反跳を考慮したニュートリノ核子散乱を計算するコードの開発

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Outline

- **Introduction**
- **Method**
- **Result**
  - **Check for thermalization**
  - **2 Checks for resolution**
- **Summary**

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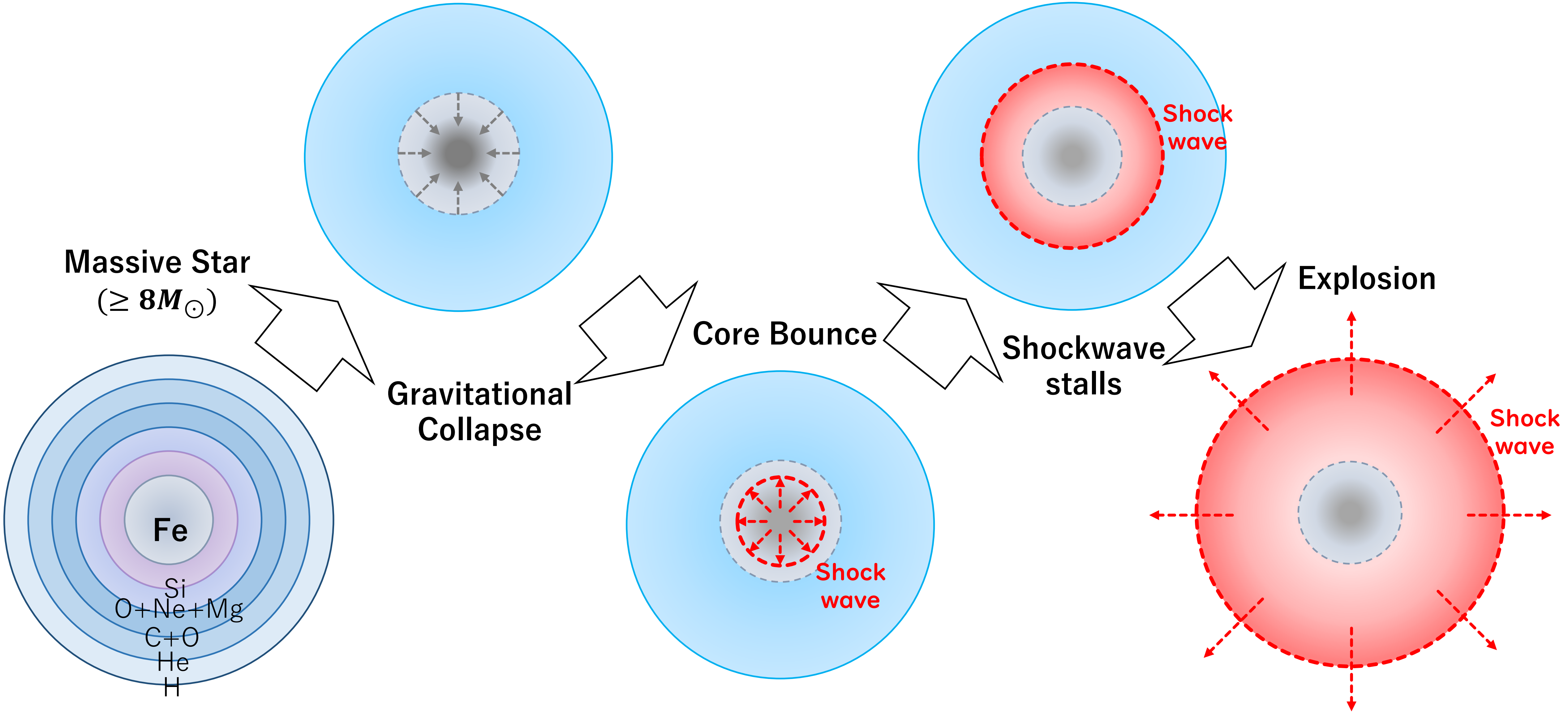
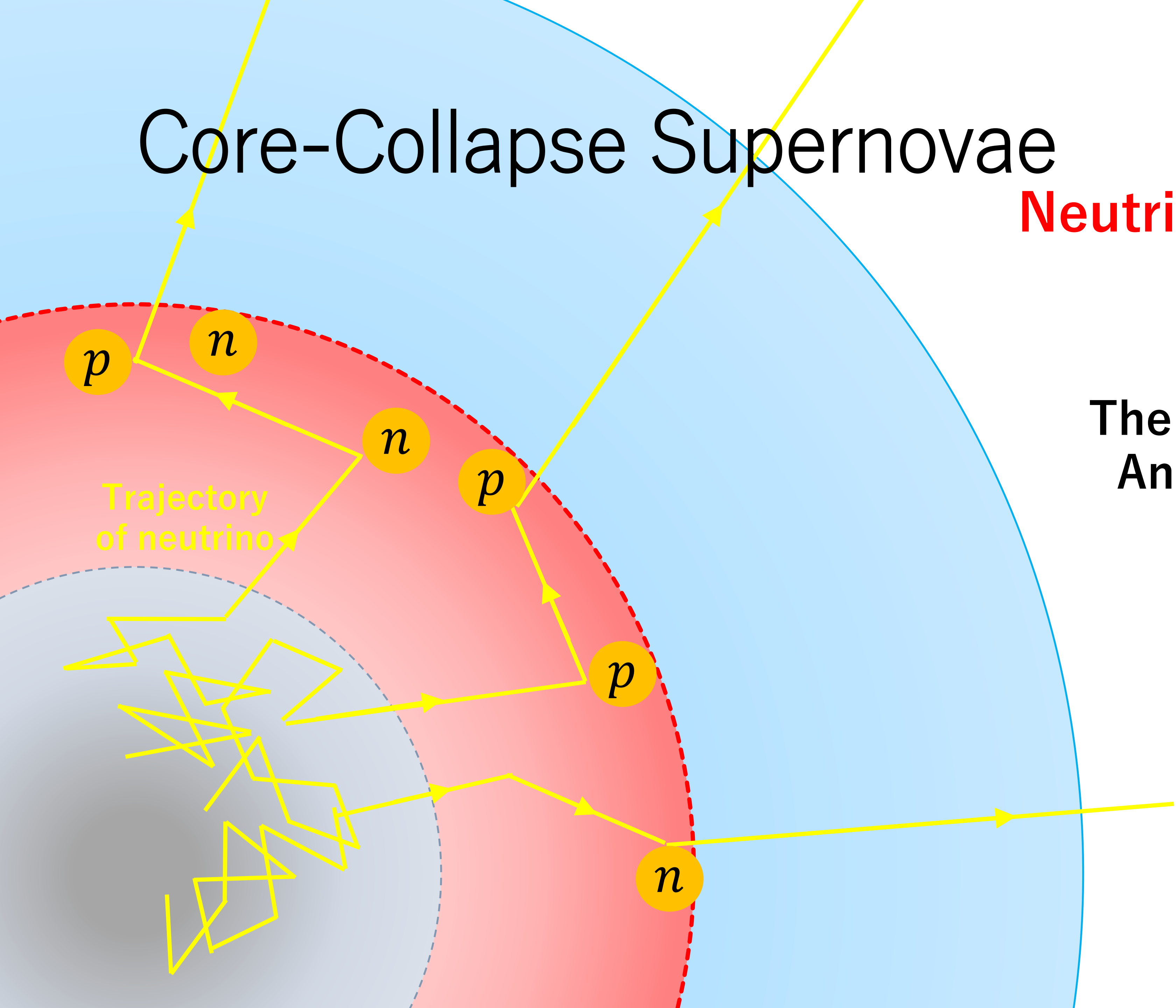
Core-Collapse Supernovae



