Neutrino Emissivity through the Nucleon Bremsstrahlung from Nuclear Matter at Finite Temperature by the Cluster Variational Method

クラスター変分法を用いた有限温度核物質からの核子制動放射によるニュートリノ放出率

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2023/03/01 第10回超新星ニュートリノ研究会

岡山大学津島キャンパス理学部1号館大会議室

1. Introduction

The aim of the study

An equation of state for supernova matter
with the cluster variational method
starting from bare nuclear forces
(Togashi EOS)

↓

Nucl. Phys. A 961 (2017) 78

Nucleon effective masses
Correlation functions between nucleons

↓

Application to the neutrino emissivity
through the nucleon-nucleon bremsstrahlung
(Weak rates that are consistent with the nuclear EOS)

An equation of state for numerical simulations of core-collapse supernovae based on the bare nuclear forces *)

H. Togashi, K. Nakazato, Y. Takehara,
S. Yamamuro, H. Suzuki, and M. Takano

An EOS table for supernova numerical simulations
constructed with the cluster variational method
based on the Argonne v18 two-body potential
and the Urbana IX three-body potential

APR-EOS

Grid point

Parameter	Minimum	Maximum	Mesh	Number
$\log_{10}(T)$ [MeV]	-1.00	2.60	0.04	$91+1$
$Y_{\rm p}$	0.00	0.65	0.01	66
$\log_{10}(\rho_{\rm B})$ [g/cm^3]	5.1	16.0	0.10	110

*) H. Togashi et al., Nucl. Phys. A 961 (2017) 78.

The Nuclear Hamiltonian

$$H = H_2 + H_3$$

$$H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij} \quad , \qquad H_3 = \sum_{i<j<k}^{N} V_{ijk}$$

V_{ij}: The AV18 potential (isoscalar), V_{ijk}: The UIX potential

The AV18 two-body potential

R. B. Wiringa et al., PRC **51** (1995) 38

$$V_{ij} = \sum_{t=0}^{1} \sum_{s=0}^{1} \Big[V_{Cts}(r_{ij}) + V_{Tt}(r_{ij}) S_{Tij} + V_{SOt}(r_{ij}) \left(\boldsymbol{s} \cdot \boldsymbol{L}_{ij}\right)$$

Central　　Tensor　　Spin-orbit

$$+ V_{qLts}(r_{ij}) \left| \boldsymbol{L}_{ij} \right|^2 + V_{qSOt}(r_{ij}) \left(\boldsymbol{s} \cdot \boldsymbol{L}_{ij}\right)^2 \Big] P_{tsij} + V_{corr}$$

Quadratic orbital angular momentum　　Quadratic spin-orbit　　t: isospin, s: spin

NN scattering data are well reproduced

Jastrow Trial Wave Function

$$\Psi(x_1,\cdots,x_N) = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}(x_1,\cdots,x_N)$$

Φ_{F}: Non-interacting Fermi-gas WF at zero-temperature

Sym[ ]: Symmetrizer

f_{ij}: two-nucleon correlation function

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\left[f^{\mu}_{\mathrm{C}ts}(r_{ij}) + s f^{\mu}_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij} + s f^{\mu}_{\mathrm{SO}t}(r_{ij})\left(\boldsymbol{s}\cdot\boldsymbol{L}_{ij}\right)\right]P^{\mu}_{tsij}$$

- $f_{\mathrm{C}ts}{}^{\mu}$: Central correlation function
- $f_{\mathrm{T}t}{}^{\mu}$: Tensor correlation function
- $f_{\mathrm{SO}t}{}^{\mu}$: Spin-orbit correlation function

$P_{tsij}{}^{\mu}$: Spin-isospin projection operater

t : isospin

s : spin

$\mu = (+, 0, -)$

for (p-p, p-n, n-n) pairs (t =1)

Cluster Variational Method

The expectation value of the Hamiltonian per nucleon

Cluster expansion

$$\frac{\langle H_2 \rangle}{N} = \frac{1}{N} \frac{\langle \Psi | H_2 | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{\langle H_2 \rangle_2}{N} + \frac{\langle H_2 \rangle_3}{N} + \ldots$$

Two-body cluster approximation

$\langle H_2 \rangle_2 / N$: Two-body cluster term

$\langle H_2 \rangle_3 / N$: Three-body cluster term

Higher order cluster terms
The FHNC methd
(APR, AM)

Two-body cluster approximation

$$\frac{E_2}{N} = \frac{\langle H_2 \rangle_2}{N} \left[f^{\mu}_{\mathrm{C}ts}(r), f^{\mu}_{\mathrm{T}t}(r), f^{\mu}_{\mathrm{SO}t}(r) \right]$$

$f_{\mathrm{C}ts}{}^{\mu}(r)$, $f_{\mathrm{T}t}{}^{\mu}(r)$, $f_{\mathrm{SO}t}{}^{\mu}(r)$ Euler-Lagrange equations

⟶ Minimized energies per nucleon

$\frac{\langle H_2 \rangle}{N}$ in the two-body cluster approximation

$$\frac{E_2}{N} = E_F(x) + 2\pi\rho \sum_{t=0}^{1} \sum_{\mu} \sum_{s=0}^{1} \int \Big[F_{ts}^{\mu}(r) V_{Cts}(r) + s F_{Tt}^{\mu}(r) V_{Tt}(r) + s F_{SOt}^{\mu}(r) V_{SOt}(r) + F_{qLts}^{\mu}(r) V_{qLts}(r) + s F_{qSOt}^{\mu}(r) V_{qSOt}(r) \Big] r^2 dr$$

$$+ \frac{2\pi\hbar^2\rho}{m} \sum_{t=0}^{1} \sum_{\mu} \sum_{s=0}^{1} \int \left[\left\{ \left[\frac{df_{Cts}^{\mu}(r)}{dr} \right]^2 + 8s \left[\frac{df_{Tt}^{\mu}(r)}{dr} \right]^2 + 48s \left[\frac{f_{Tt}^{\mu}(r)}{r} \right]^2 \right\} F_{Fts}^{\mu}(r) + \frac{2}{3} s \left[\frac{df_{SOt}^{\mu}(r)}{dr} \right]^2 F_{qFt}^{\mu}(r) \right] r^2 dr$$

$$F_{ts}^{\mu}(r) = \left[f_{Cts}^{\mu}(r)\right]^2 F_{Fts}^{\mu}(r) + 8s \left[f_{Tt}^{\mu}(r)\right]^2 F_{Ft1}^{\mu}(r) + \frac{2}{3} s \left[f_{SOt}^{\mu}(r)\right]^2 F_{qFt1}^{\mu}(r)$$

$$F_{Tt}^{\mu}(r) = 16 \left\{ f_{Ct1}^{\mu}(r) f_{Tt}^{\mu}(r) - \left[f_{Tt}^{\mu}(r)\right]^2 \right\} F_{Ft1}^{\mu}(r) - \frac{2}{3} s \left[f_{SOt}^{\mu}(r)\right]^2 F_{qFt1}^{\mu}(r)$$

$$F_{SOt}^{\mu}(r) = -24 \left[f_{Tt}^{\mu}(r)\right]^2 F_{Ft1}^{\mu}(r) + \frac{4}{3} \left\{ f_{Ct1}^{\mu}(r) - \frac{1}{4} f_{SOt}^{\mu}(r) - f_{Tt}^{\mu}(r) \right\} f_{SOt}^{\mu}(r) F_{qFt1}^{\mu}(r)$$

$$F_{qLts}^{\mu}(r) = \left[f_{Cts}^{\mu}(r)\right]^2 F_{qFts}^{\mu}(r) + 8s \left[f_{Tt}^{\mu}(r)\right]^2 \left[6F_{Ft1}^{\mu}(r) + F_{qFts}^{\mu}(r) \right] + \frac{2}{3} s \left[f_{SOt}^{\mu}(r)\right]^2 F_{bFt1}^{\mu}(r)$$

$$F_{qSOt}^{\mu}(r) = \frac{2}{3} \left[f_{Ct1}^{\mu}(r)\right]^2 F_{qFt1}^{\mu}(r) - \frac{2}{3} f_{Ct1}^{\mu}(r) \left[2 f_{Tt}^{\mu}(r) + f_{SOt}^{\mu}(r) \right] F_{qFt1}^{\mu}(r)$$

$$+ 8s \left[f_{Tt}^{\mu}(r)\right]^2 \left[72 F_{Ft1}^{\mu}(r) + \frac{20}{3} F_{qFt1}^{\mu}(r) \right] + \frac{8}{3} f_{Tt}^{\mu}(r) f_{SOt}^{\mu}(r) F_{qFt1}^{\mu}(r) + \frac{2}{3} s \left[f_{SOt}^{\mu}(r)\right]^2 F_{bFt1}^{\mu}(r)$$

