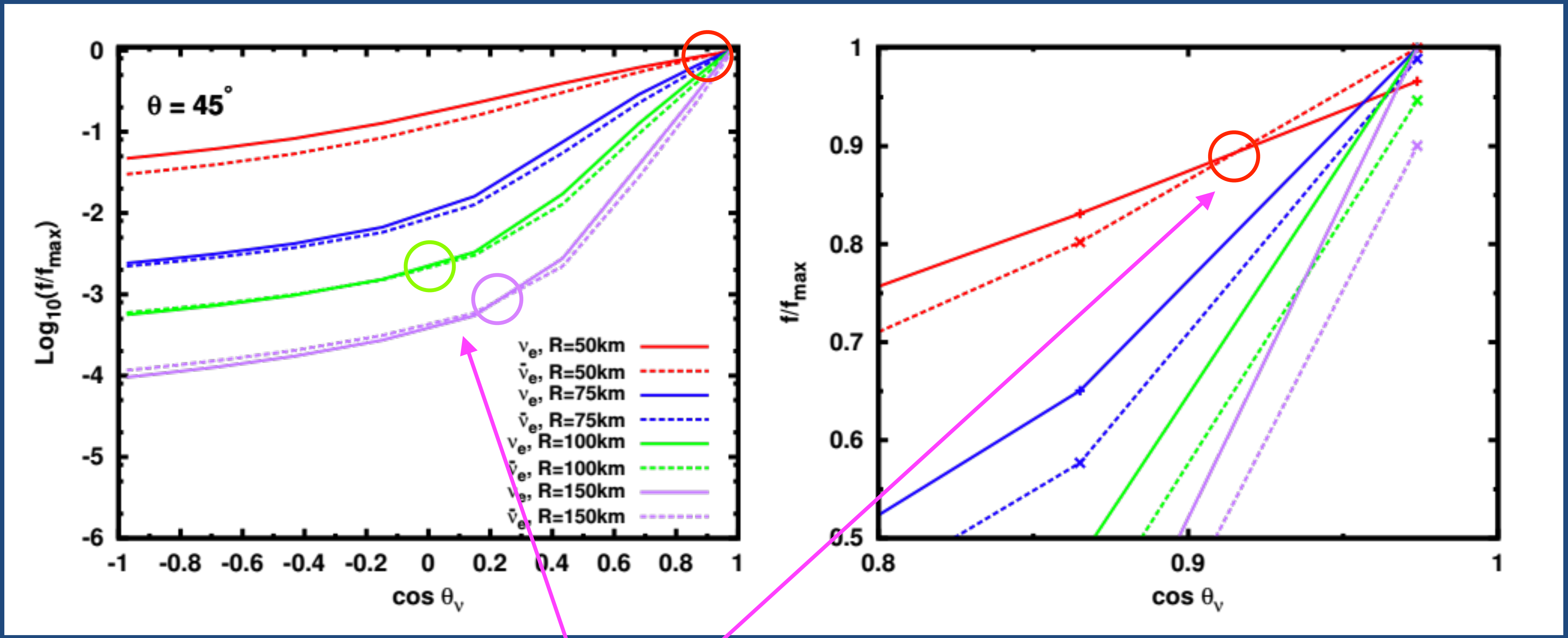
1. Breaking of up-down symmetry in matter distributions by PNS motions
2. Appearing of sustained lateral circular matter motion in the envelope of PNS
3. Sustained asymmetries in the Ye distribution and neutrino emissions

