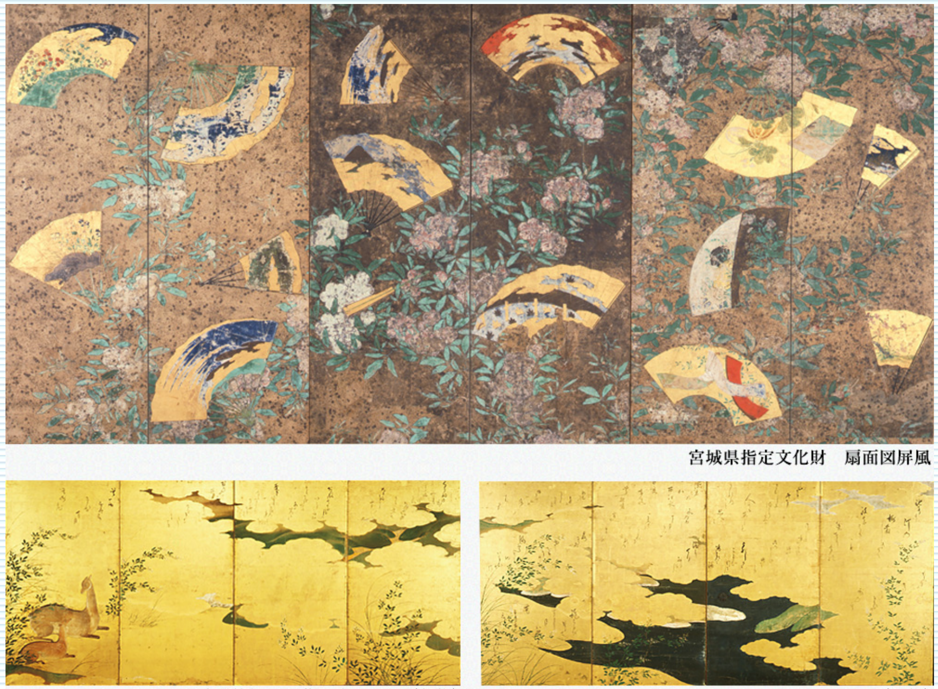
Scientific Research on Innovative Areas:

Revealing the history of the universe with underground particle and nuclear research

Tohoku University, Japan, March 7-9, 2019



宮城県指定文化財　扇面図屏風

重要美術品　萩に鹿図屏風（左隻）　（右隻）

0νββ-decay mechanisms, NMEs and quenching of g_A

Fedor Šimkovic

OUTLINE

I. ***Introduction***

Majorana, Pontecorvo, Weinberg

II. ***The $0\nu\beta\beta$-decay scenarios due neutrinos exchange***

(simpliest, sterile ν, LR-symmetric model, interpolating formula)

III. ***DBD NMEs – Current status***

(deformed QRPA versus ISM, …)

IV. ***Is there a proportionality between $0\nu\beta\beta$- and $2\nu\beta\beta$-decay NMEs?***

(role of SU(4) symmetry …)

V. ***Quenching of g_A (Improved formalism of $2\nu\beta\beta$-decay, KamLAND-Zen constraining of g_A)***

VI. ***New modes of the double-beta decay with emission of a single electron from an atom***

Acknowledgements: **A. Faessler (Tuebingen), P. Vogel (Caltech), S. Kovalenko (Valparaiso U.), M. Krivoruchenko (ITEP Moscow), D. Štefánik, R. Dvornický (Comenius U.), A. Babič, A. Smetana (IEAP CTU Prague), …**

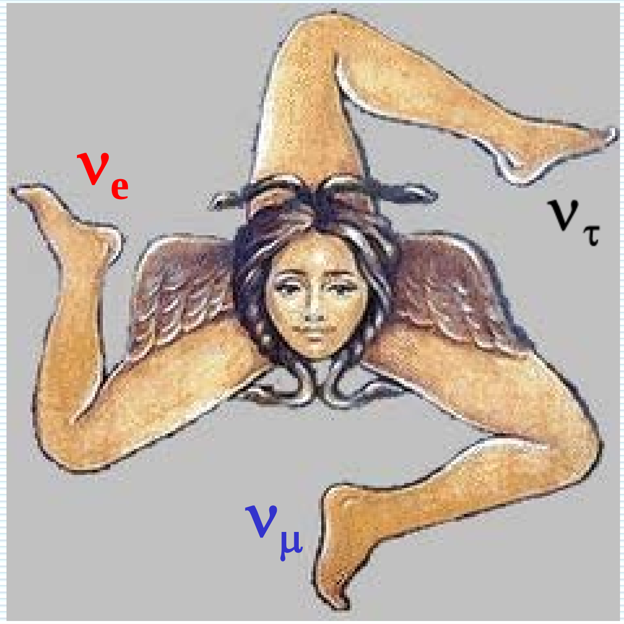
I. Introduction

After 62 years we know

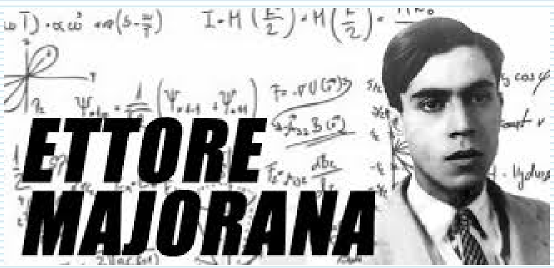
Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

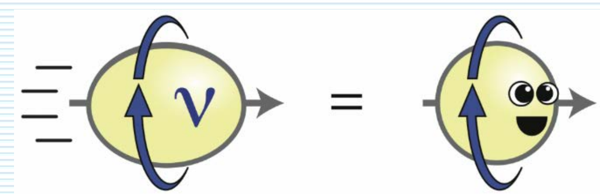


- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?