Two Constraints for the Variational Calculation

1. Mayer Condition (Nucleon number conservation)

$$4\pi\rho \int_0^\infty \left[F^{\mu}_{ts}(r) - F^{\mu}_{\mathrm{F}ts}(r)\right] r^2 dr = 0$$

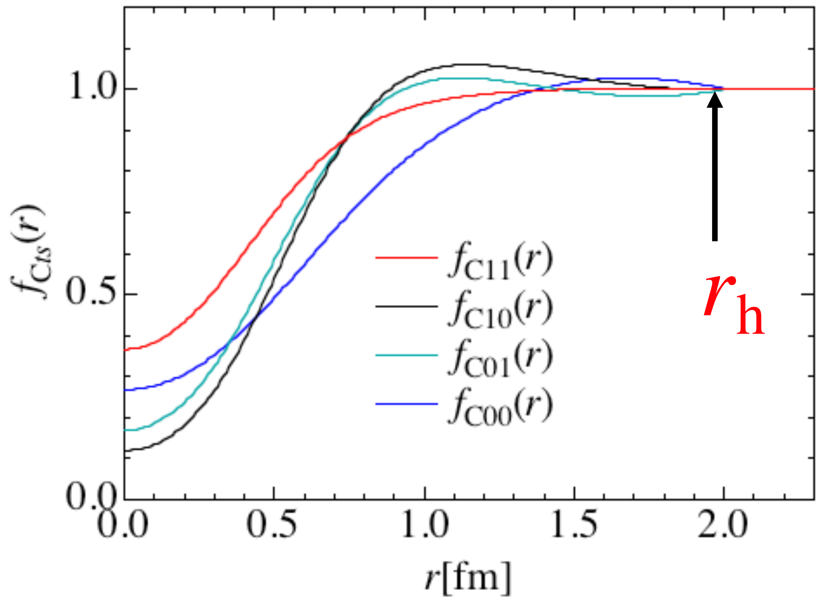
2. Healing distance: r_h

$$f_{\mathrm{C}ts}{}^{\mu}(r) = 1,\ f_{\mathrm{T}t}{}^{\mu}(r) = 0,\ f_{\mathrm{SO}t}{}^{\mu}(r) = 0 \quad (r > r_\mathrm{h})$$

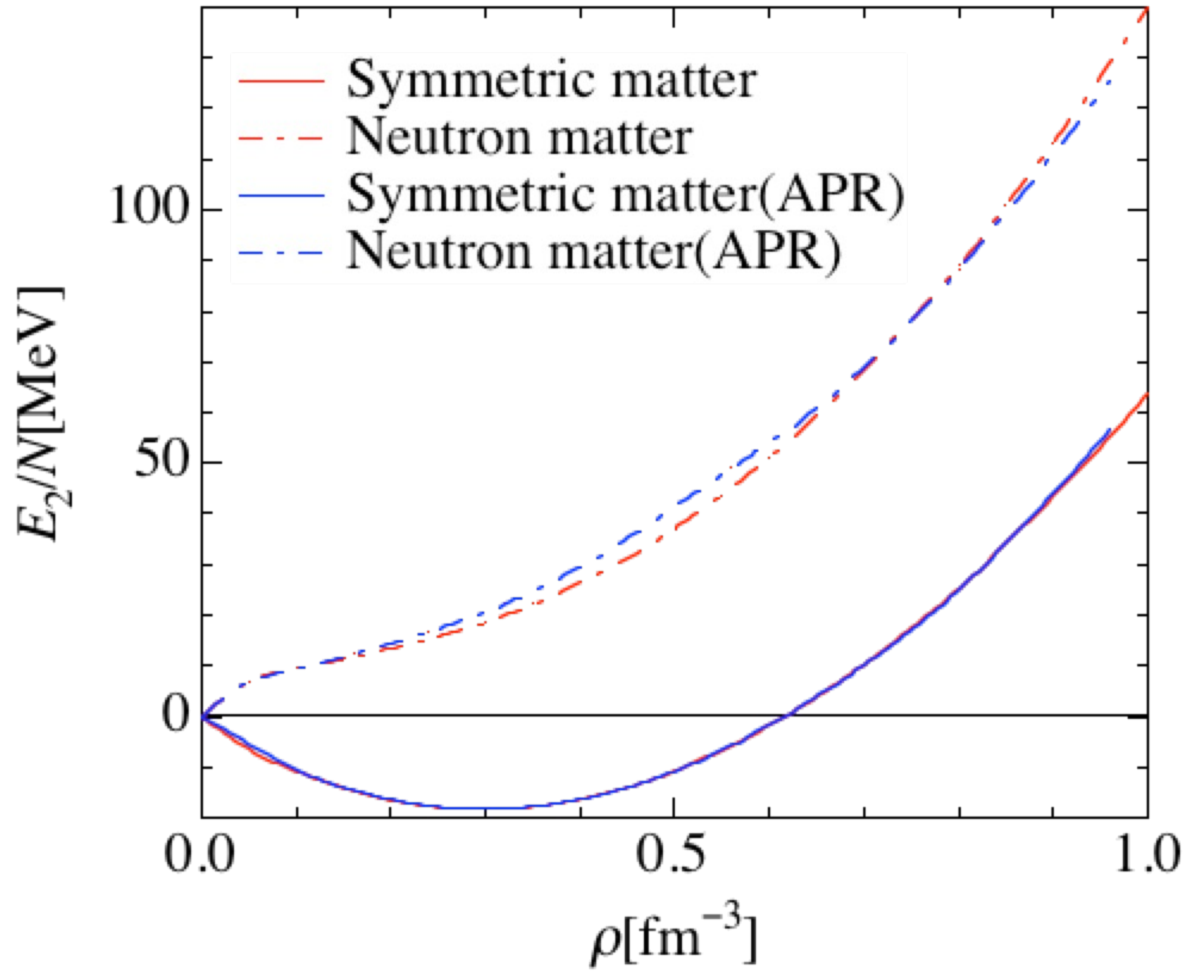
$$r_\mathrm{h} = a_\mathrm{h}\, r_0 \qquad \frac{4\pi r_0^3}{3} = \frac{1}{\rho}$$

a_h: Adjustable parameter

The value of ah is chosen so as to reproduce the result for symmetric nuclear matter by APR (FHNC).