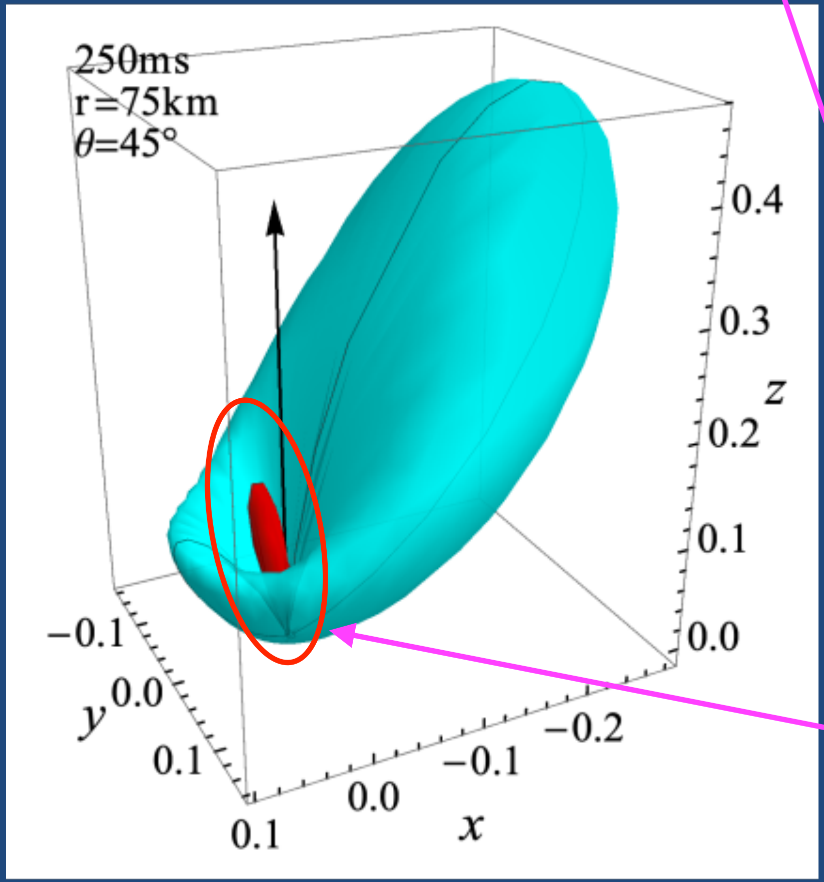
Locations of Fast Flavor Conversion in the Post-Shock Region



The domains of possible fast flavor conversion are expanding with time in the direction of the stronger shock expansion.

