Development of a neutrino transport equation calculation code using the M1-closure method to improve the proto-neutron star cooling calculation code

Tokyo University of Science Takaki Shimura, Hideyuki Suzuki

1.Objective and Background

- A code describing the cooling process of proto-neutron stars (PNSC) was created by H.Suzuki (1993) using multi-energy flux limited diffusion (FLD) scheme to solve Boltzmann equation approximately.
- To prepare for future observations, the current code needs to be improved and updated to make it more accurate and capable of long-time calculations.
- Therefore, taking these points into account, I am aiming to solve the Boltzmann equation using M1-closure scheme.

2.Formalism

Moment scheme

Remove the full angular dependence of the Boltzmann equation by expanding the neutrino distribution function as a series of moments

✓ **Flux limited diffusion (original code)**

Closes the moment expansion after the zeroth moment

✓ **M1-closure (Created Code)**

Evolves both the energy density and the moment density but assumes an analytic closure for higher moments.

Details

I. Metric

$$ds^2 = e^{2\phi(m,t)}dt^2 - e^{-2\lambda(m,t)}dm^2 - r(m,t)^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

II. Boltzmann equation (Lindquist equation)

$$D_t f_\nu + \mu D_m f_\nu - \omega\left[\mu D_m\phi + \mu^2 D_t\Lambda + (1-\mu^2)\frac{U}{r}\right]\frac{\partial f_\nu}{\partial\omega} + (1-\mu^2)\left[-D_m\phi + \frac{\Gamma}{r} + \mu\left(\frac{U}{r} - D_t\Lambda\right)\right]\frac{\partial f_\nu}{\partial\mu} = Coll[f_\nu]$$

$$D_t = e^{-\Phi}\partial_t \quad D_m = e^{-\lambda}\partial_m \quad \Gamma = D_m r \quad U = D_t r$$

III. Equation for moments (Steady-flow for simplicity)

$$\partial_t \vec{\mathcal{U}} + \partial_m(4\pi r^2\rho\mathcal{F}) = \mathcal{S}$$

Moment

$$n_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu d\Omega$$

$$F_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu d\Omega$$

$$P_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu^2 d\Omega$$

Closure : Levermore, ME, etc.

Variables

$$\vec{\mathcal{U}} = \frac{1}{\rho}(n_\nu, F_\nu) \qquad \vec{\mathcal{F}} = 4\pi c r^2(F_\nu, P_\nu)$$

$$\vec{\mathcal{S}} = \frac{1}{\rho}\left(S^0, S^1 - 4\pi r c\rho e^{\phi}(n_\nu - P_\nu)\partial_m(re^{-\phi})\right)$$

IV. Neutrino-Matter reaction

NuLib (O'Connor 2015)

Absorption and emission	$\nu_e + n \to e^- + p$ $\overline{\nu_e} + p \to e^+ + n$	$\nu_e + A \to e^- + A$
Elastic scattering	$\nu + n \to \nu + n$ $\nu + p \to \nu + p$	$\nu + A \to \nu + A$ $(\nu + \alpha \to \nu + \alpha)$
Thermal pair production/annihilation	$e^+ + e^- \to \nu + \bar{\nu}$	
Inelastic scattering	$\nu + e \to \nu + e$	

3.Numerical methods and test

< Outline >

✓ Solve implicitly

✓ Use Newton-Raphson method

$$\left[\Delta\mathcal{U}_j^{(k)} + \sum_{q,l}\frac{\partial\mathcal{F}_{j+1/2}^{(k)}}{\partial q_l}\Delta q_l^{(k)} - \sum_{q,l}\frac{\partial\mathcal{F}_{j-1/2}^{(k)}}{\partial q_l}\Delta q_l^{(k)} - \sum_{q,l}\frac{\partial\mathcal{S}_j^{(k)}}{\partial q_l}\Delta q_l^{(k)}\right] = -\mathcal{U}_j^{(k)} + \mathcal{U}_j^n + \mathcal{S}_j^{(k)} - \left(\mathcal{F}_{j+1/2}^{(k)} - \mathcal{F}_{j-1/2}^{(k)}\right)$$

k: iteration number

Details

I. Advection term (position space)

HLL flux + correlation (E.Audit et al. 2002)