E_2/N: Two-body energy



Kanzawa et al.
NPA791(2007)232

$$r_h = a_h\, r_0$$

$a_h = 1.76$

Two-body cluster approximation + Healing distance condition

⇩

Results by APR(FHNC) are well reproduced

*) A. Akmal et al., Phys. Rev. C58(1998)1804

The three-body energy

UIX potential $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$

$V_{ijk}^{2\pi}$: 2π exchange term

V_{ijk}^{R} : Repulsive term

Expectation values with the non-interacting FG-WF.

$$\frac{E_3^{2\pi}}{N} = \frac{1}{N} \sum_{i<j<k}^{N} \left\langle V_{ijk}^{2\pi} \right\rangle_F \qquad \frac{E_3^{R}}{N} = \frac{1}{N} \sum_{i<j<k}^{N} \left\langle V_{ijk}^{R} \right\rangle_F$$

The energy caused by the three-body nuclear force

$$\frac{E_3}{N}(x) = \alpha \frac{E_3^{R}}{N}(x) + \beta \frac{E_3^{2\pi}}{N}(x) + \gamma \rho^2 e^{-\rho\delta} \left[1 - (1-2x)^2\right]$$

The correction term

Total energy $\frac{E}{N} = \frac{E_2}{N} + \frac{E_3}{N}$

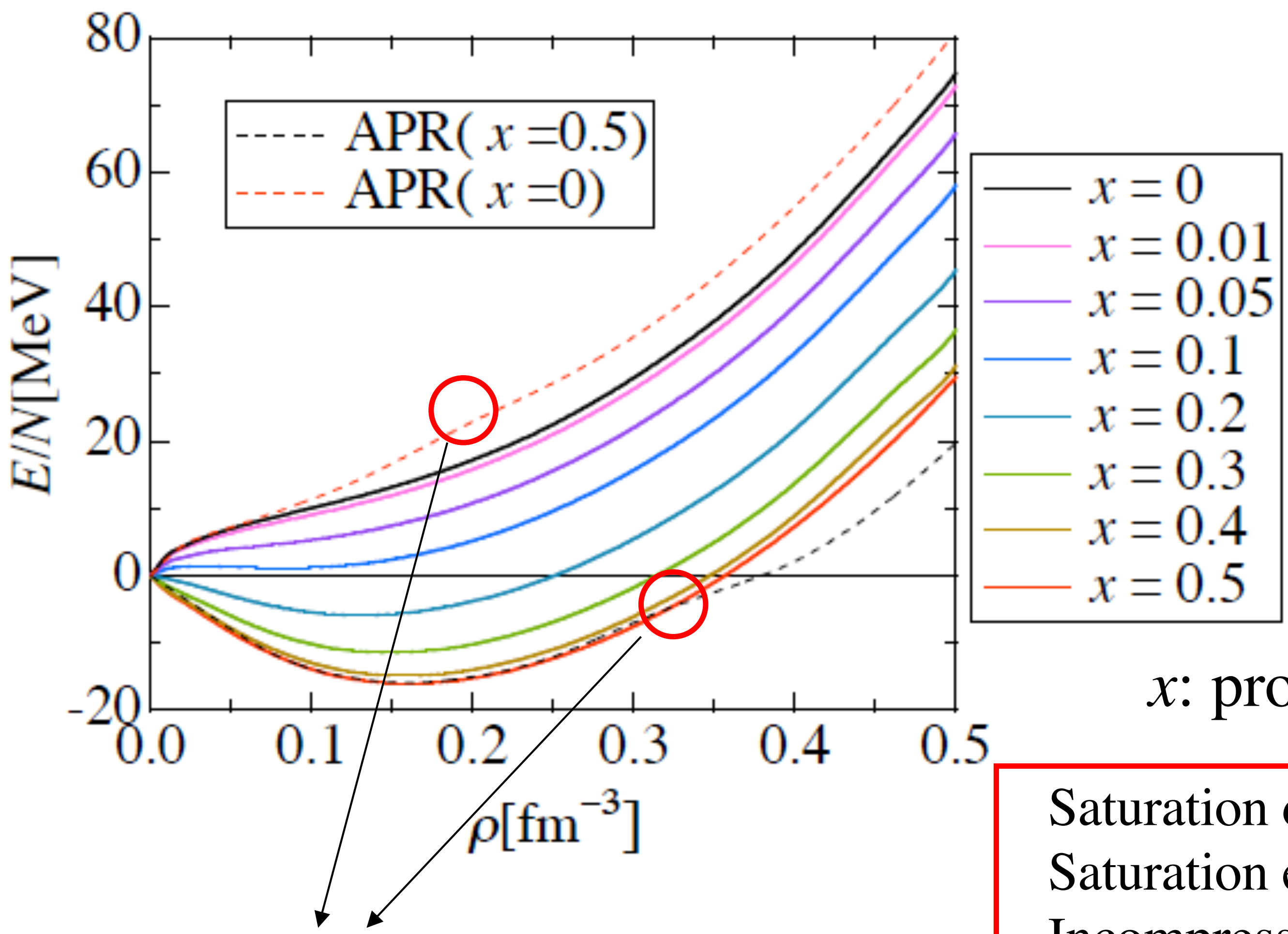
x: proton fraction

Parameters: $\alpha, \beta, \gamma, \delta$

Chosen so as to reproduce the empirical saturation point

$\alpha = 0.43, \beta = -0.34, \gamma = -1804\ \mathrm{MeVfm^6}, \delta = 14.62\ \mathrm{fm^3}$

Energy of asymmetric nuclear matter at zero temeprature



x: proton fraction

Pion condensation

Saturation density: $0.16\ \mathrm{fm}^{-3}$
Saturation energy: –16.1 MeV
Incompressibility: 250 MeV
Symmetry energy: 30 MeV

Uniform Nuclear Matter at Finite Temperatures

K. E. Schmidt and V. R. Pandharipande: Phys. Lett. 87B(1979) 11.

Free energy $$\frac{F}{N} = \frac{E_0}{N} - T\frac{S_0}{N}$$

E_0/N : Internal energy
S_0/N : Entropy

$$\frac{E_0}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$

E_2/N: In E_2/N at zero temperature, occupation probabilities of single-nucleon states are replaced by the averaged occupation probabilities $n_i(k)$ at the temperature T.