Currently main issue

Nature, Mass hierarchy,
CP-properties, sterile ν



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

Majorana fermion

https://en.wikipedia.org/wiki/File:Ettore_Majorana.jpg

CNNP 2018, Catania, October 15-21, 2018

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di Ettore Majorana

Symmetric Theory of Electron and Positron
Nuovo Cim. 14 (1937) 171

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di Dirac ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.*

L'interpretazione dei cosidetti « stati di energia negativa » proposta da Dirac [1] conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accord ... isfacenti; sia perchè s ... trica, sia perchè la si ... iante tali procedimenti ... che possibilmente dov ... uova via che conduce più direttamente alla meta.

Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

(1) P. A. M. Dirac, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. Heisenberg, « ZS. f. Phys. », **90**, 209, 1934.

MESONIUM AND ANTIMESONIUM

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 549-551 (August, 1957)

INVERSE BETA PROCESSES AND NONCONSERVATION OF LEPTON CHARGE

B. PONTECORVO

Joint Institute for Nuclear Research

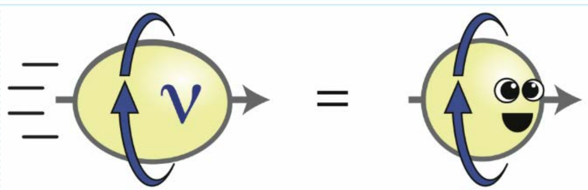
Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 247-249 (January, 1958)

$\nu \leftrightarrow \bar{\nu}$ **oscillation**

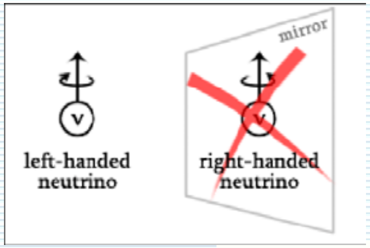
(neutrinos are Majorana particles)

It follows from the above assumptions that in vacuum a neutrino can be transformed into an antineutrino and vice versa. This means that the neutrino and antineutrino are "mixed" particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 of different combined parity.[5]

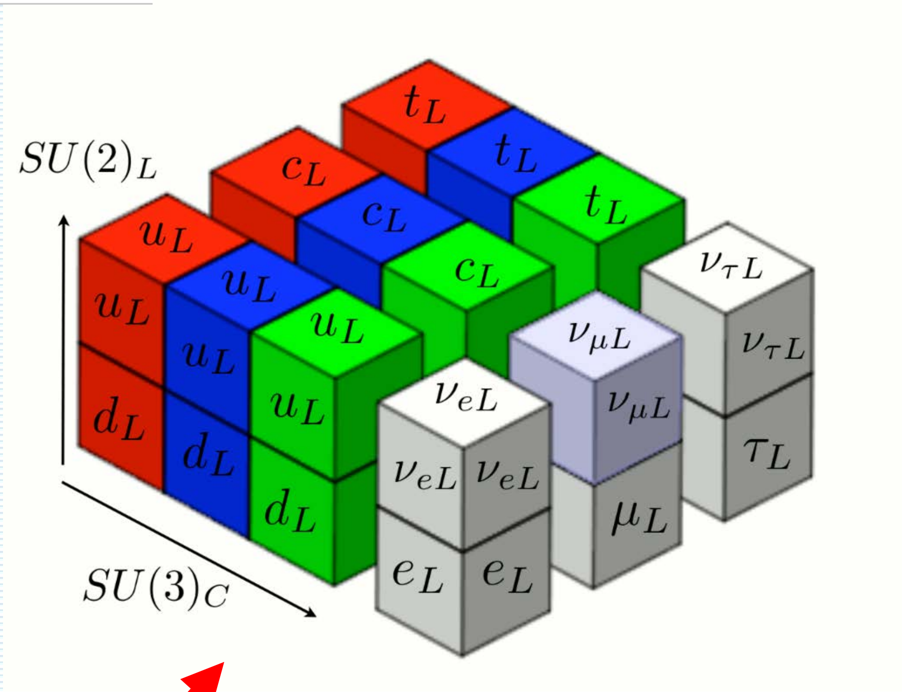


1968 Gribov, Pontecorvo [PLB 28(1969) 493]

oscillations of neutrinos - a solution of deficit of solar neutrinos in Homestake exp.



Beyond the Standard model physics *(EFT scenario)*



The absence of the right-handed neutrino fields in the SM is the simplest, most economical possibility. In such a scenario Majorana mass term is the only possibility for neutrinos to Be massive and mixed. This mass term is generated by the Lepton number violating Weinberg effective Lagrangian.