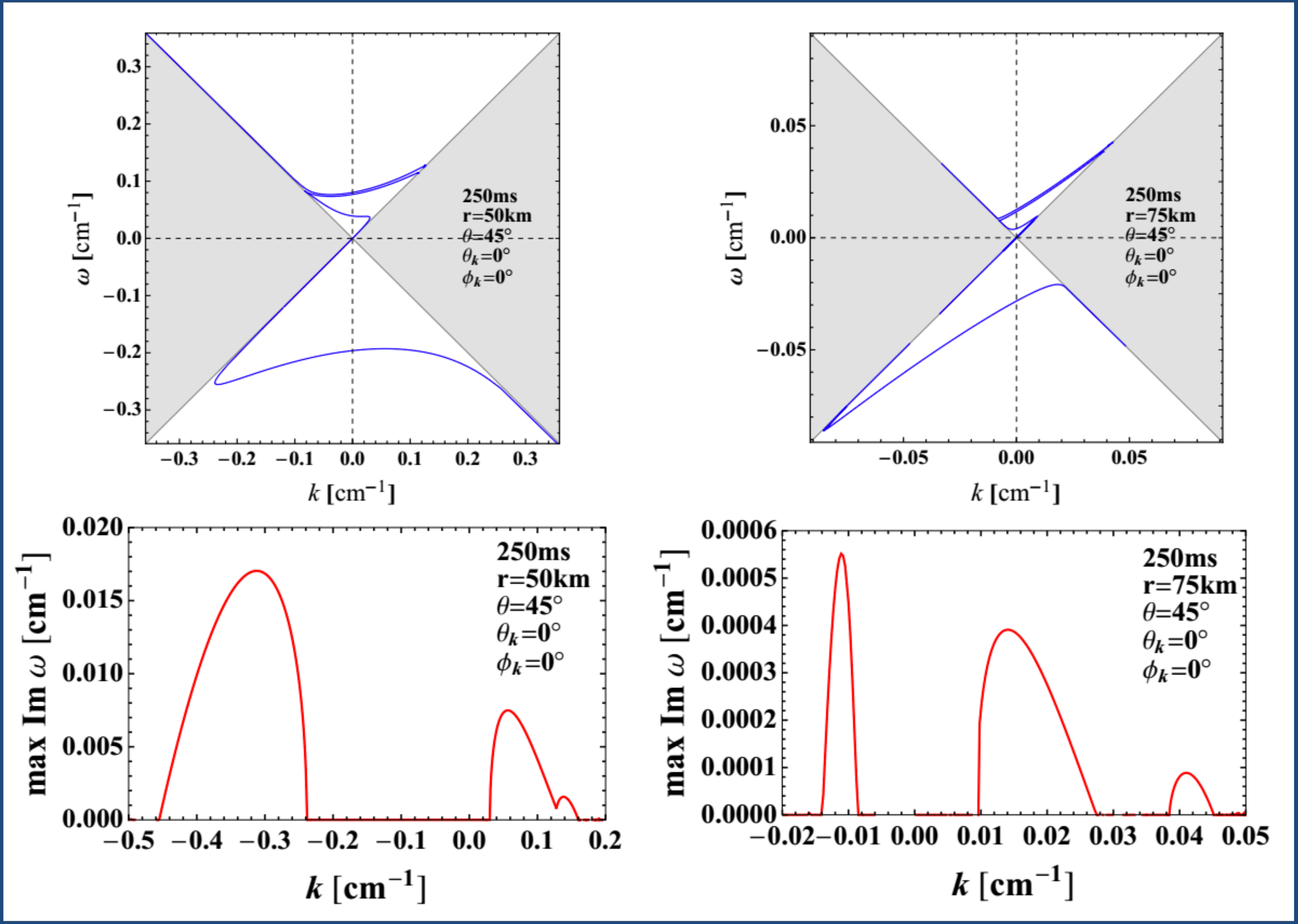
ELN Crossings in the Post-Shock Region





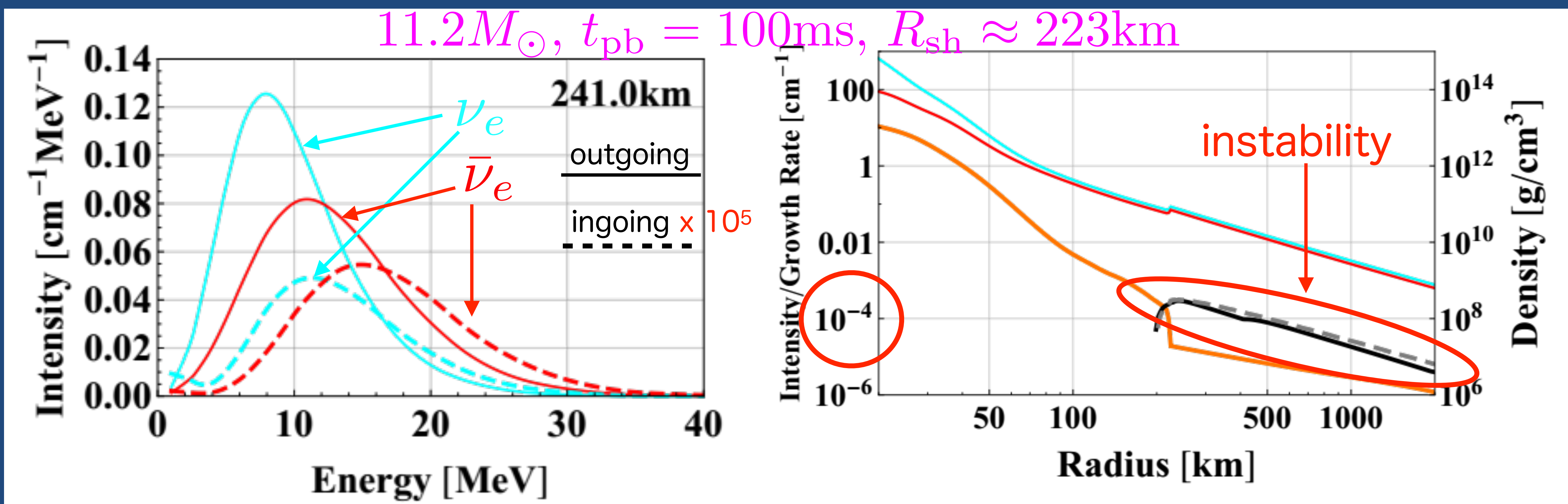
- ✓ $\bar{\nu}_e$ is dominant in the outward direction at r ~ 50km, since it is more forward-peaked.
- ✓ It is dominant in the inward direction at r ≳ 100km, since it is emitted or scattered more frequently.
- ✓ It is dominant in the non-radial direction at r ~ 75km.