E_3/N: Thermal effects are ignored

Averaged occupation probabilities: $n_i(k)$

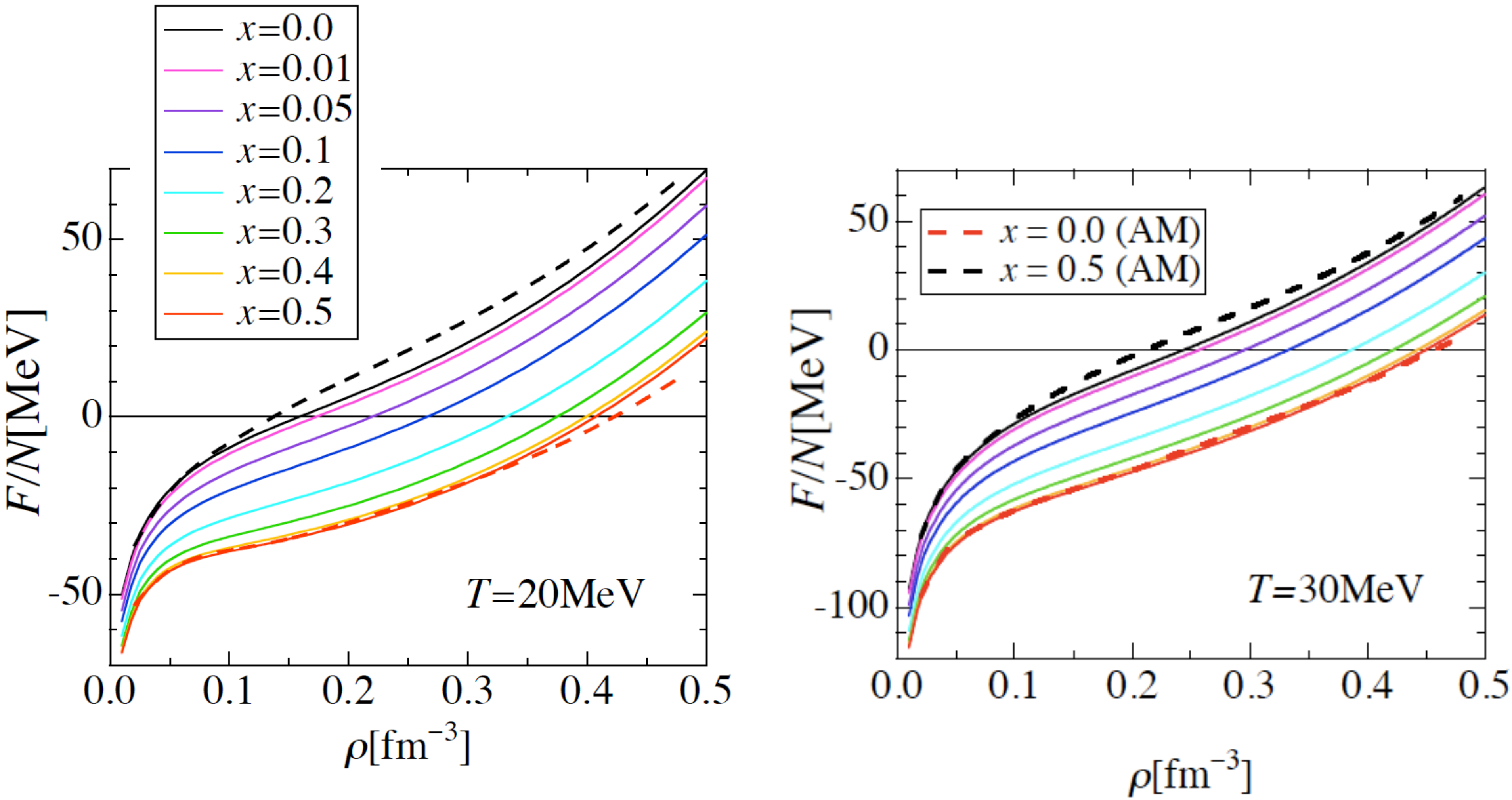
$$n_i(k) = \left\{1 + \exp\left[\left[e_i(k) - \mu_i\right]/k_B T\right]\right\}^{-1}, \quad e_i(k) = \frac{\hbar^2 k^2}{2m_i^*} \quad (i = \mathrm{p}, \mathrm{n})$$

← Effective mass

$$\frac{S_0}{N} = -\frac{k_B}{N}\sum_{i=\mathrm{p,n}}\sum_j \left\{\left[1 - n_i(k_j)\right]\ln\left[1 - n_i(k_j)\right] + n_i(k_j)\ln\left[n_i(k_j)\right]\right\}$$

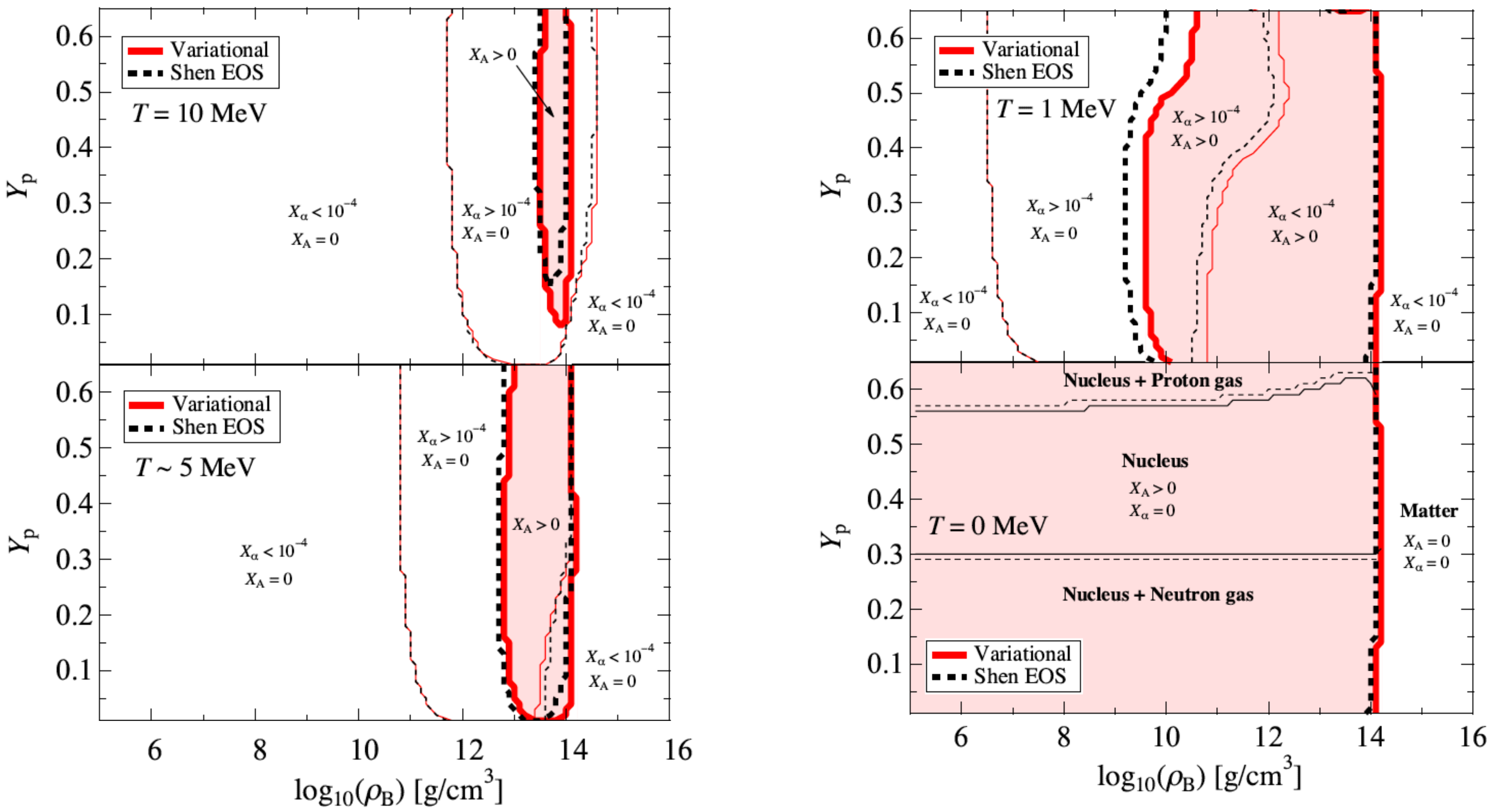
Free energy is minimized with respect to m_p^* and m_n^*

Free energy of asymmetric nuclear matter



H. Togashi et al., Nucl. Phys. A902 (2013) 53.

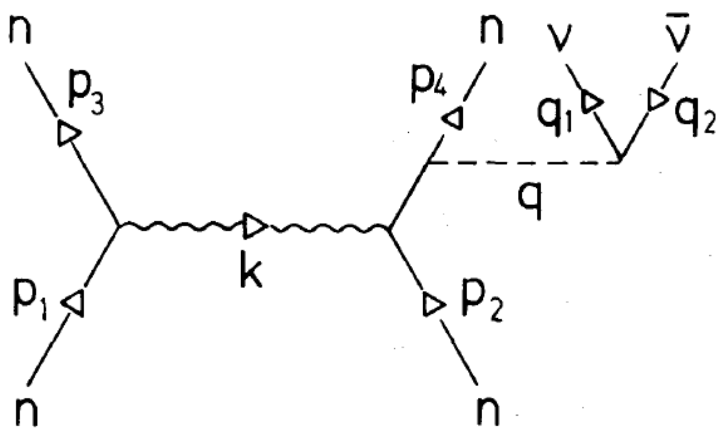
Phase Diagram of Nuclear Matter



H. Togashi et al., Nucl. Phys. A961 (2017)78

3. Application to Bremsstrahlung

O. V. Maxwell, Astrophys. J. 316 (1987) 691.
B. L. Friman and O. V. Maxwell, Astrophys. J. 232 (1979)451.



Friman & Maxwell(1979)