$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \boxed{\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O(\frac{1}{\Lambda^3})}$$

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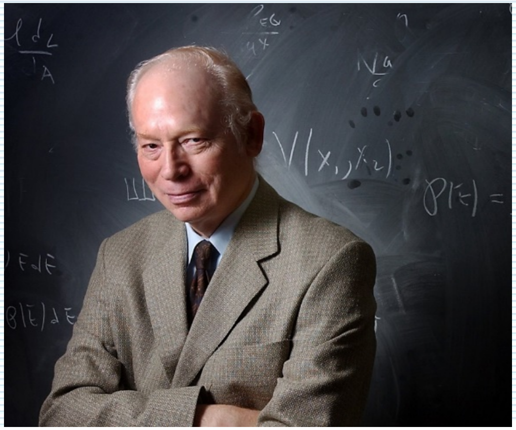
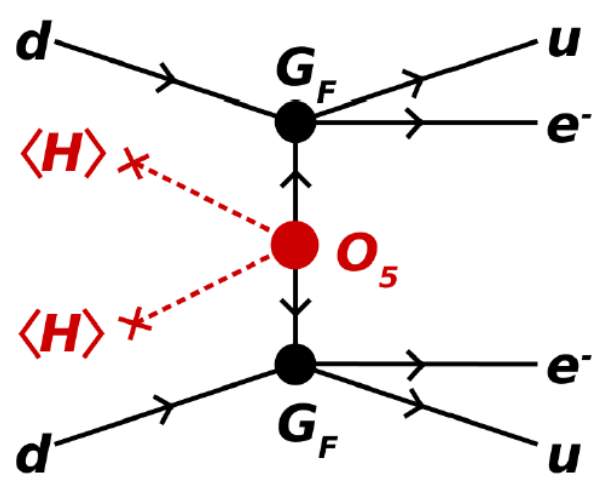


Weinberg, 1979: d=5

$0\nu\beta\beta$ decay:



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

. Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

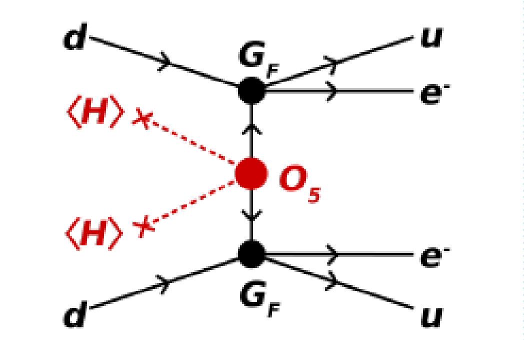
II. The 0νββ-decay scenarios due neutrinos exchange

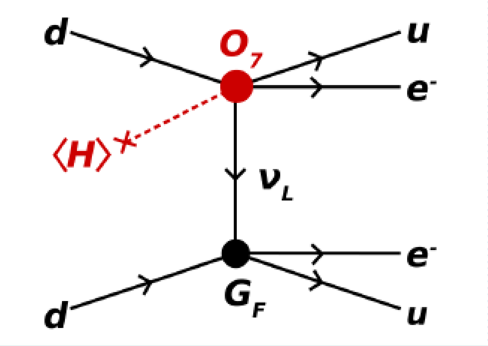
Amplitude for (A,Z)→(A,Z+2)+2e⁻ can be divided into:

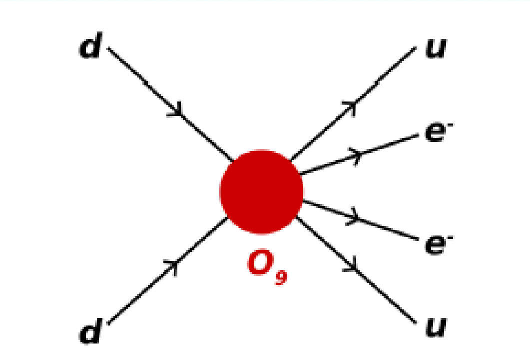
M. Hirsch, Pontecorvo school 2015

short range: d=9 (d=11)

mass mechanism: d=5 + **long range: d=7** +







$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda}(L_i H)(L_j H)$$

Weinberg, 1979

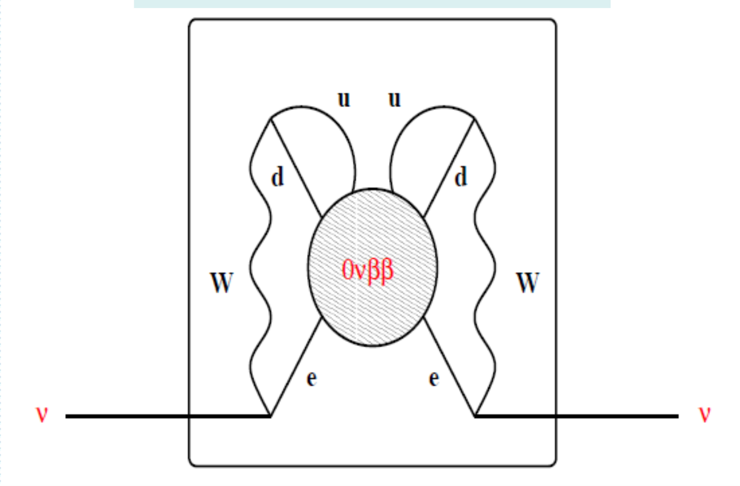
$$\mathcal{O}_2 \propto LLLe^cH$$
$$\mathcal{O}_3 \propto LLQd^cH$$
$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^cH$$
$$\mathcal{O}_8 \propto L\bar{e}^c\bar{u}^cd^cH$$