Linear Growth rates of Fast Flavor Conversion in the Post-Shock Region



Fast Flavor Conversion Ahead of Shock Wave

✓ We found the sign change in ELN for the ingoing ν's outside the stalled shock at $t_{pb} \gtrsim$ 100ms in our 1D simulations.



✓ We found it also in the 1D models of the MPA group that Tamborra et al. '17 employed in their survey.

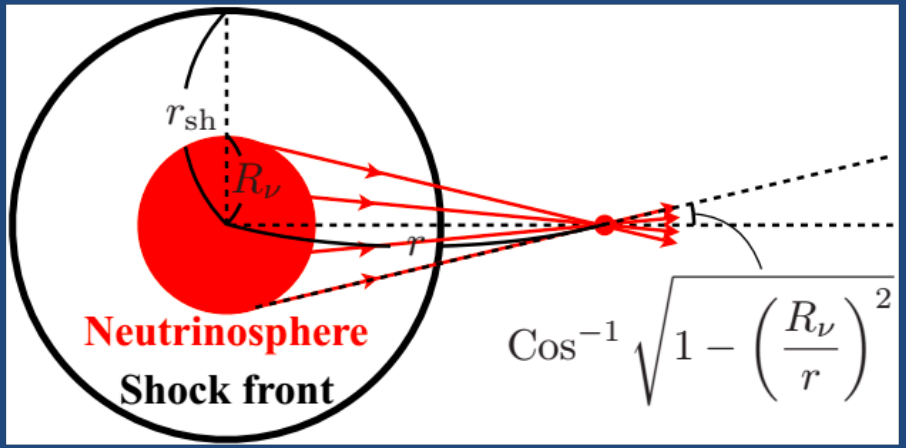
Morinaga et al. '19

✓ It turns out that the crossing is induced by coherent back-scatterings of neutrinos on heavy nuclei.

Bulb Model

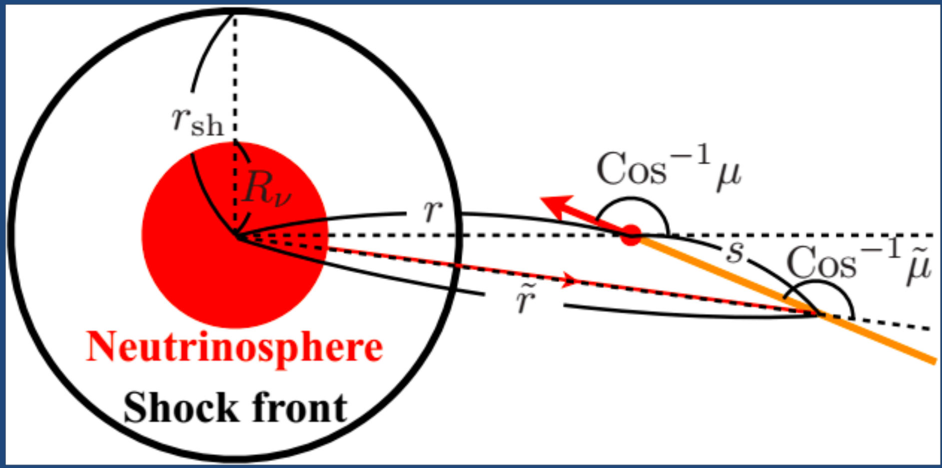
outgoing component

$$\mathscr{G}_\nu^{\rm bulb}(\mu) = 2\,{\rm cm}^{-1}\left(\frac{50\,{\rm km}}{R_\nu}\right)^2\left(\frac{L_\nu}{10^{52}\,{\rm erg/s}}\right)\left(\frac{10\,{\rm MeV}}{\bar{E}_\nu}\right) \times \Theta\left(\mu - \sqrt{1-(R_\nu/r)^2}\right)$$