N-N interaction:

One Pion Exchange (OPE)

$$V_{12}(\mathbf{k}) = -\left(\frac{f}{m_\pi}\right)^2 (\tau_1 \cdot \tau_2) \frac{(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})}{k^2 + m_\pi^2}$$

Emissivity

$$\epsilon_{\nu\bar{\nu}}^{NN} = \frac{1}{\hbar} \int \prod_{i=1}^{4} \frac{d^3 p_i}{(2\pi)^3} \frac{d^3 q_1}{(2\pi)^3 2\omega_1} \frac{d^3 q_2}{(2\pi)^3 2\omega_2} (2\pi)\delta(E_\mathrm{f} - E_\mathrm{i})(2\pi)^3 \delta(\mathbf{P}_\mathrm{f} - \mathbf{P}_\mathrm{i})$$
$$(\omega_1 + \omega_2) n_1 n_1 (1 - n_3)(1 - n_4) S \sum_{\mathrm{spins}} \left| M^{NN}(\mathbf{k}, \mathbf{k}') \right|^2$$

$$\sum_{\mathrm{spins}} \left| M^{NN}(\mathbf{k}, \mathbf{k}') \right|^2 = 128 \frac{G_\mathrm{F}^2}{2} \left(\frac{f_\pi}{m_\pi}\right)^4 F^{NN}(\mathbf{k}, \mathbf{k}')$$

$$\mathbf{k} = \mathbf{p_1} - \mathbf{p_3} \quad \mathbf{k}' = \mathbf{p_1} - \mathbf{p_4}$$

$$F^{\mathrm{nn}}(\mathbf{k}, \mathbf{k}') = \left(\frac{k^2}{k^2 + m_\pi^2}\right)^2 + \left(\frac{k'^2}{k'^2 + m_\pi^2}\right)^2 + \frac{1}{(k^2 + m_\pi^2)(k'^2 + m_\pi^2)} \left[k^2 k'^2 - 3(\mathbf{k} \cdot \mathbf{k}')^2\right]$$

$$F^{\mathrm{np}}(\mathbf{k}, \mathbf{k}') = \left(\frac{k^2}{k^2 + m_\pi^2}\right)^2 - 2\left(\frac{k'^2}{k'^2 + m_\pi^2}\right)^2 - \frac{2}{(k^2 + m_\pi^2)(k'^2 + m_\pi^2)} \left[k^2 k'^2 - (\mathbf{k} \cdot \mathbf{k}')^2\right]$$

Degenerate Limit

Emissivity

$$\epsilon_{\nu\bar{\nu}}^{nn} = \frac{G_F^2}{4\pi S}\left(\frac{f_\pi}{m_\pi}\right)^4 \frac{82}{14175}(k_B T)^8 (m_n^*)^4 \left[2J_1(2k_{Fn},0) + J_2(k_{Fn},0)\right]$$

$$\epsilon_{\nu\bar{\nu}}^{np} = \frac{G_F^2}{32\pi}\left(\frac{f_\pi}{m_\pi}\right)^4 \frac{82}{14175}(k_B T)^8 (m_n^* m_p^*)^2 \left[J_1(2k_{Fp},0) + J_1(k_{Fn}+k_{Fp},|k_{Fn}-k_{Fp}|) - J_2(2k_{Fp},0)\right]$$

$$J_1(k_2,k_1) = \int_{k_1}^{k_2}\left(\frac{k^2}{k^2+m_\pi^2}\right)^2 dk \qquad J_2(k_2,k_1) = \int_{k_1}^{k_2}\frac{k^2}{k^2+m_\pi^2}\left(1-\frac{m_\pi^2}{\sqrt{a^2-b^2}}\right)dk$$

$$a = 2p_{Fn}^2 - k^2 + m_\pi^2 - 2p_{Fn}^2\cos\theta_1\cos\theta_2 \qquad b = p_{Fn}^2\sin\theta_1\sin\theta_2$$

$$\cos\theta_1 = \frac{k}{2p_{Fn}} \qquad \cos\theta_2 = -\frac{k}{2p_{Fn}}$$

Interaction rate $\Phi^{NN}(\omega)$

$$\epsilon_{\nu\bar{\nu}}^{NN} = \frac{1}{2}(4\pi)^2\frac{1}{30}\int_0^\infty \Phi^{NN}(\omega)\exp\left(-\frac{\omega}{k_B T}\right)\omega^6 d\omega$$

Degenerate Limit

c.f., H. Suzuki, Int. Symp. Neutrino Astrophys. (1993) p219.

$$\phi(\omega) = \frac{1}{6\pi^9}c(\hbar c)^2\left[\frac{G_F}{(\hbar c)^3}\right]^2 g_A^2\left(\frac{f_\pi^2}{4\pi}\right)^2\left(\frac{1}{m_\pi c^2}\right)^4(m_N c^2)^4 k_B T\left\{4\pi^2\frac{k_B T}{\omega}+(\frac{\omega}{k_B T})\right\}\frac{1}{1-exp(-\frac{\omega}{k_B T})}C_{NN}$$

Non-Degenerate Limit

$$\Phi^{nn}(\omega) = \frac{1}{8\pi^{11/2}}G_F^2 g_A^2\left(\frac{f_\pi}{m_\pi}\right)^4\frac{1}{(4\pi)^2}\left(\frac{m_n^*}{k_B T}\right)^{3/2} n_B^2(1-Y_p)^2\exp\left(\frac{\omega}{2k_B T}\right)\int_1^\infty \exp\left(-\frac{\omega t}{2k_B T}\right)\sqrt{t^2-1}F_{nn}(t)dt$$

$$F_{nn}(t) = 3+\frac{2\alpha_n^2}{1+\alpha_n^2+2\alpha_n t}+\frac{1}{2}\frac{5\alpha_n^2+6\alpha_n t+3}{(\alpha_n+t)\sqrt{t^2-1}}\ln\left(\frac{\alpha_n+t-\sqrt{t^2-1}}{\alpha_n+t+\sqrt{t^2-1}}\right) \qquad \alpha_n = \frac{m_\pi^2}{m_n^*\omega}$$

$$\Phi^{np}(\omega) = \frac{1}{2\pi^{11/2}}G_F^2 g_A^2\left(\frac{f_\pi}{m_\pi}\right)^4\frac{1}{(4\pi)^2}\left(\frac{2m_n^* m_p^*}{m_n^*+m_p^*}\frac{1}{k_B T}\right)^{3/2} n_B^2(1-Y_p)Y_p\exp\left(\frac{\omega}{2k_B T}\right)\int_1^\infty \exp\left(-\frac{\omega t}{2k_B T}\right)\sqrt{t^2-1}F_{np}(t)dt$$

$$F_{np}(t) = 1+\frac{3\alpha_{np}^2}{1+\alpha_{np}^2+2\alpha_{np}t}+\frac{2\alpha_{np}^2+\alpha_{np}t-1}{(\alpha_{np}+t)\sqrt{t^2-1}}\ln\left(\frac{\alpha_{np}+t-\sqrt{t^2-1}}{\alpha_{np}+t+\sqrt{t^2-1}}\right) \qquad \alpha_{np} = \frac{m_\pi^2}{\frac{2m_n^* m_p^*}{m_n^*+m_p^*}\omega}$$