Babu, Leung: 2001
de Gouvea, Jenkins: 2007

$$\mathcal{O}_5 \propto LLQd^cHHH^\dagger$$
$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^cHH^\dagger H$$
$$\mathcal{O}_7 \propto LQ\bar{e}^c\bar{Q}HHH^\dagger$$
$$\mathcal{O}_9 \propto LLLe^cLe^c$$
$$\mathcal{O}_{10} \propto LLLe^cQd^c$$
$$\mathcal{O}_{11} \propto LLQd^cQd^c$$
$$\cdots\cdots$$

Physics at LHC (Jose Valle talk)

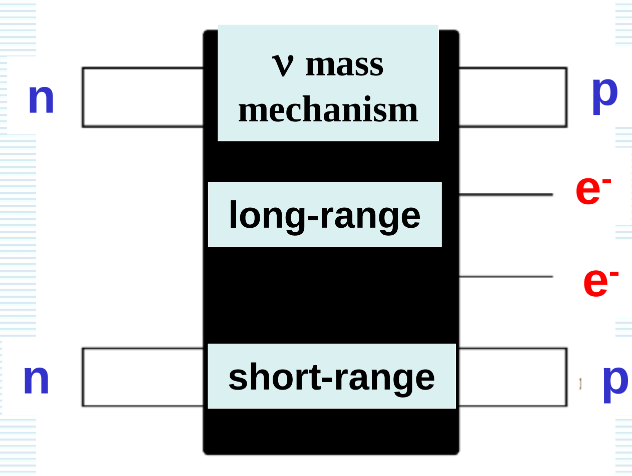
Schechter, Valle: PRD 1982



If 0νββ is observed the ν is
a Majorana particle

Different 0νββ-decay scenarios

Can we say something about content of the black box?

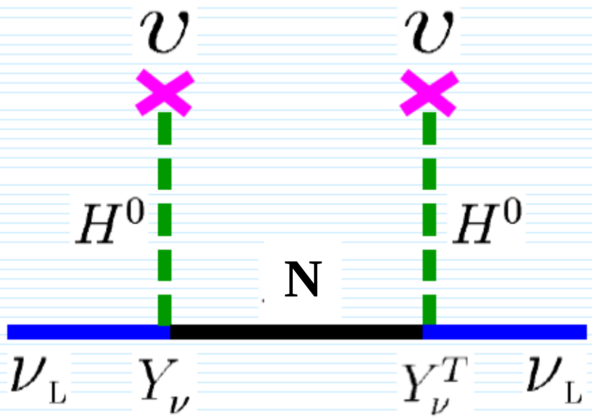


Considering

i. Sterile ν
ii. Different LNV scales
iii. Right-handed currents
iv. Non-standard ν-interactions

II.a. *The simplest $0\nu\beta\beta$-decay scenario: LHC & LNV scale Λ is too large*

$$\mathcal{L}_5^{eff} = -\frac{1}{\Lambda} \sum_{l_1 l_2} \left(\overline{\Psi}^{lep}_{l_1 L} \tilde{\Phi} \right) \acute{Y}_{l_1 l_2} \left(\tilde{\Phi}^T (\Psi^{lep}_{l_2 L})^c \right)$$



Heavy Majorana leptons N_i ($N_i = N^c_i$) singlet of $SU(2)_L \times U(1)_Y$ group Yukawa lepton number violating int.

$$m_i = \frac{v}{\Lambda} (y_i v), \quad i = 1, 2, 3$$

$\Lambda \geq 10^{15}$ GeV

S.M. Bilenky, Phys.Part.Nucl.Lett. 12 (2015) 453-461

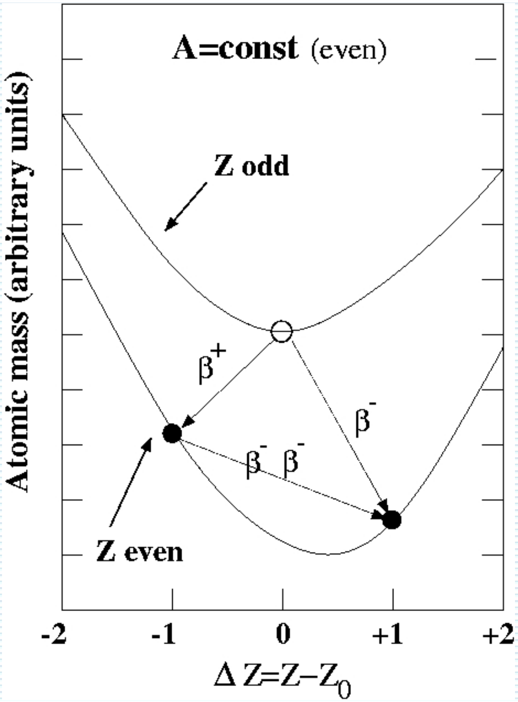
The three Majorana neutrino masses are suppressed by the ratio of the electroweak scale and a scale of a lepton-number violating physics.