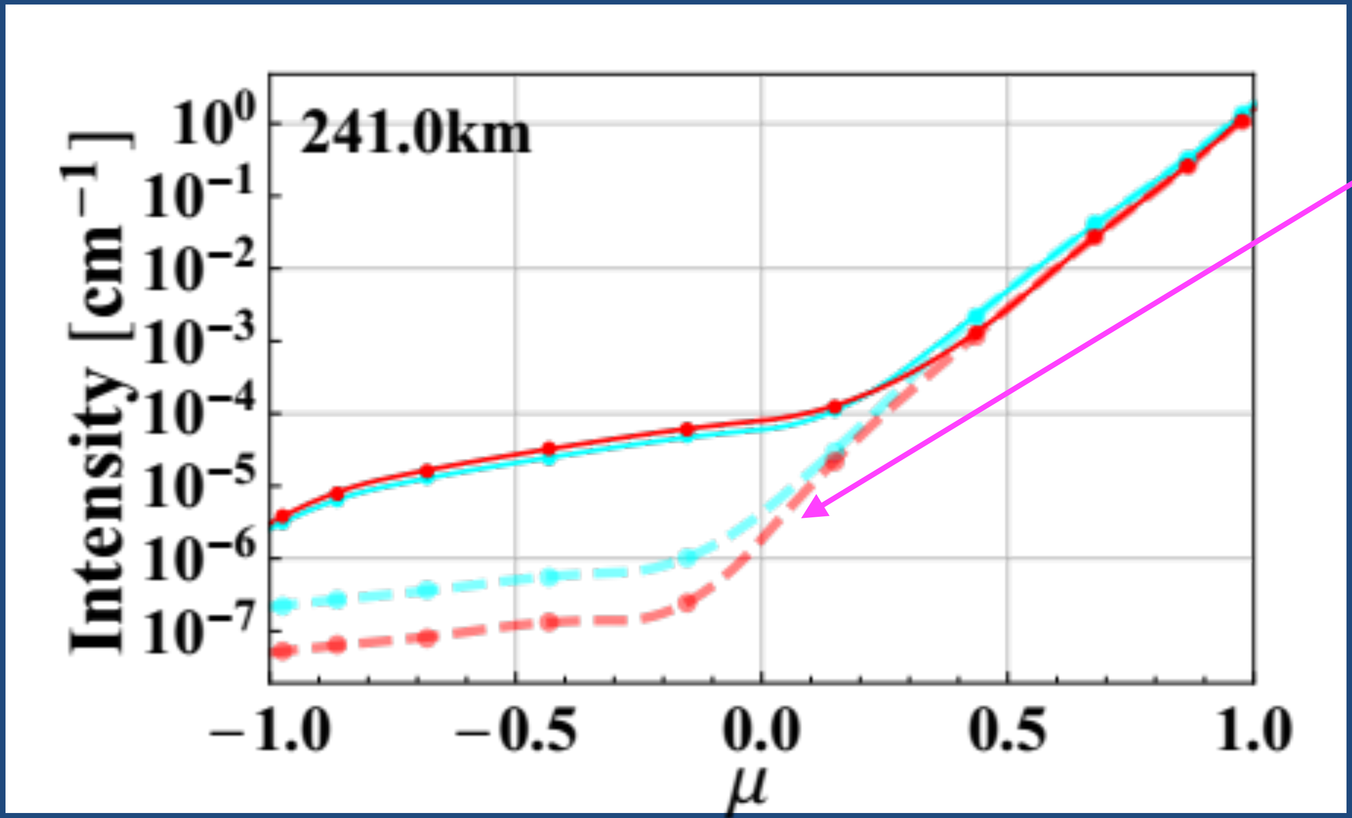
ingoing component

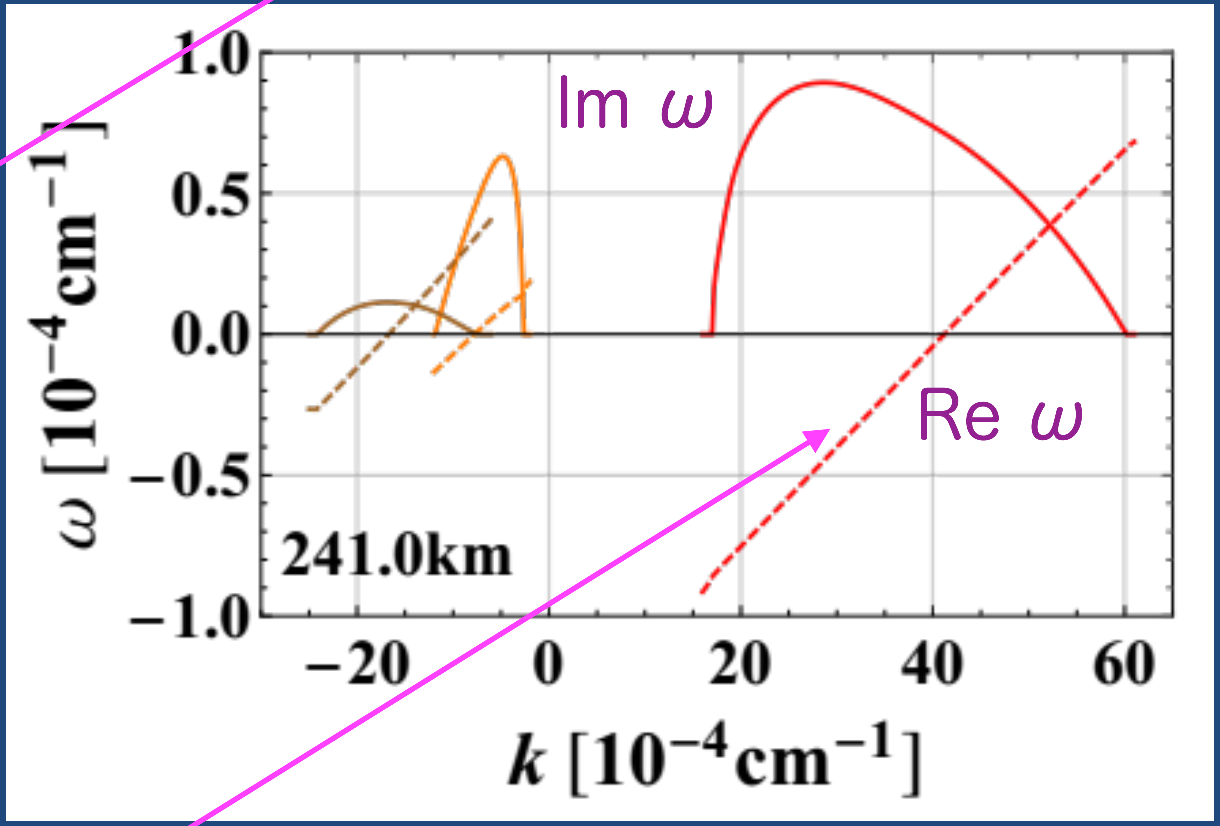
$$\mathscr{G}_\nu^{\rm scat}(\mu) \simeq 2\times10^{-4}\,{\rm cm}^{-1}\,\frac{4+\alpha_\nu}{(3+\alpha_\nu)(3+\beta)}\left(\frac{A}{56}\right) \times\left(\frac{\rho_{\rm sh}}{10^7\,{\rm g/cm^3}}\right)\left(\frac{R_{\rm sh}}{200\,{\rm km}}\right)^\beta\left(\frac{200\,{\rm km}}{r}\right)^{1+\beta} \times\left(\frac{L_\nu}{10^{52}\,{\rm erg/s}}\right)\left(\frac{\bar{E}_\nu}{10\,{\rm MeV}}\right)\left[(\mu+1)+\frac{1}{4}\left(\frac{R_\nu}{r}\right)^2\right]$$



→ If $L_{\nu_e}\bar{E}_{\nu_e} < L_{\bar{\nu}_e}\bar{E}_{\bar{\nu}_e}$, then ELN becomes negative.

✓ When the coherent scattering is turned off in the simulation, then the ELN crossing disappears.