Core-Collapse Supernovae



Neutrino heating mechanism

⇩

The stalled shock is revived
And the explosion occurs

Boltzmann Eq.

$$\frac{1}{c}\frac{\partial f}{\partial t}+\frac{\partial f}{\partial s}=\frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$$

Boltzmann Equation

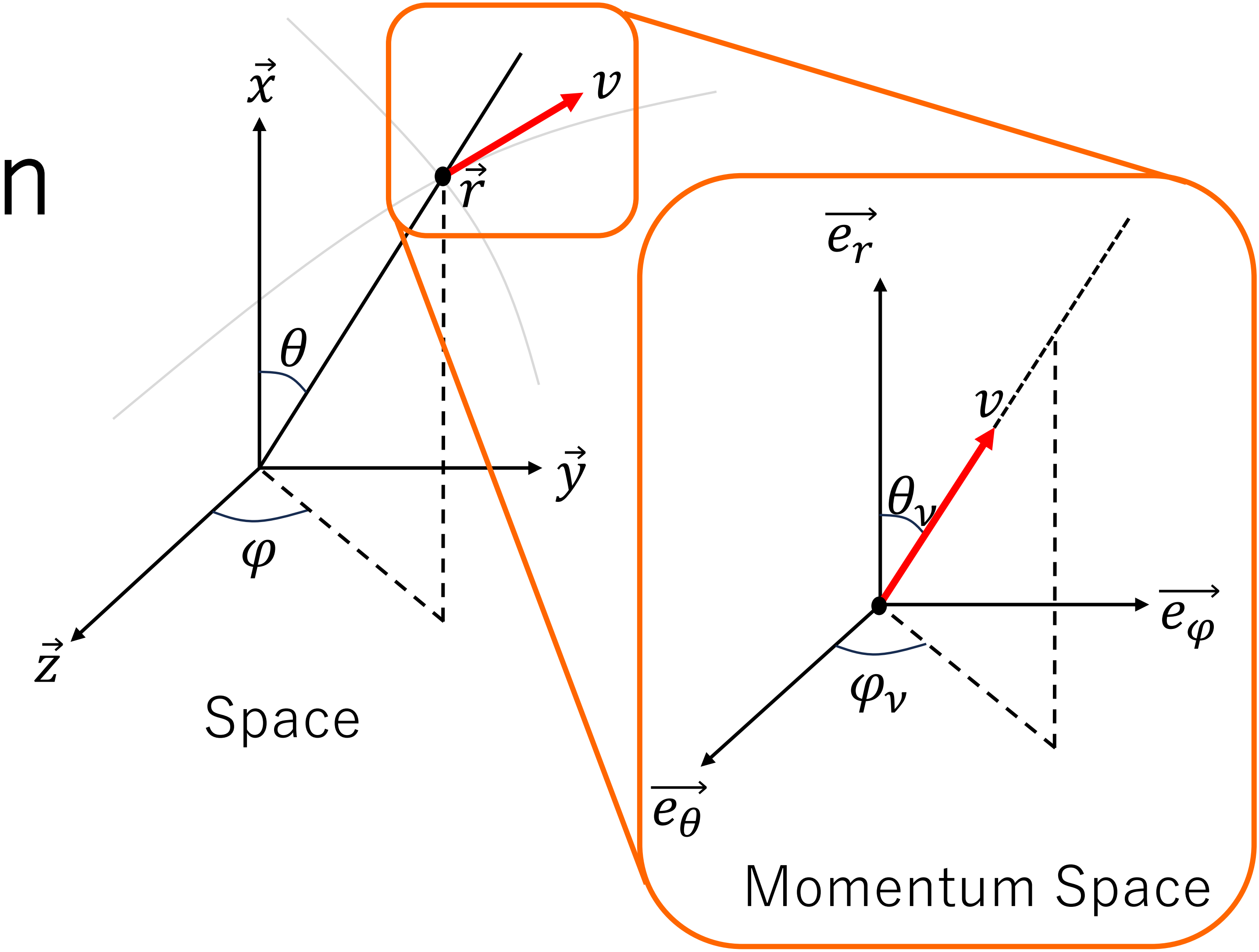
$$\frac{1}{c}\frac{\partial f}{\partial t} + \frac{\partial f}{\partial s} = \frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$$

Distribution function :

$$f(\underbrace{r,\theta,\varphi}_{\text{space}},\underbrace{\epsilon,\theta_\nu,\varphi_\nu}_{\text{Momentum space}};t)$$

$\frac{\partial f}{\partial s}$: Advection term

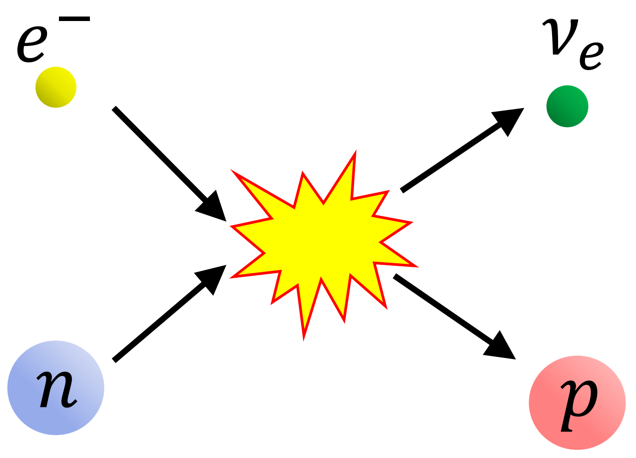
$\frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$: Collision term represents the neutrino reaction



Neutrino Reaction

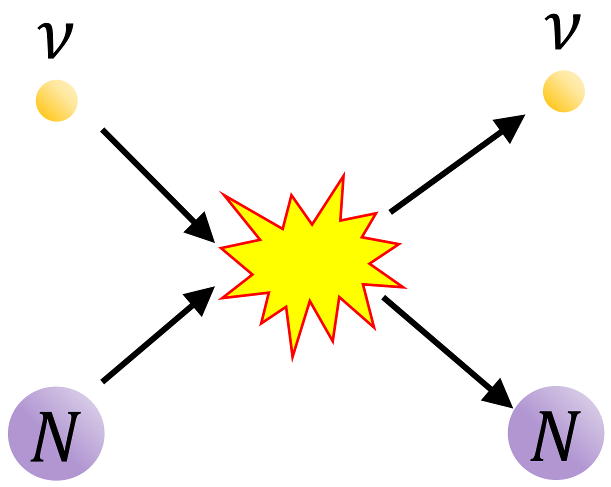
Emission and Absorption

$$e^- + p \leftrightarrow \nu_e + n$$
$$e^+ + n \leftrightarrow \overline{\nu_e} + p$$
$$e^- + A \leftrightarrow \nu_e + A$$



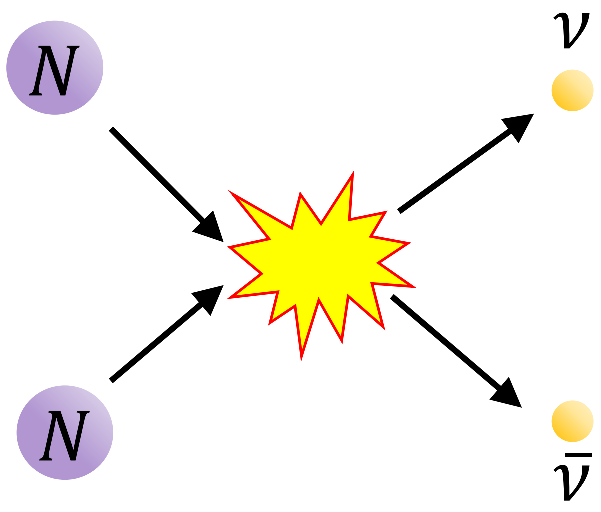
Scattering

$$\nu + N \leftrightarrow \nu + N$$
$$\nu + A \leftrightarrow \nu + A$$



Pair Process

$$e^- + e^+ \leftrightarrow \nu_i + \overline{\nu_i}$$
$$N + N \leftrightarrow N + N + \nu_i + \overline{\nu_i}$$