The discovery of the ββ-decay and absence of transitions of flavor neutrinos into sterile states would be evidence in favor of this minimal scenario.

(A,Z) → (A,Z+2) + e⁻ + e⁻

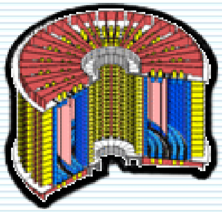
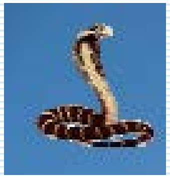
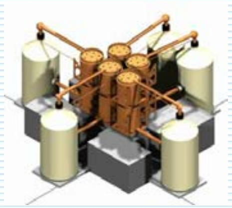
$$\left(T_{1/2}^{0\nu}\right)^{-1} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 g_A^4 \left|M_\nu^{0\nu}\right|^2 G^{0\nu}$$

$$m_{\beta\beta} = \left|c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3\right|$$

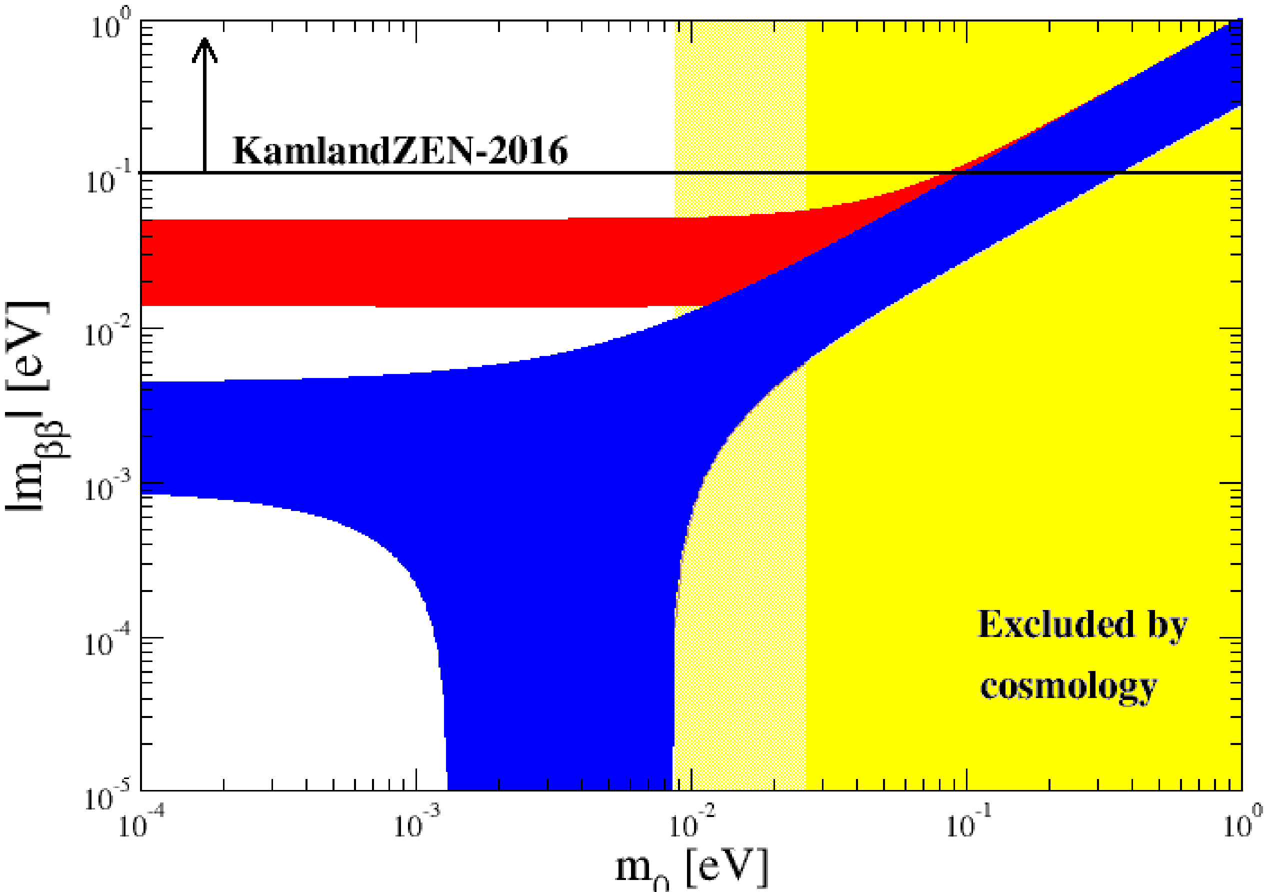


transition	$G^{01}(E_0, Z)$ $\times 10^{14} y$	$Q_{\beta\beta}$ [MeV]	Abund. (%)	$\|M^{0\nu}\|^2$
$^{150}Nd \to {}^{150}Sm$	26.9	3.667	6	?
$^{48}Ca \to {}^{48}Ti$	8.04	4.271	0.2	?
$^{96}Zr \to {}^{96}Mo$	7.37	3.350	3	?
$^{116}Cd \to {}^{116}Sn$	6.24	2.802	7	?
$^{136}Xe \to {}^{136}Ba$	5.92	2.479	9	?
$^{100}Mo \to {}^{100}Ru$	5.74	3.034	10	?