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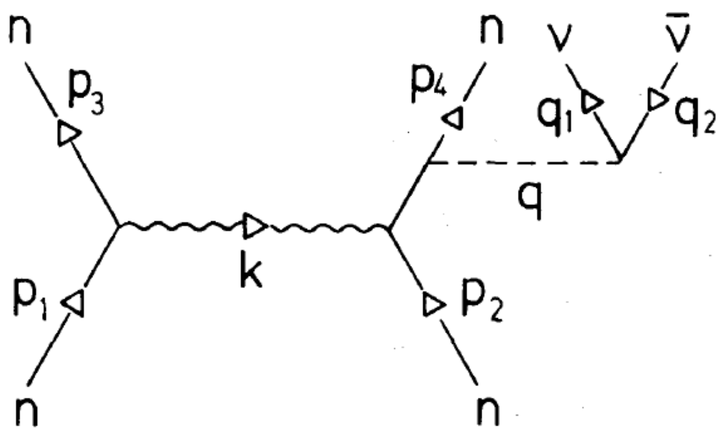


Friman&Maxwell(1979)

N-N interaction:

One Pion Exchange (OPE)

$$V_{12}(\mathbf{k}) = -\left(\frac{f}{m_\pi}\right)^2 (\tau_1 \cdot \tau_2) \frac{(\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k})}{k^2 + m_\pi^2}$$

Replaced by the two-body cluster approximation of the long-range part of the AV18 potential (OPEP)

Effects of the nuclear correlations are included in f_π and m_π

Introduction of the effective OPE potential

$$V_{ij}^{\rm OPE}(f,m_\pi)=f^2(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)\Big[(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)V_{\rm c}(m_\pi r_{ij})+S_{{\rm T}ij}V_{\rm T}(m_\pi r_{ij})\Big]$$

$$V_{\rm c}(x)=\frac{{\rm e}^{-x}}{x}\qquad\qquad V_{\rm T}(x)=(1+\frac{3}{x}+\frac{3}{x^2})\frac{{\rm e}^{-x}}{x}$$

For $\nu = ({\rm pp}, {\rm pn}, {\rm nn})$, $(f_\nu^*, m_{\pi\nu}^*)$ are introduced as follows:

$$\begin{aligned}\frac{<V^{\rm OPE}>}{N}&=\sum_\nu\int\cdots\int dx_1\cdots dx_N\Psi^\dagger(x_1,\cdots,x_N)\sum_{i>j}V_{ij}^{\rm OPE}(f,m_\pi)P_{\nu ij}\Psi(x_1,\cdots,x_N)\\&=\sum_\nu\int\cdots\int dx_1\cdots dx_N\Phi_{\rm F}^\dagger(x_1,\cdots,x_N)\sum_{i>j}g_{ij}^\nu V_{ij}^{\rm OPE}(f,m_\pi)P_{\nu ij}\Phi_{\rm F}(x_1,\cdots,x_N)\\&=\sum_\nu\int\cdots\int dx_1\cdots dx_N\Phi_{\rm F}^\dagger(x_1,\cdots,x_N)\sum_{i>j}V_{ij}^{\rm OPE}(f_\nu^*,m_{\pi\nu}^*)P_{\nu ij}\Phi_{\rm F}(x_1,\cdots,x_N)\end{aligned}$$

$\Psi(x_1,\cdots,x_N)$: WF of uniform nuclear matter

$\Phi_{\rm F}(x_1,\cdots,x_N)$: WF of non-interacting Fermi gas

$P_{\nu ij}$: Projection operator with respect to $\nu = ({\rm pp}, {\rm pn}, {\rm nn})$

Introduction of the effective OPE potential

The two-body cluster approximation

$$\frac{< V^{\mathrm{OPE}} >}{N} = 2\pi\rho \sum_{t=0}^{1} \sum_{\mu} \sum_{s=0}^{1} \int_0^\infty r^2 dr \left[F^{\mu}_{ts}(r) V^{\mathrm{OPE}}_{\mathrm{C}ts}(r) + s F^{\mu}_{\mathrm{T}t}(r) V^{\mathrm{OPE}}_{\mathrm{T}t}(r) \right]$$

$$F^{\mu}_{ts}(r) = \left\{ \left[f^{\mu}_{\mathrm{C}ts}(r) \right]^2 + 8s \left[f^{\mu}_{\mathrm{T}t}(r) \right]^2 \right\} F^{\mu}_{\mathrm{F}ts}(r) + \frac{2}{3} s \left[f^{\mu}_{\mathrm{SO}t}(r) \right]^2 F^{\mu}_{\mathrm{qF}ts}(r),$$

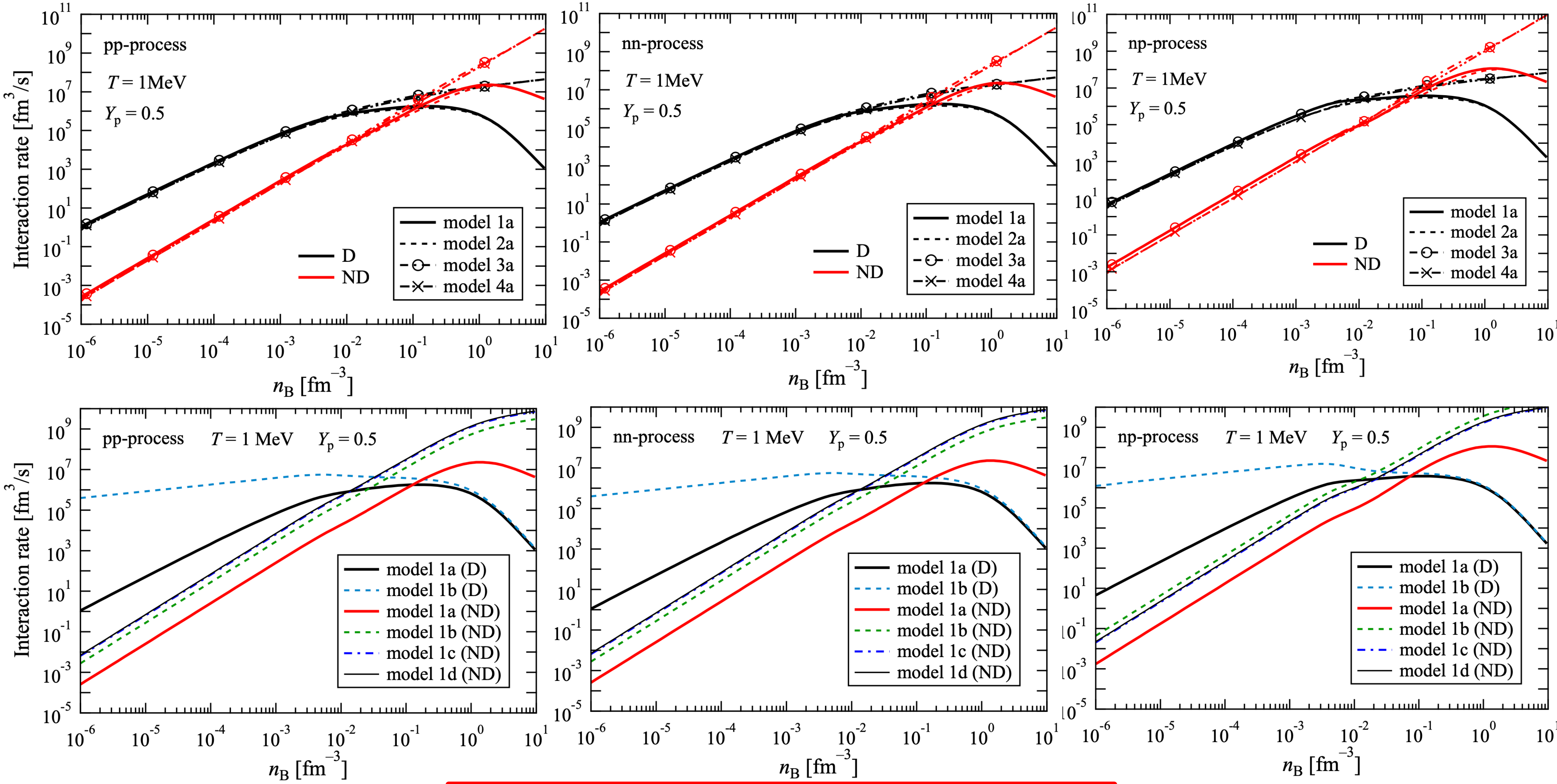
$$F^{\mu}_{\mathrm{T}t}(r) = 16 f^{\mu}_{\mathrm{T}t}(r) \left[f^{\mu}_{\mathrm{C}t1}(r) - f^{\mu}_{\mathrm{T}t}(r) \right] F^{\mu}_{\mathrm{F}t1}(r) - \frac{2}{3} s \left[f^{\mu}_{\mathrm{SO}t}(r) \right]^2 F^{\mu}_{\mathrm{qF}t1}(r),$$