Neutrino Reaction

Emission and Absorption

$$e^- + p \leftrightarrow \nu_e + n$$
$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$
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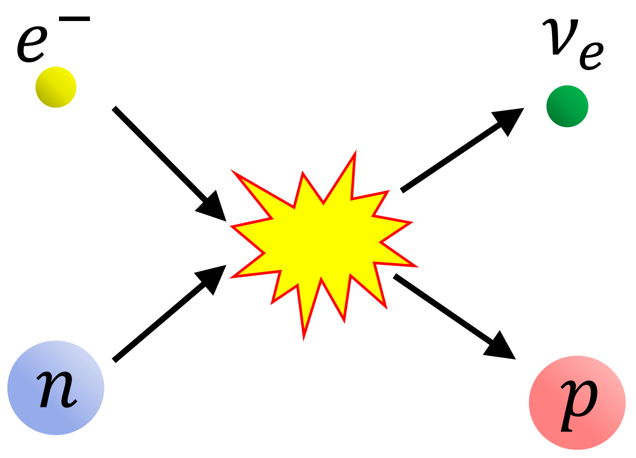

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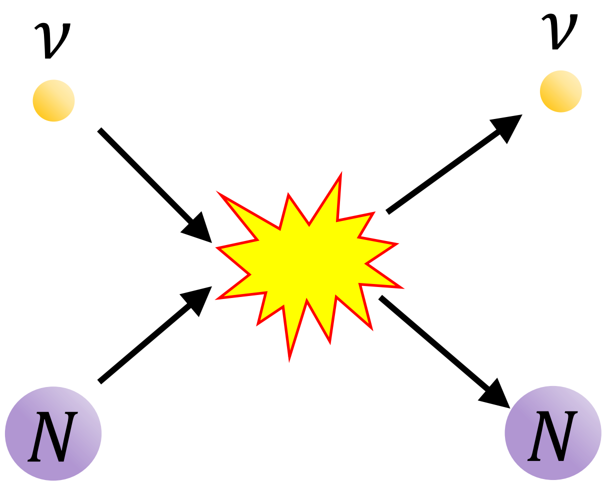

Pair Process

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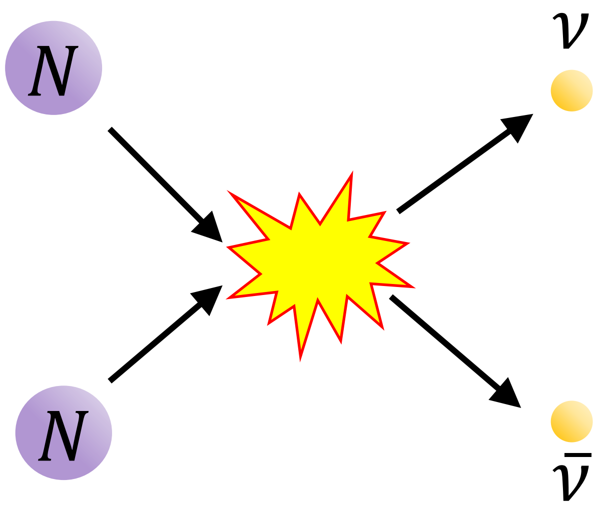

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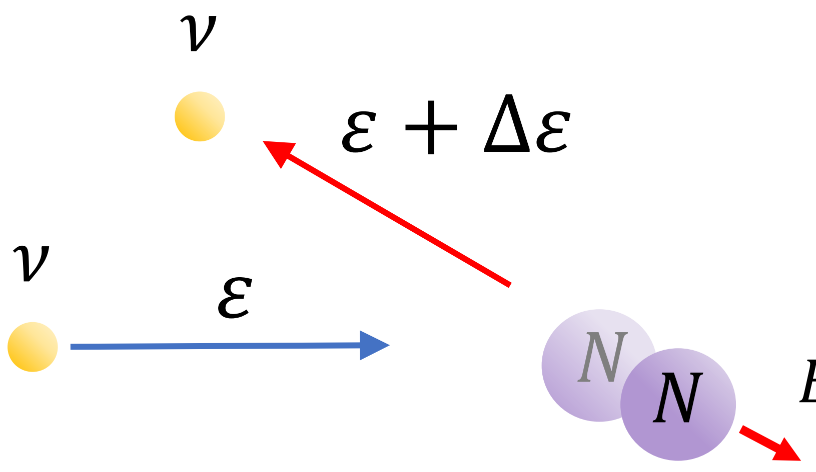
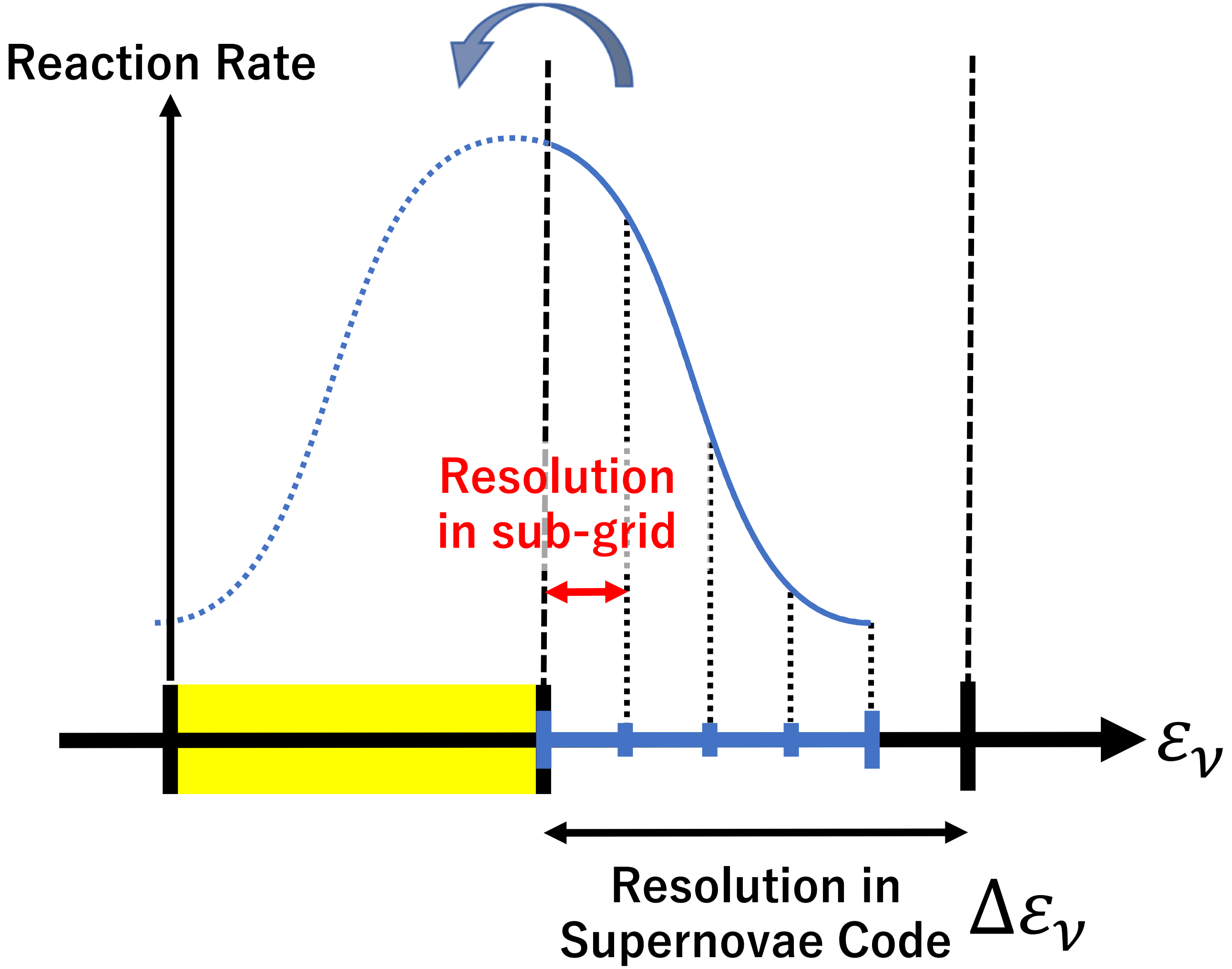
Neutrino Reaction