$^{130}Te \to {}^{130}Xe$	5.55	2.533	34	?
$^{82}Se \to {}^{82}Kr$	3.53	2.995	9	?
$^{76}Ge \to {}^{76}Se$	0.79	2.040	8	?

The NMEs for 0νββ-decay must be evaluated using tools of nuclear theory

  GERDA  CUORE 

Effective mass of Majorana neutrinos



GUT's

m_1, m_2, m_3, θ_{12}, θ_{13}, α_1, α_2
(3 unknown parameters)

3/7/2019

Complementarity of 0νββ-decay, β-decay and cosmology

β-decay (Mainz, Troitsk)

$$m_\beta^2 = \sum_i |U_{ei}^L|^2 m_i^2 \le (2.2\ \mathrm{eV})^2$$

KATRIN: $(0.2\ \mathrm{eV})^2$

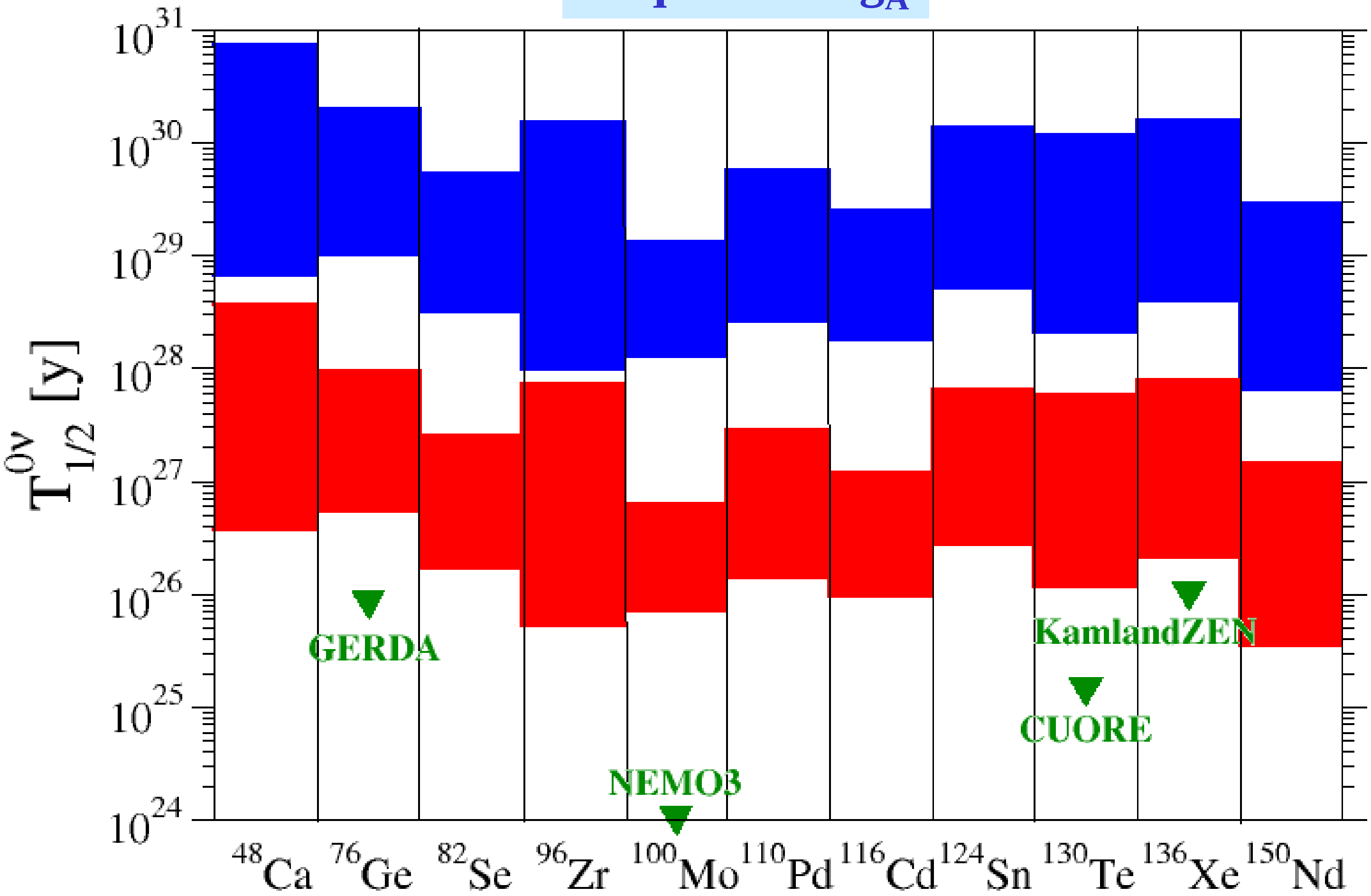
Cosmology (Planck)

$$\Sigma < 110\ \mathrm{meV}$$

$$m_0 > 26\ \mathrm{meV\ (NS)}$$
$$87\ \mathrm{meV\ (IS)}$$

0νββ –half lives for NH and IH with included undertainties in NMEe

unquenched g_A



NH: $m_1 \ll m_2 \ll m_3$ $m_3 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}$. $m_2 \simeq \sqrt{\delta m^2}$