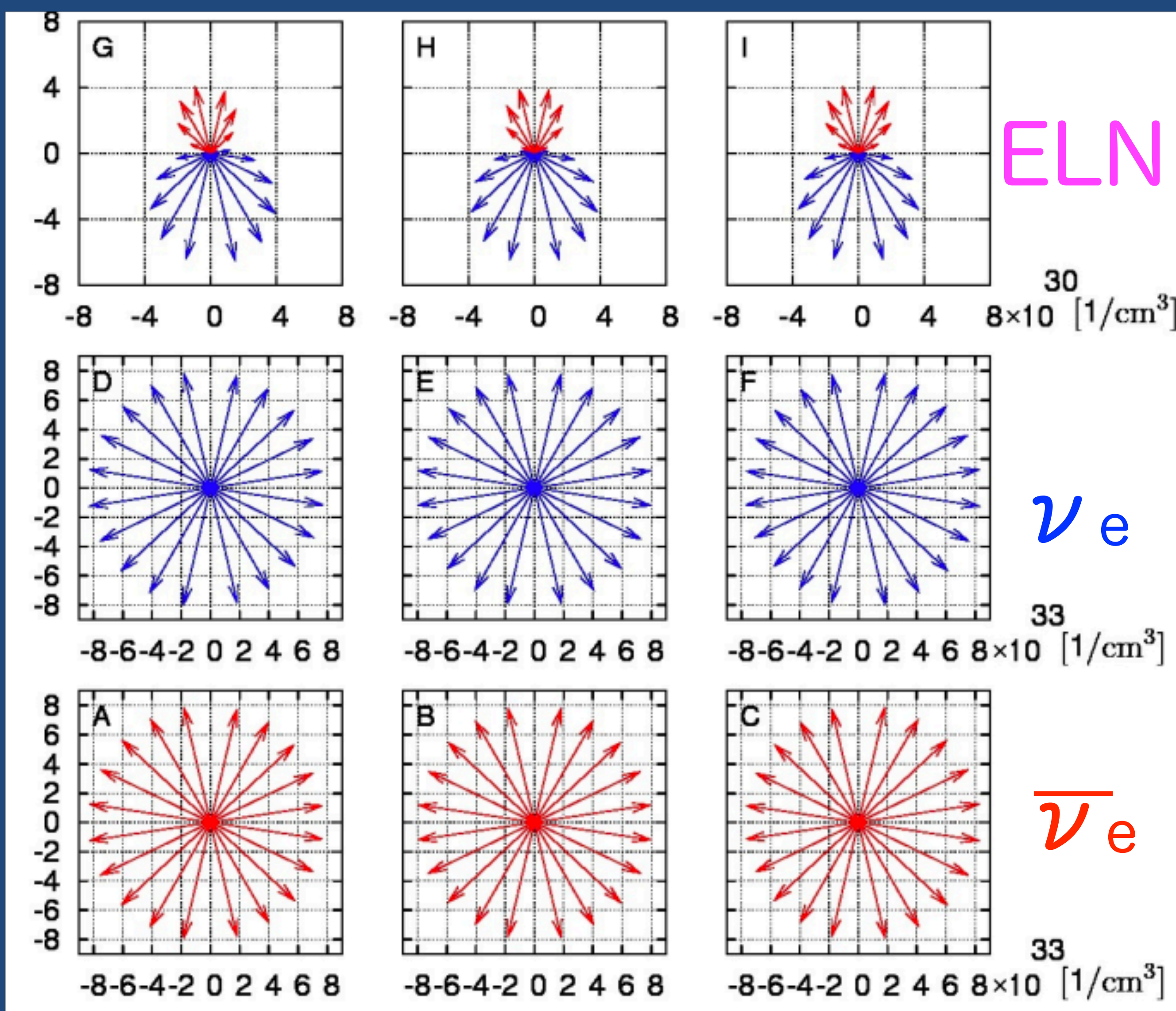




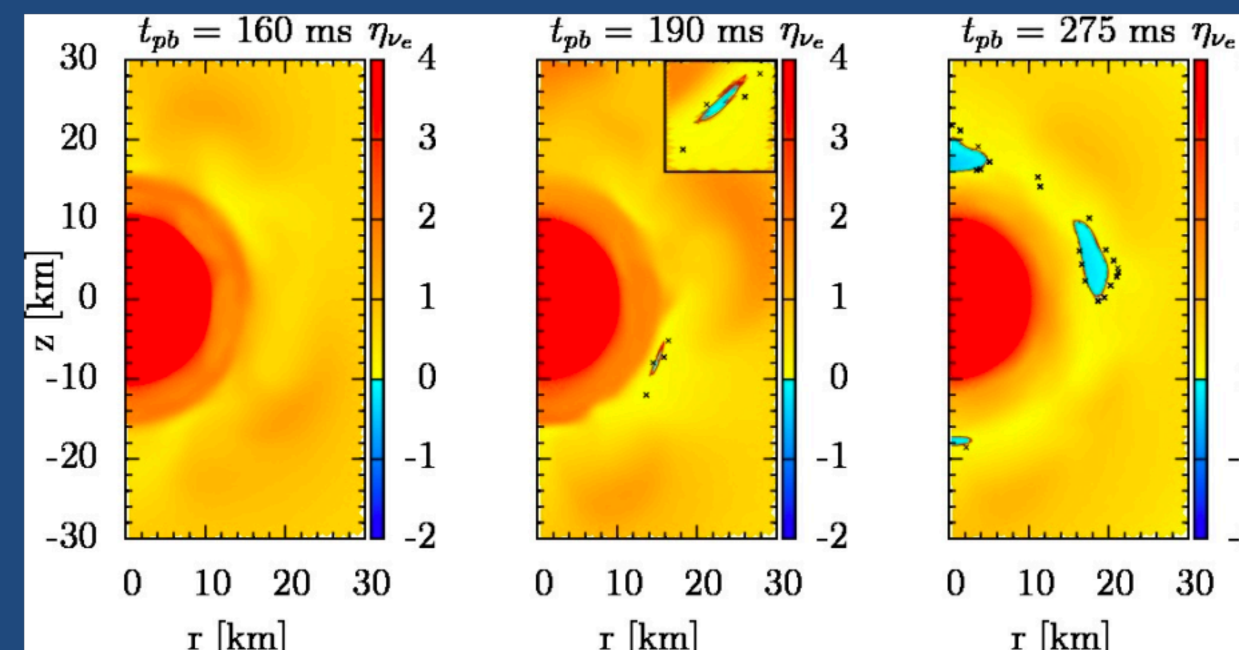
✓ Interestingly, the group velocity is always positive irrespective of the phase velocity and hence the flavor conversion may have an observational impact.