Neutrino Nucleon Scattering

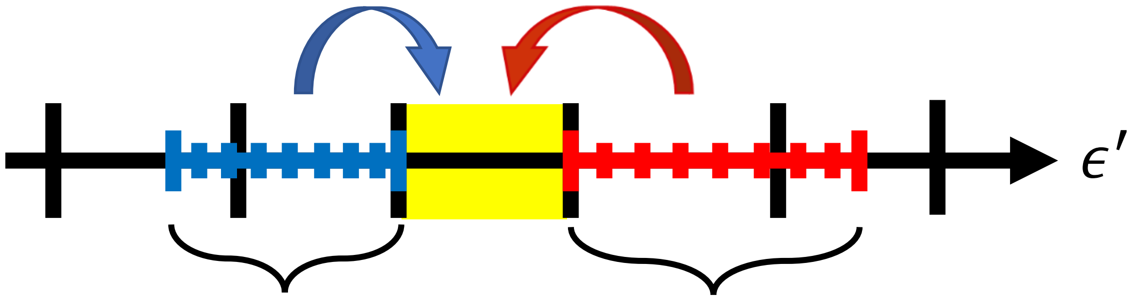


$$\frac{\Delta\varepsilon}{\varepsilon} \simeq 0.1 \sim 0.2 < 0.3 \sim 0.4 \simeq \frac{\Delta\varepsilon_\nu}{\varepsilon_\nu}$$

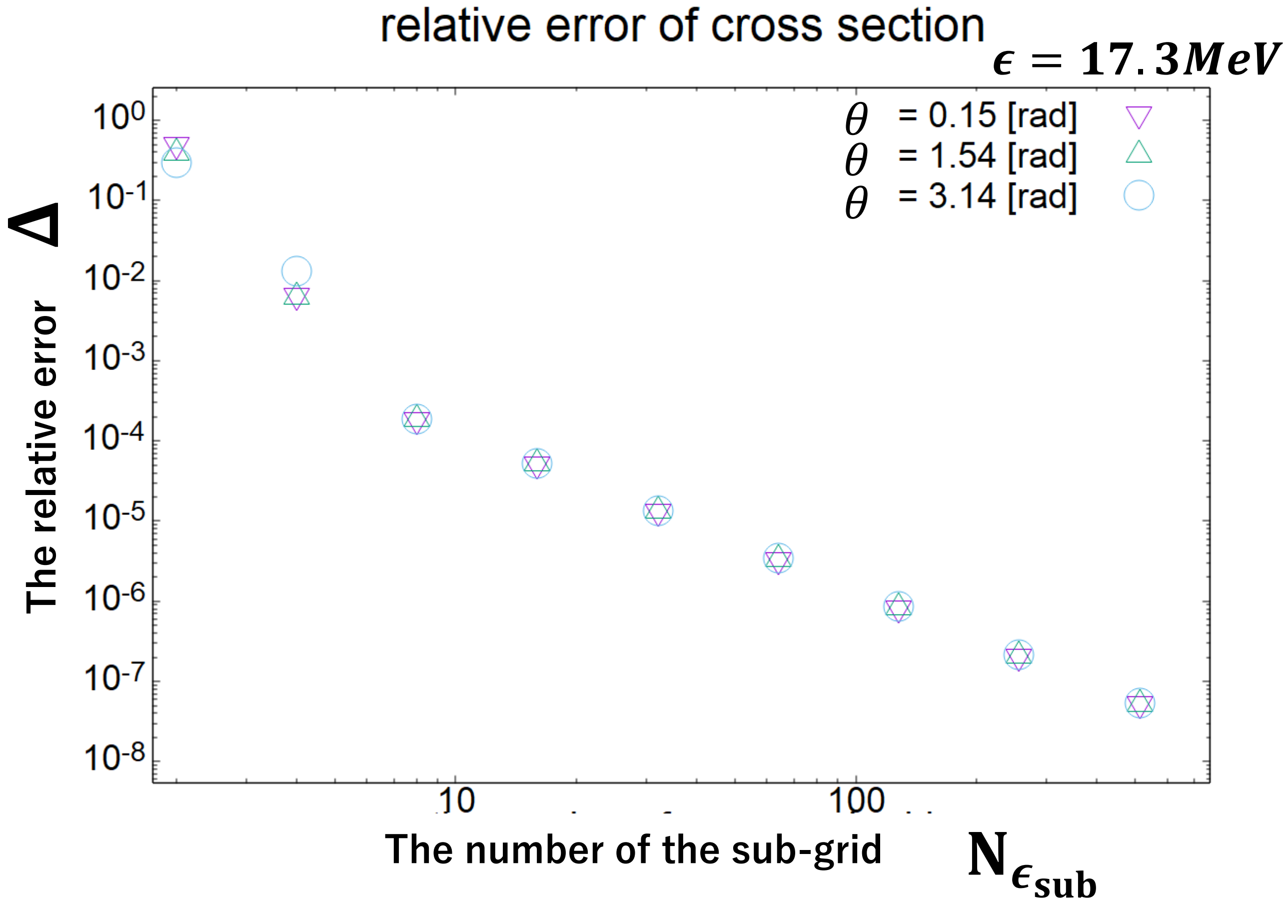
Subgrid Model



Accuracy



$\rho \simeq 10^{10}$g/cc , $T \simeq 2.698$MeV , $Ye \simeq 0.2447$



If $N_{\epsilon_{sub}} = 8$, the relative error is

$$\Delta = \frac{|\sigma_{N_{\epsilon_{sub}}} - \sigma_{true}|}{\sigma_{true}} \simeq 10^{-4}$$

where

$$\sigma = \int \epsilon'^2 d\epsilon' \, R_{scat}(\epsilon, \epsilon', \theta)$$

(**Reaction Rate :** R_{scat})

Energy sub-Grids: $N_{\epsilon_{sub}} = 8$
is adopted hereafter

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Model of first test for thermalization

OneZone Calculation ： Neglecting the space dependence in Boltzmann's equation

$$\frac{1}{c}\frac{\partial f}{\partial t} = -\frac{1}{c}\int \frac{d\epsilon' \epsilon'^2}{(2\pi)^3}\int d\Omega'\, R_{\mathrm{scat}}^{subgrid}(\epsilon,\Omega;\epsilon',\Omega')\, f(\epsilon,\Omega)\,[1-f(\epsilon',\Omega')]$$
$$+\frac{1}{c}\int \frac{d\epsilon' \epsilon'^2}{(2\pi)^3}\int d\Omega'\, R_{\mathrm{scat}}^{subgrid}(\epsilon',\Omega';\epsilon,\Omega)\, f(\epsilon',\Omega')\,[1-f(\epsilon,\Omega)]$$