$1.4 \text{ meV} \leq m_{\beta\beta} \leq 3.6 \text{ meV}$

IH: $m_3 \ll m_1 < m_2$ $m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

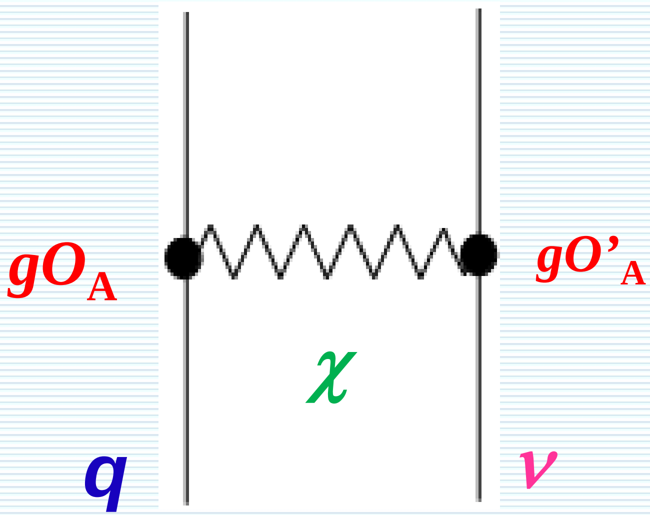
$m_3 \ll \sqrt{\Delta m^2}$

Lightest ν-mass equal to zero

$20 \text{ meV} \leq m_{\beta\beta} \leq 49 \text{ meV}$

Nuclear medium effect on the light neutrino mass exchange mechanism of the $0\nu\beta\beta$-decay

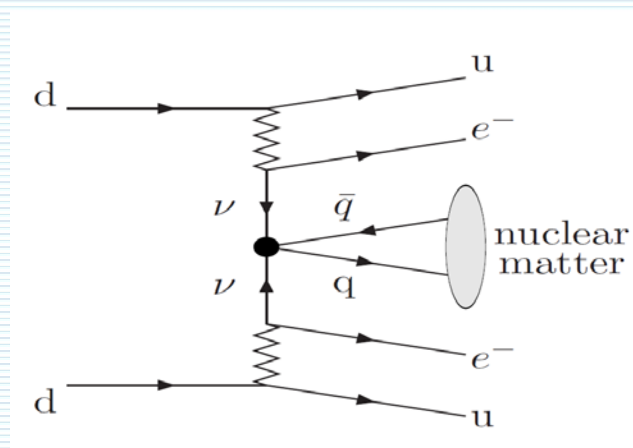
S.G. Kovalenko, M.I. Krivoruchenko, F. Š., Phys. Rev. Lett. 112 (2014) 142503



➡

Low energy 4-fermion
$\Delta L \neq 0$ Lagrangian

$$L_{\text{eff}} = \frac{g^2}{m_\chi^2}\sum_A (\bar{q}O_A q)(\bar{\nu}O'_A \nu), \qquad m_\chi \gtrsim M_W.$$

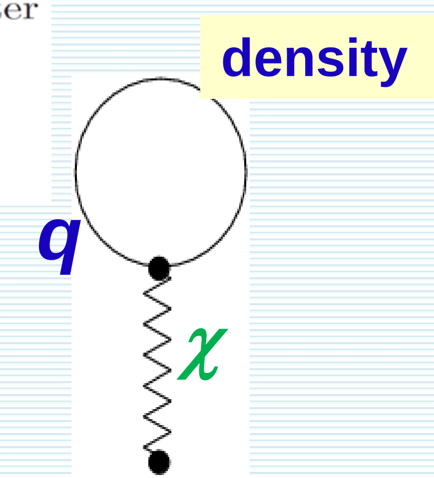


oscillation experiments
tritium β-decay, cosmology

$$\Sigma_\nu^{\text{vac}} = -\!\times\!-,$$

0νββ-decay

$$\Sigma_\nu^{\text{medium}} = -\!\times\!- \; +$$



Mean field: $\bar{q}q \rightarrow \langle \bar{q}q \rangle$ **and** $\langle \bar{q}q \rangle \approx 0.5 \langle q^\dagger q \rangle \approx 0.25\,\mathrm{fm}^{-3}$

The effect depends on $\langle \chi \rangle = -\frac{g_\chi}{m_\chi^2} \langle \bar{q}q \rangle$