$$\begin{aligned} F^{\mu}_{\mathrm{F}ts}(r_{12}) &\equiv \Omega^2 \sum_{\text{isospin}} \sum_{\text{spin}} \int \Phi^{\dagger}_{\mathrm{F}} P^{\mu}_{ts12} \Phi_{\mathrm{F}} \, d\boldsymbol{r}_3 \, d\boldsymbol{r}_4 \cdots d\boldsymbol{r}_N \\ &= \frac{2s+1}{4} \{ \xi_i \xi_j - (-1)^{t+s} l_i(r_{12}) l_j(r_{12}) \}, \end{aligned}$$

$$\begin{aligned} F^{\mu}_{\mathrm{qF}ts}(r_{12}) &\equiv \Omega^2 \sum_{\text{isospin}} \sum_{\text{spin}} \int \Phi^{\dagger}_{\mathrm{F}} |\boldsymbol{L}_{12}|^2 P^{\mu}_{ts12} \Phi_{\mathrm{F}} \, d\boldsymbol{r}_3 \, d\boldsymbol{r}_4 \cdots d\boldsymbol{r}_N \\ &= \frac{2s+1}{4} \left\{ \frac{r_{12}^2}{10} \xi_i \xi_j (k^2_{\mathrm{F}i} + k^2_{\mathrm{F}j}) - (-1)^{t+s} \frac{r_{12}}{2} \left[l_i(r_{12}) \frac{dl_j(r_{12})}{dr_{12}} + l_j(r_{12}) \frac{dl_i(r_{12})}{dr_{12}} \right] \right\}, \end{aligned}$$

Interaction Rate

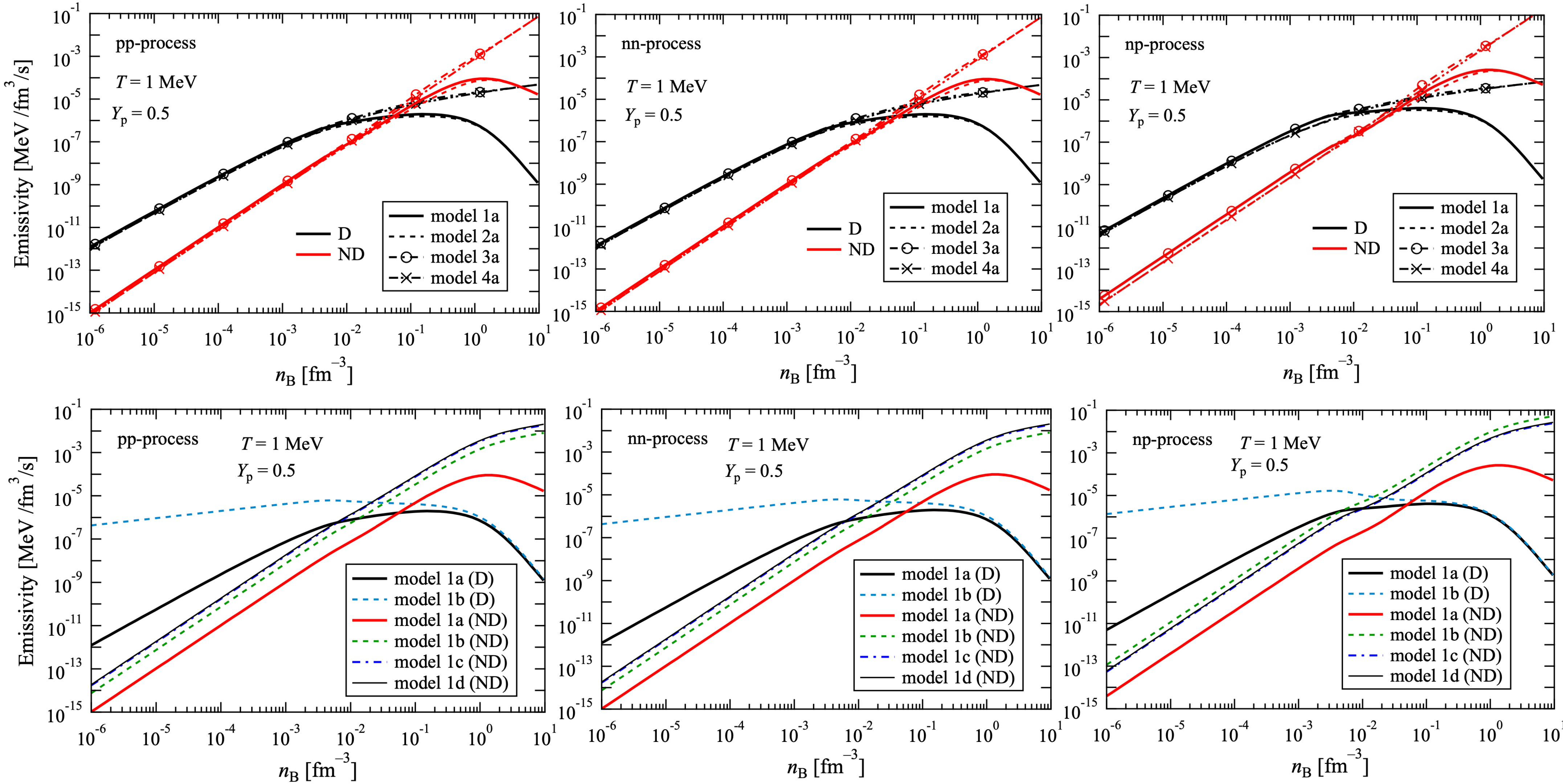
$T = 1$ MeV, $Y_p = 0.5$, $\omega = 5$ MeV