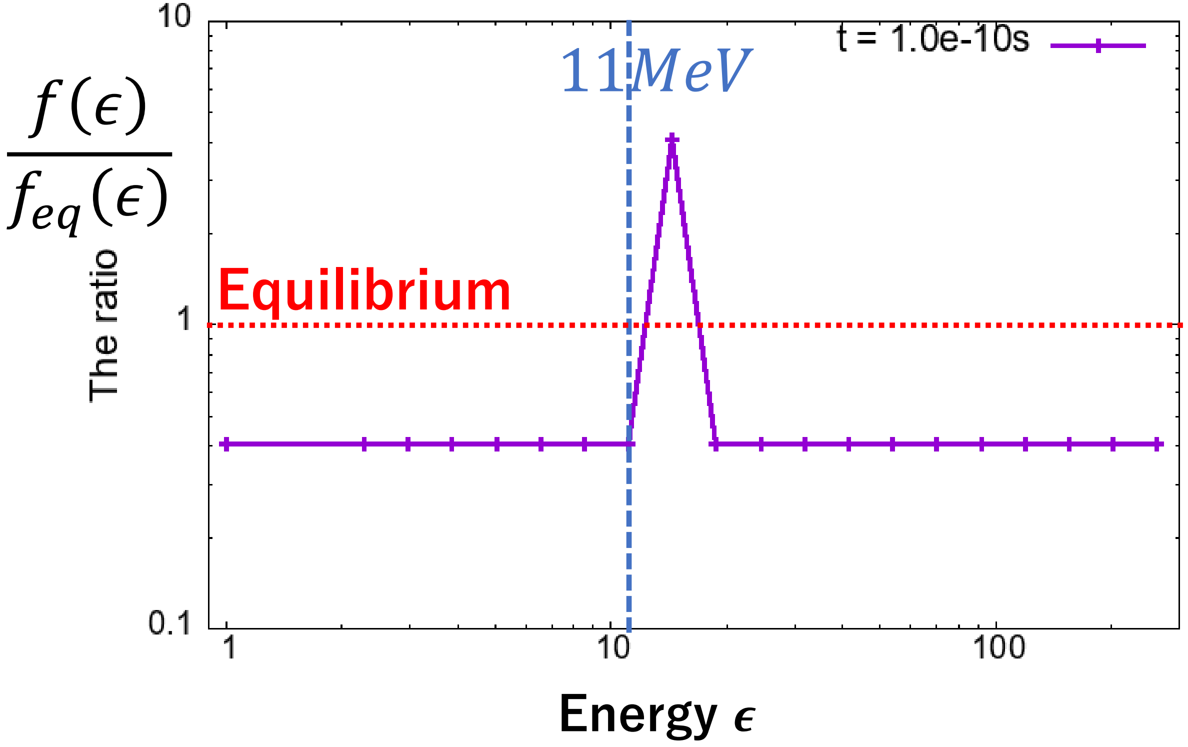
I input non-equilibrium distribution in energy ϵ and angle θ_ν

Energy Grid : $N_\epsilon = 20 \;\; (0\ \mathrm{MeV} \leq \epsilon \leq 300\ \mathrm{MeV})$

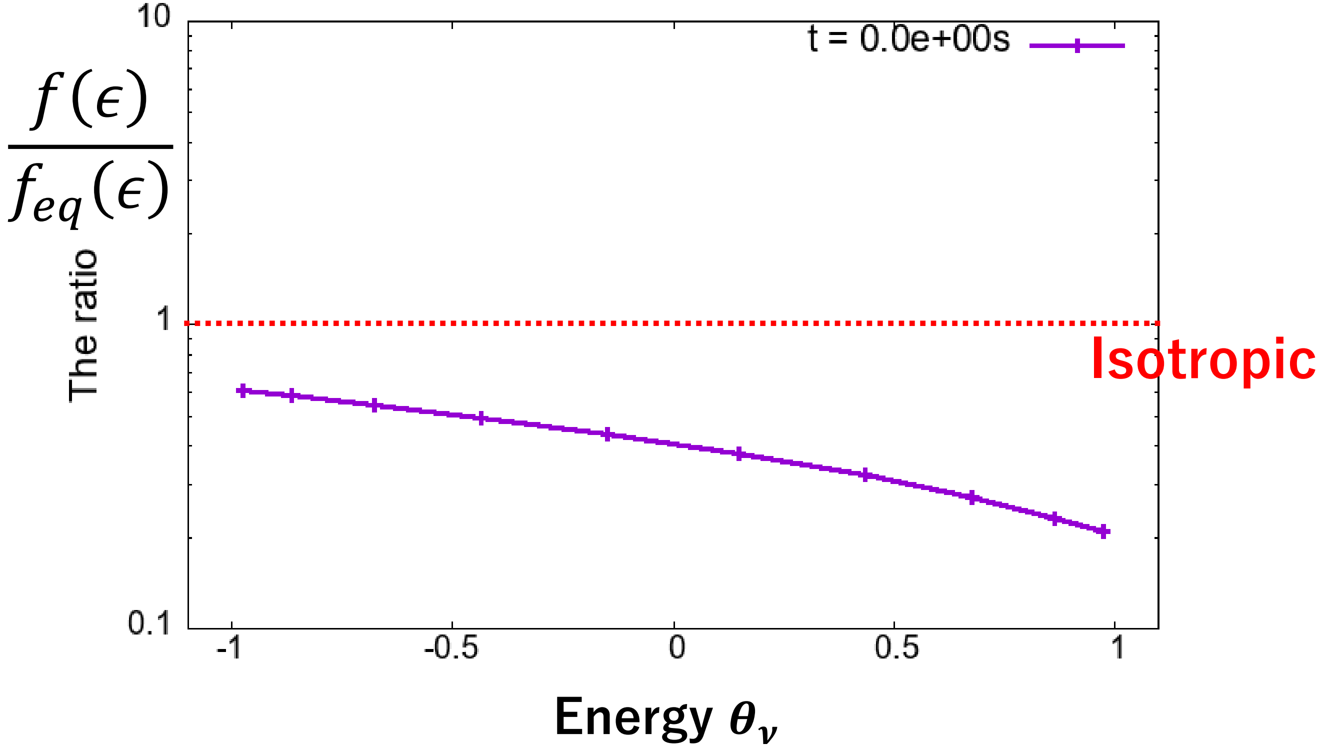
Angular Grid : $N_{\theta_\nu} = 10 \;\; (0 \leq \theta_\nu \leq \pi\,)$

Test of Thermalization

Energy Dependence



Angular Dependence at $E = 11MeV$



Distribution reach the equilibrium state

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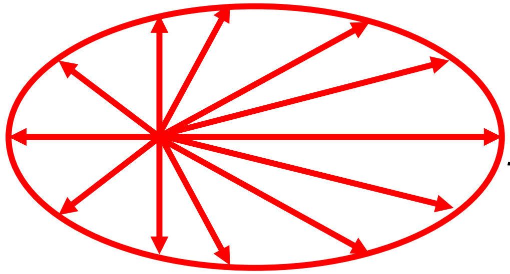
Check for resolution

OneZone calculation + **the source term** $\tilde{\boldsymbol{f}}(\boldsymbol{\epsilon},\boldsymbol{\Omega})$

$$\frac{1}{c}\frac{\partial f}{\partial t} = -\frac{1}{c}\int \frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' \, R_{\mathrm{scat}}(\epsilon,\Omega;\epsilon',\Omega')\, f(\epsilon,\Omega)\,[1-f(\epsilon',\Omega')]$$
$$+\frac{1}{c}\int \frac{d\epsilon'\epsilon'^2}{(2\pi)^3}\int d\Omega' \, R_{\mathrm{scat}}(\epsilon',\Omega';\epsilon,\Omega)\, f(\epsilon',\Omega')\,[1-f(\epsilon,\Omega)] + \tilde{\boldsymbol{f}}(\boldsymbol{\epsilon},\boldsymbol{\Omega})$$

$$\tilde{f}(\epsilon,\Omega) \propto \cos\theta_\nu \;, \; \frac{f_{min}(\epsilon)}{f_{max}(\epsilon)} \sim \frac{1}{3}$$



