Development of a neutrino transport equation calculation code using the M1-closure method to improve the proto-neutron star cooling calculations

Tokyo university of science, Takaki Shimura, Hideyuki Suzuki

✓ Objective・background

- **Proto-neutron star cooling** :) code (H.Suzuki 1993)

  using multi-energy flux limited diffusion (**FLD**) scheme to solve Boltzmann equation approximately

▼ For future observations,
the current code needs to be improved and updated to make it more accurate.

- Solve the Boltzmann equation using **M1-closure** scheme.

✓ Formalism

- Metric

$$ds^2 = e^{2\phi(m,t)}dt^2 - e^{-2\lambda(m,t)}dm^2 - r(m,t)^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- Equation for moment (steady flow for simplicity)

$$\boldsymbol{\partial_t \vec{\mathcal{U}} + \partial_m \vec{\mathcal{F}} = \vec{\mathcal{S}}}$$

$$\vec{\mathcal{U}} = \frac{1}{\rho}(n_\nu, F_\nu) \qquad \vec{\mathcal{F}} = 4\pi c r^2 (F_\nu, P_\nu)$$

$$\vec{\mathcal{S}} = \frac{1}{\rho}\left(S^0, S^1 - 4\pi r c \rho \mathrm{e}^{\phi}(n_\nu - P_\nu)\partial_m\left(r\mathrm{e}^{-\phi}\right)\right)$$

✓ Numerical methods

outline

- Solve implicitly
- Newton-Raphson method

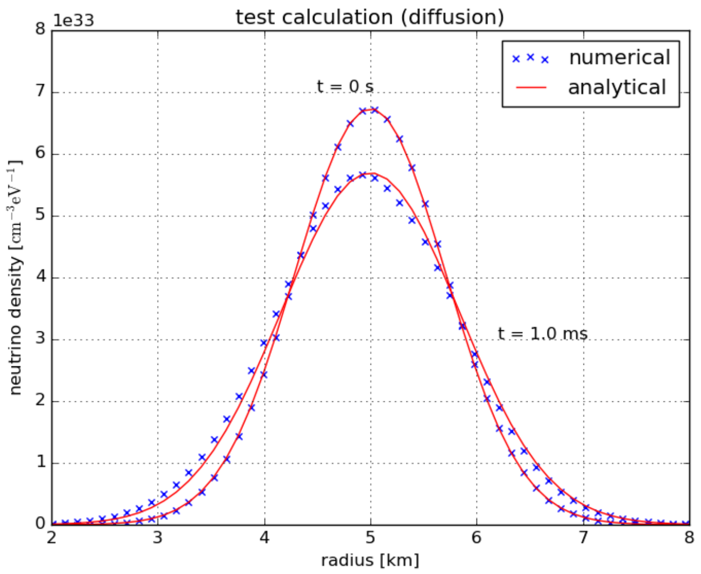
$$\left[\Delta\mathcal{U}_j^{(k)} + \sum_{q,l}\frac{\partial\mathcal{F}_{j+1/2}^{(k)}}{\partial q_l}\Delta q_l^{(k)} - \sum_{q,l}\frac{\partial\mathcal{F}_{j-1/2}^{(k)}}{\partial q_l}\Delta q_l^{(k)} - \sum_{q,l}\frac{\partial\mathcal{S}_j^{(k)}}{\partial q_l}\Delta q_l^{(k)}\right]$$
$$= -\mathcal{U}_j^{(k)} + \mathcal{U}_j^n + \mathcal{S}_j^{(k)} - \left(\mathcal{F}_{j+1/2}^{(k)} - \mathcal{F}_{j-1/2}^{(k)}\right)$$

Advection (position space)

- HLL + correlation

to reproduce
the diffusion limit

Test 1 : Diffusion limit



Test 2 : Free-streaming limit