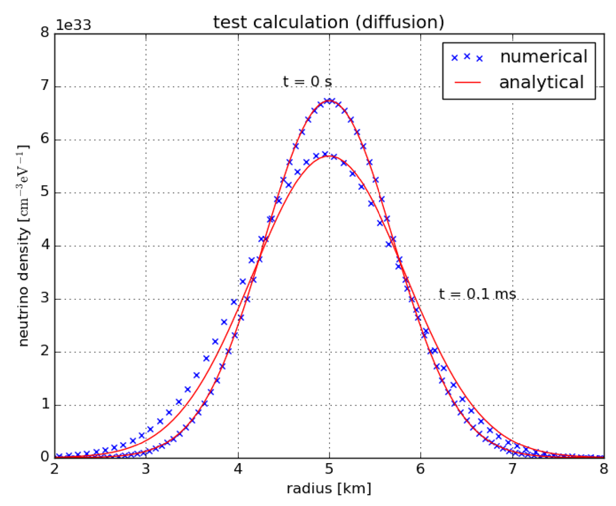
It is known from previous studies that HLL fluxes cannot reproduce the diffusion limit in optically thick regions.

$$\mathcal{F}_n = \frac{\psi^+\mathcal{F}_L - \psi^-\mathcal{F}_R + a\psi^+\psi^-(\mathcal{U}_R - \mathcal{U}_L)}{\psi^+ - \psi^-} \qquad a = \tanh\left(\frac{1}{\kappa\Delta r}\right)$$

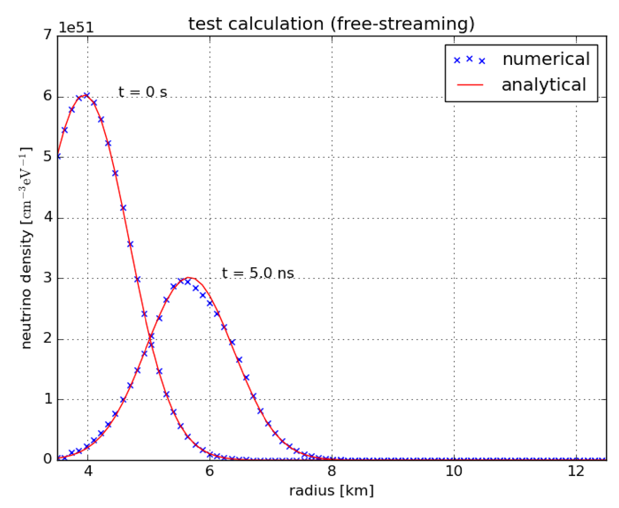
$$\mathcal{F}_F = \frac{a^2(\psi^+\mathcal{F}_L - \psi^-\mathcal{F}_R) + a\psi^+\psi^-(\mathcal{U}_R - \mathcal{U}_L)}{\psi^+ - \psi^-} + (1-a^2)\frac{\mathcal{F}_L + \mathcal{F}_R}{2}$$

k: opacity (AE + IS)

Test1 :) Diffusion limit , neutrino spread out from center



Test2 :) Free-streaming limit, neutrino proceeds at the speed of light



II. Advection term (energy space)

Upwind scheme

If the time variation of density is negative (contraction), the energy is considered to increase.
Consider the reverse in the same way.

III. Source term (matter/geometric)

Time derivative：forward difference

Mass derivative：central difference

4. Conclusion/Future Plans

- I developed a code to calculate steady-state flow using the M1-closure scheme and performed a test calculation of the advection term.
- Comparison with the steady flow calculated by the original code (with FLD).
- Incorporating the created code into the original code.
- Consider additional reactions.