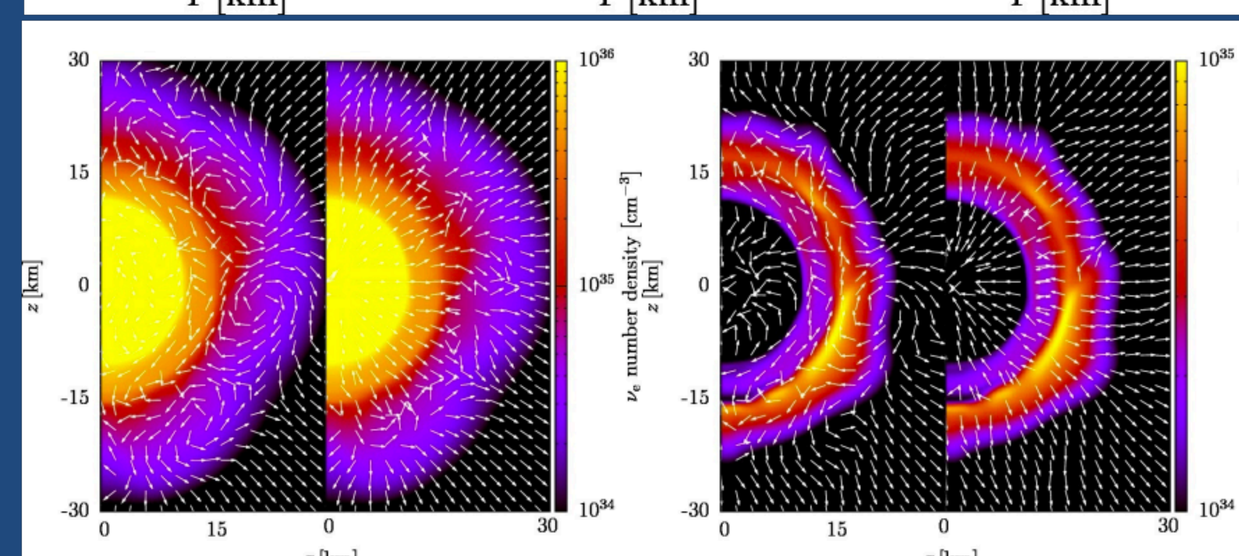
Fast Flavor Conversion inside the Neutrino Sphere

✓ We found the ELN crossing inside the neutrino sphere (r ~15-20km) at $t_{pb} \gtrsim$ 200ms in one of our 2D simulations for 11.2M model.



Delfan Azari et al. '19





Summary

✓ We have been developing the radiation-hydrodynamics code that solves the Boltzmann

✓ The occurrence of the fast conversion is one thing and its consequences are quite another!

✓ The latter will be the next focus.

- identification of modes
- nonlinear evolution and saturation
- feedback to transport and hydrodynamics
- observational implications

further investigations.

– pre- and post-shock regions, inside ν sphere