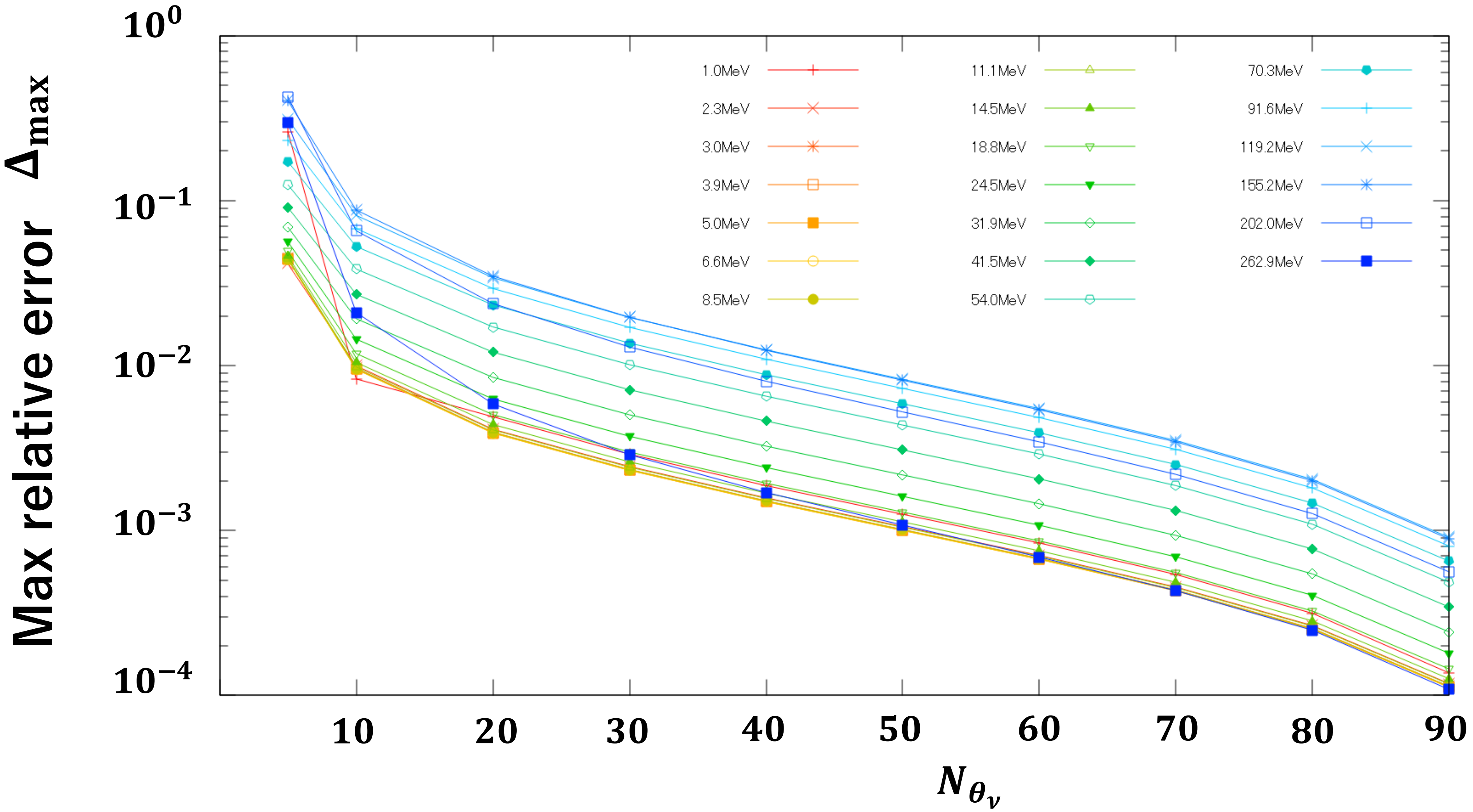
Energy Grid : $\boldsymbol{N_\epsilon = 20 \;\; (0\ \mathrm{MeV} \le \epsilon \le 300\ \mathrm{MeV})}$

Angular Grid : $\boldsymbol{N_{\theta_\nu} = \{5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}}$

1st Check for resolution

Max relative error is defined as

$$\Delta_{\max} = \max_{\epsilon,\theta_\nu}\left(\frac{|f_{N_\theta}(\epsilon,\theta_\nu) - f_{N_\theta=100}(\epsilon,\theta_\nu)|}{f_{N_\theta=100}(\epsilon,\theta_\nu)}\right)$$