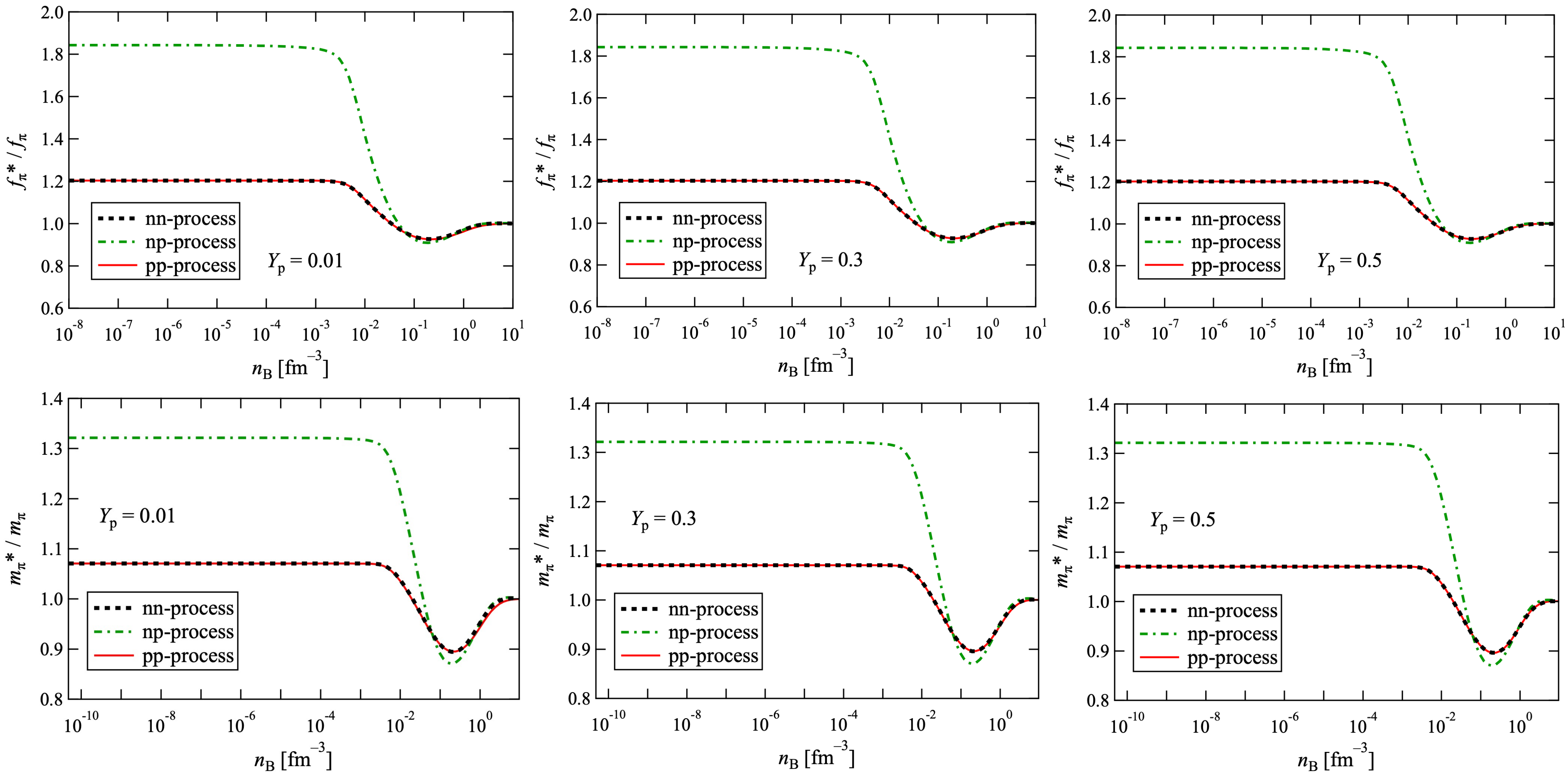
$$\Phi^{\nu}(\omega) = \mathrm{Min}\left[\Phi^{\nu}_{\mathrm{D}}(\omega), \Phi^{\nu}_{\mathrm{ND}}(\omega)\right]$$

Emissivity

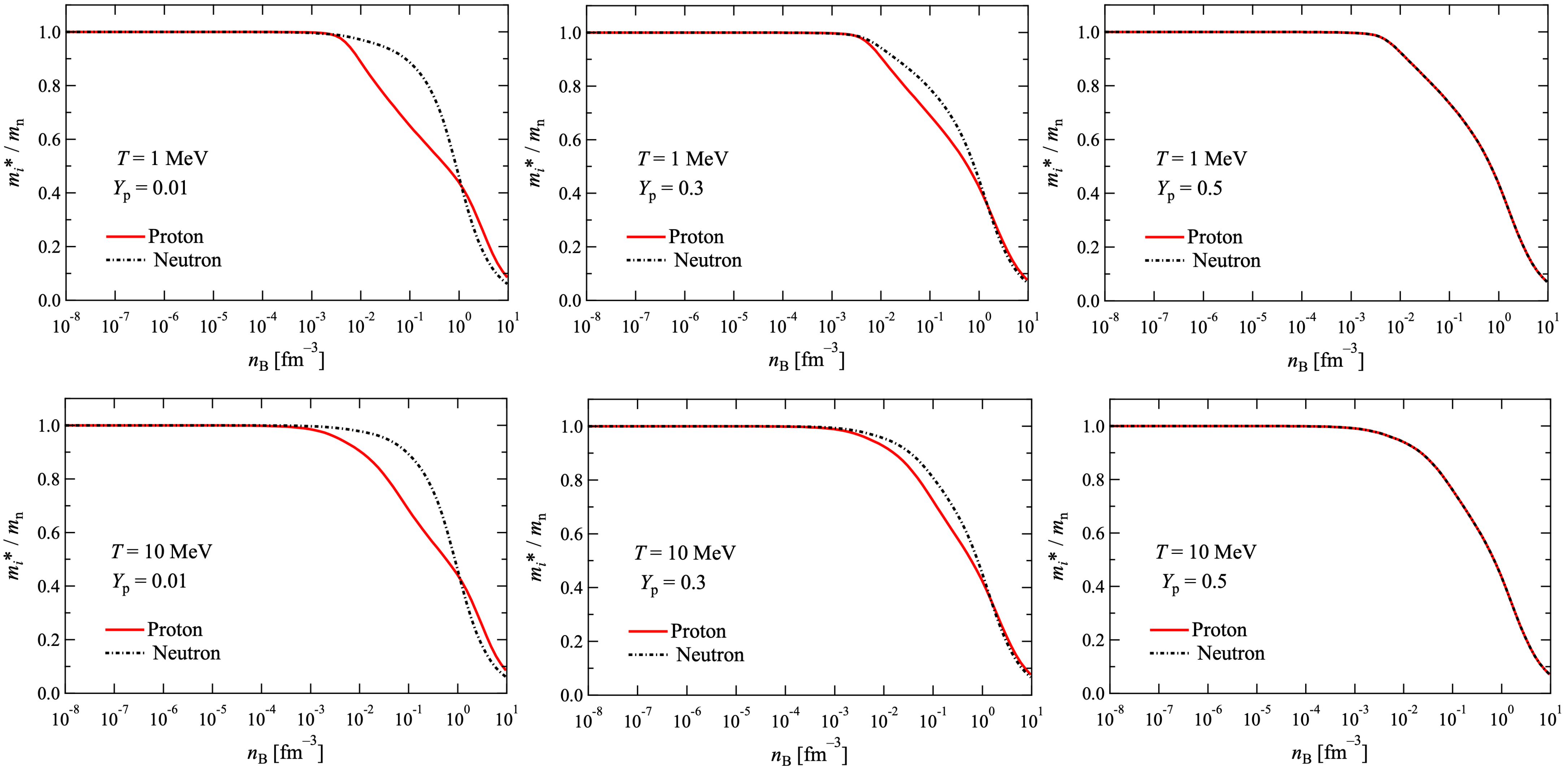
$T = 1$ MeV, $Y_p = 0.5$



Pion effective coupling constant and effective mass



Nucleon Effective Mass



Summary

The correlation between nucleons and nucleon effective masses obtained through the cluster variational calculations are applied to neutrino emissivity by nucleon-nucleon bremsstrahlung.

The interaction rates are calculated in the degenerate limit and non-degenerate limit.

Future Plans

Analytic fitting formula will be prepared for numerical simulations of core-collapse supernovae.

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Weak rates that are consistent with nuclear equation of state.