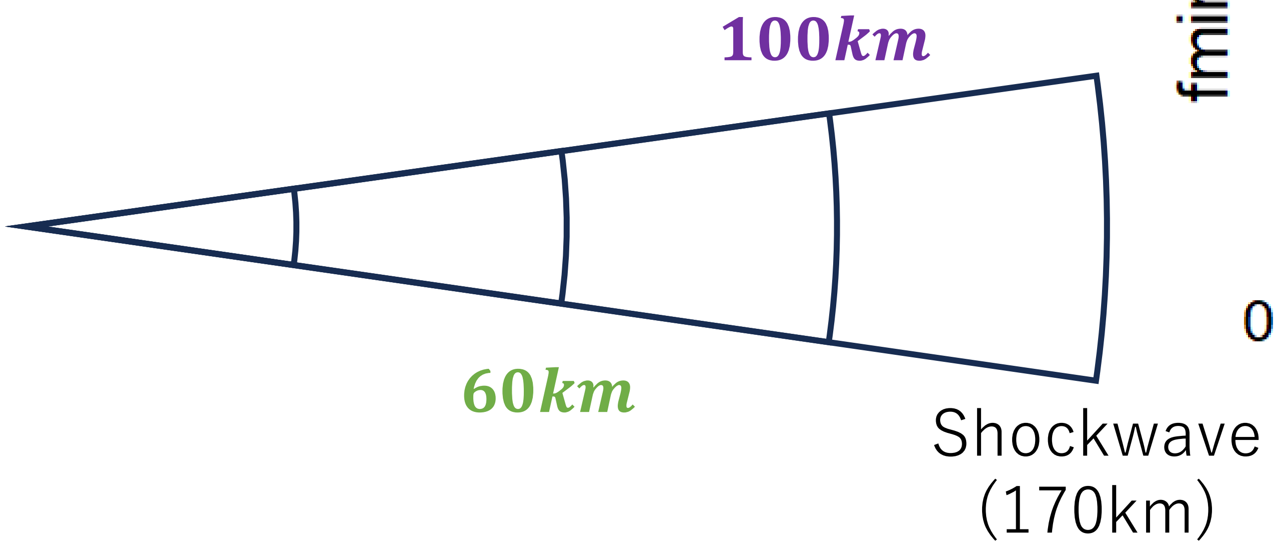
2nd Check for resolution

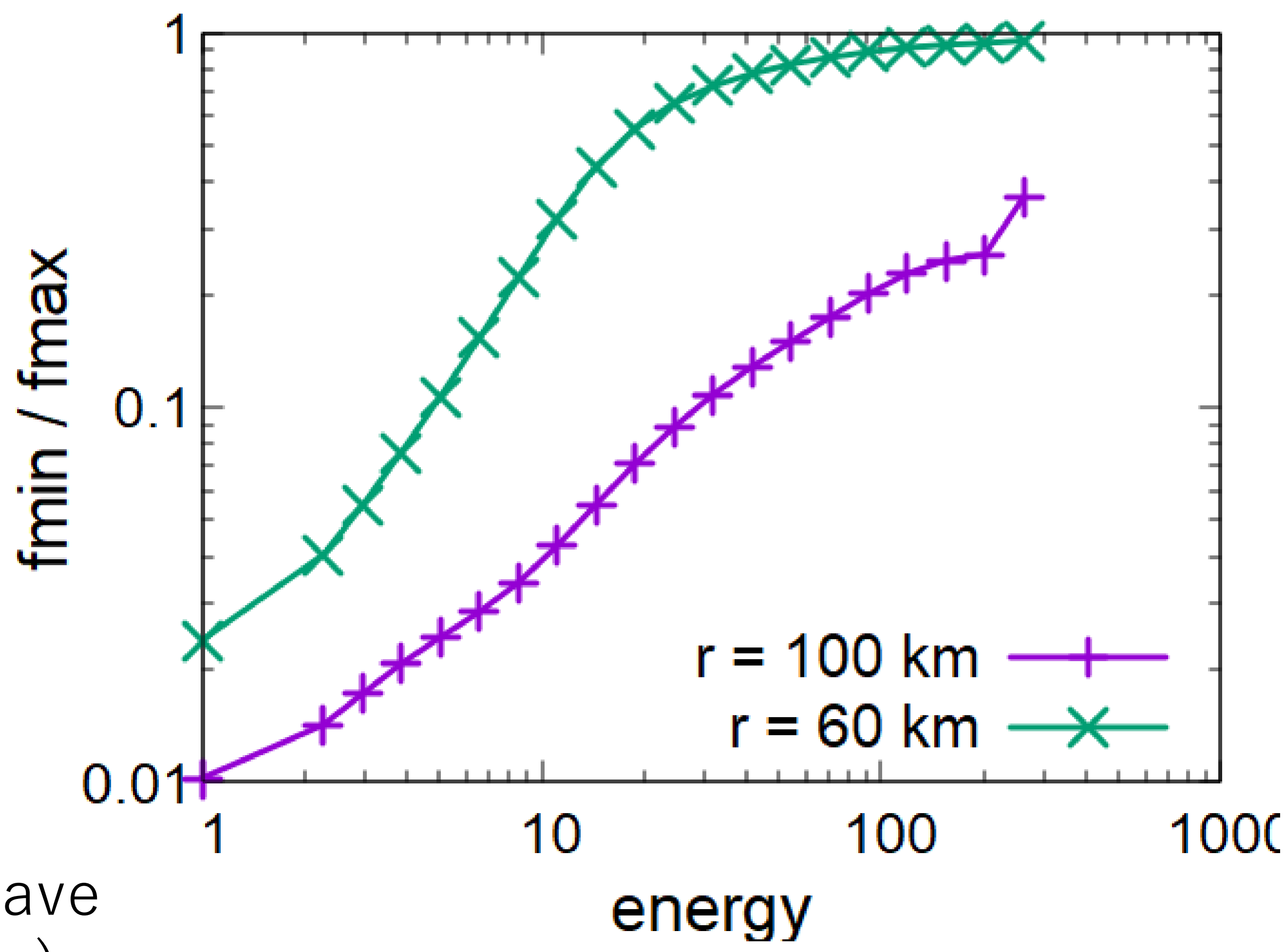
Progenitor : 15$M_{\odot}$

Data : t = 100ms

$(N_e, N_{\theta_\nu}, N_{\phi_\nu}) = (20, 10, 6)$

Comparison : $(N_e, N_{\theta_\nu}, N_{\phi_\nu}) =$ $(20, 10, 6)$, $(20, 40, 6)$

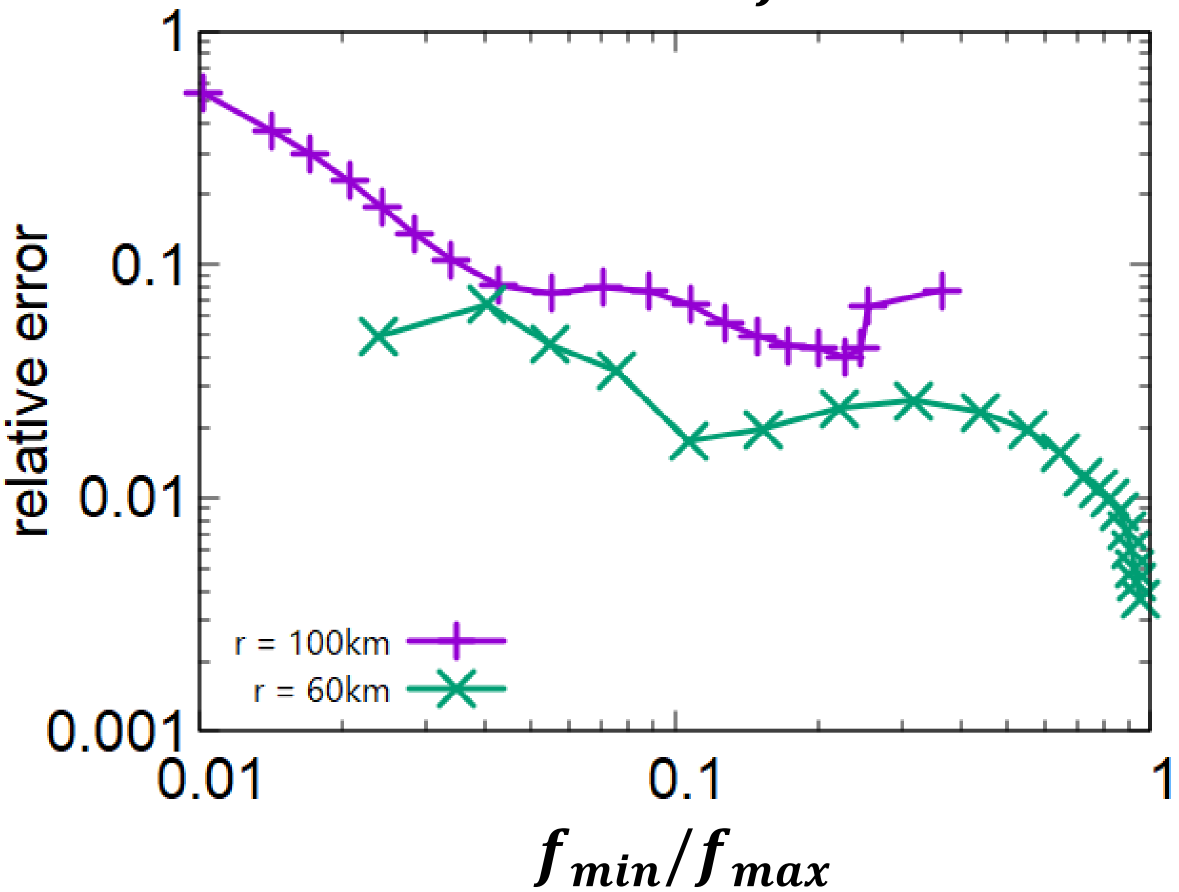




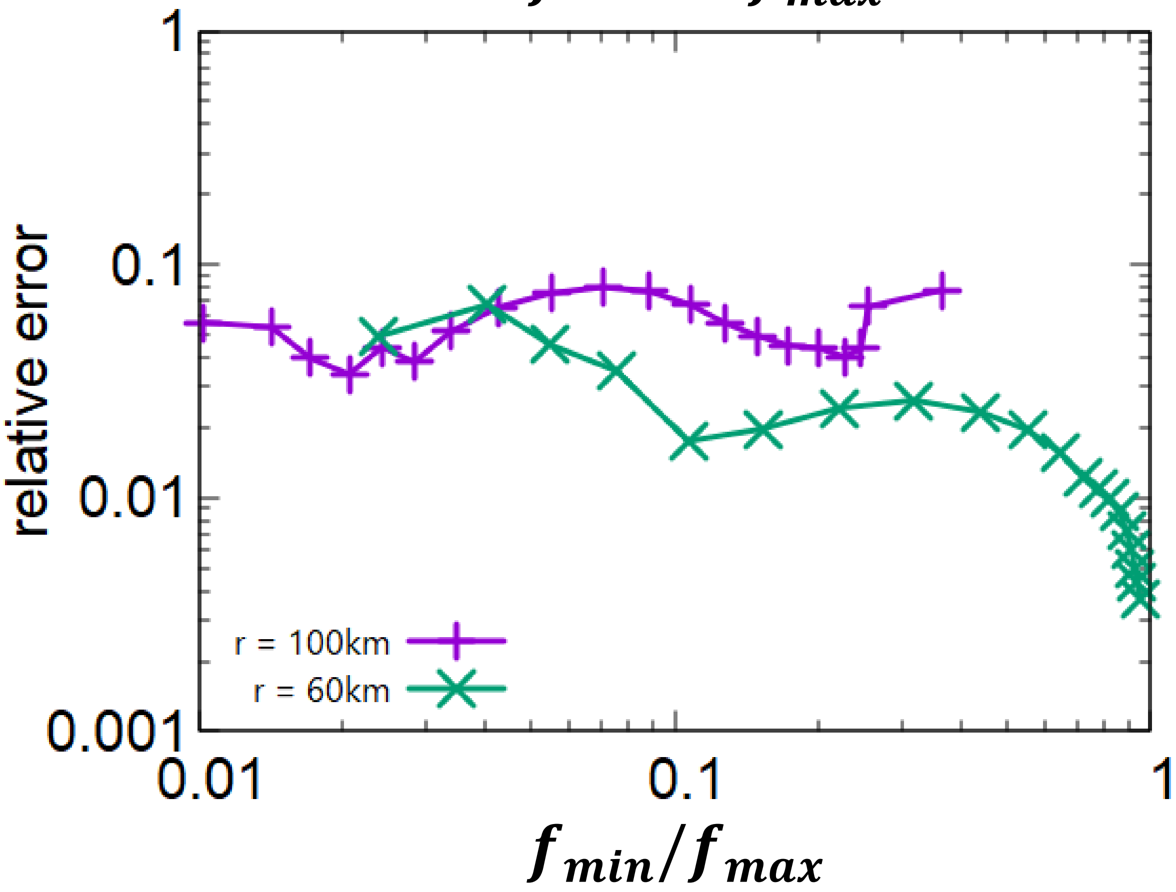
2^{nd} Check for resolution

$$\Delta_{\mathrm{max}} = \max_{\epsilon,\theta_\nu}\left(\frac{|f_{N_{\theta_\nu}}(\epsilon,\theta_\nu) - f_{N_{\theta_\nu}=40}(\epsilon,\theta_\nu)|}{f_{N_{\theta_\nu}=40}(\epsilon,\theta_\nu)}\right)$$

All of f



$f > 0.1 f_{max}$



Prospect

Advection + Collision

$$\frac{1}{c}\frac{\partial f}{\partial t} + \frac{\partial f}{\partial s} = \frac{1}{c}\left[\frac{\partial f}{\partial t}\right]_{coll}$$

Advection (High resolution)

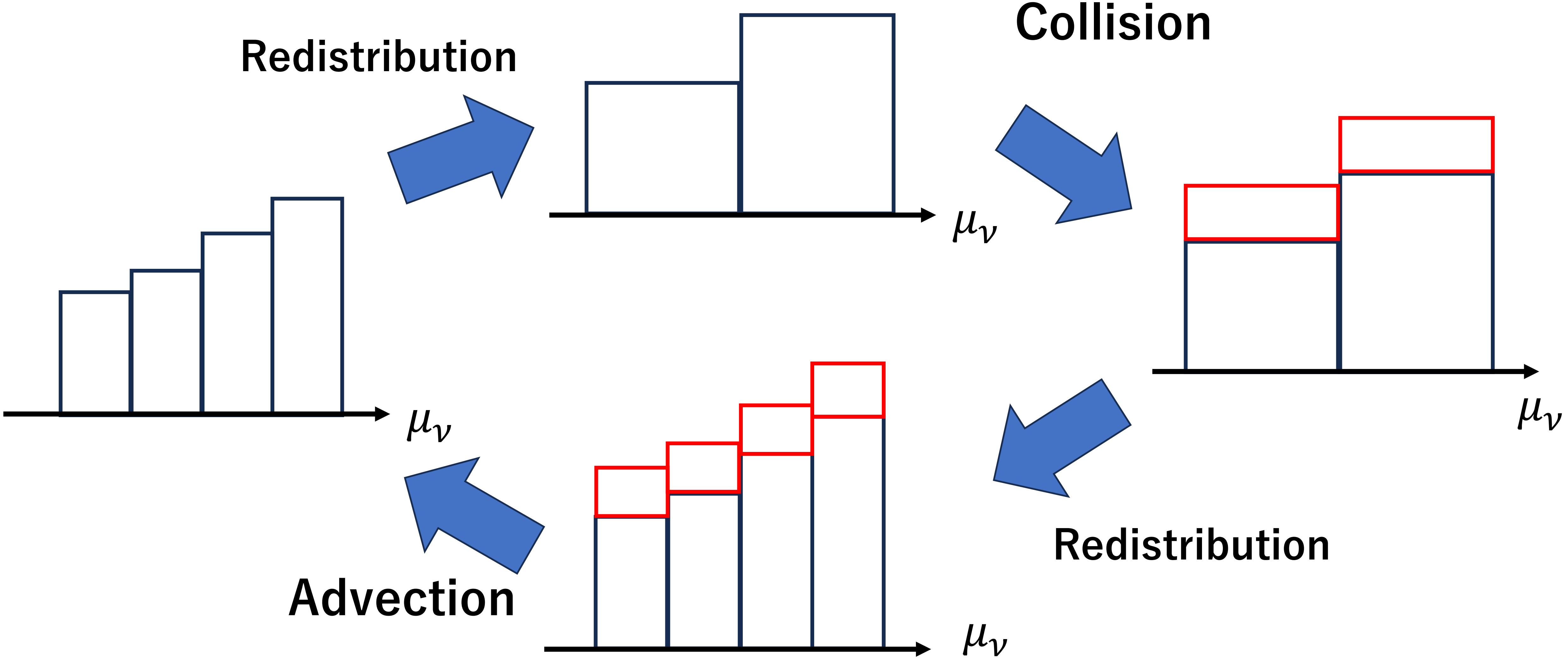
$$\boldsymbol{f^* = f - \frac{\partial f}{\partial s}}$$

+

$$\boldsymbol{\frac{\partial f^*}{\partial t} = \left[\frac{\partial f^*}{\partial t}\right]_{coll}}$$

Collision

Prospect



Redistribution
Collision
Redistribution
Advection
μ_ν
μ_ν
μ_ν
μ_ν

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Summary

- **I developed the subgrid model** to treat the small energy exchange due to neutrino-nucleon scattering.
- By inputting non-equilibrium distribution, it was found that the distribution **finally reaches equilibrium**.
- I also performed the tests with **the source term**. The result were found to **converge with increasing angular resolution**.
- It is shown that the relative error is based on the ratio of minimum distribution to maximum distribution with respect to θ_ν .
- In the future, OneZone calculation can invent **the way to capture the effect of high angular resolution** with small number of angular meshes.