A comparison with G_F:

$$\frac{g_\chi g_{ij}^a}{m_\chi^2} = \frac{G_F}{\sqrt{2}} \varepsilon_{ij}^a$$

Typical scale: $\langle \chi \rangle g_{ij}^a = -\frac{G_F}{\sqrt{2}} \langle \bar{q}q \rangle \varepsilon_{ij}^a \approx -25\, \varepsilon_{ij}^a \ \mathrm{eV}$

We expect:

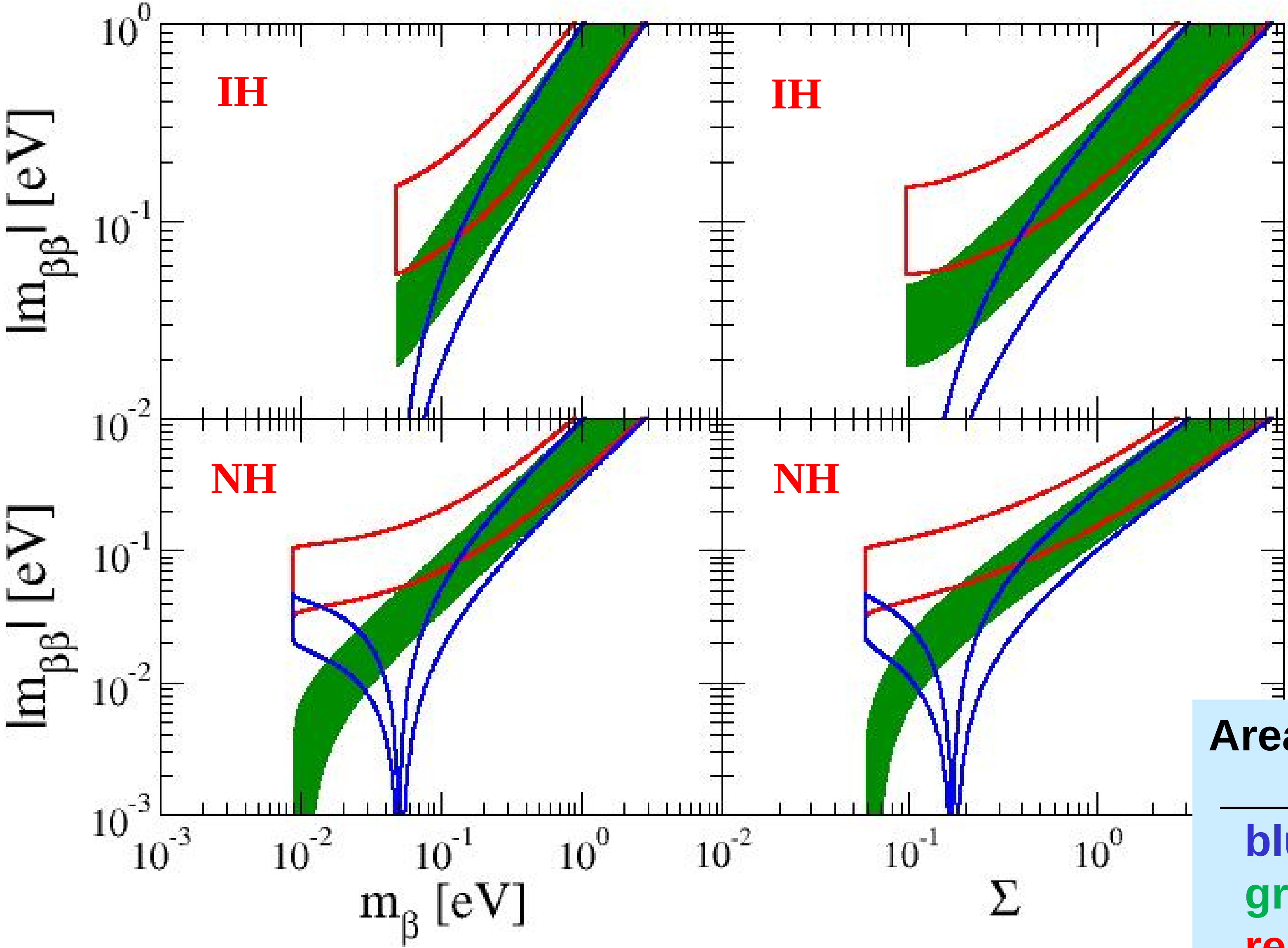
$$25\, \varepsilon_{ij}^a < 1 \rightarrow m_\chi^2 > 25 \frac{g_\chi g_{ij}^a \sqrt{2}}{G_F} \sim 1\,\mathrm{TeV}^2$$

Universal scalar interaction $g_{ij}^a = \delta_{ij} g_a \quad \varepsilon_{ij}^a = \delta_{ij} \varepsilon_a$

In medium effective Majorana ν mass

$$m_{\beta\beta} = \sum_{i=1}^{n} U_{ei}^2 \xi_i \frac{\sqrt{(m_i + \langle \chi \rangle g_1)^2 + (\langle \chi \rangle g_2)^2}}{(1 - \langle \chi \rangle g_4)^2}.$$

Complementarity between β-decay, 0νββ –decay and cosmological measurements might be spoiled



Area	$\langle\chi\rangle g_1$ [eV]
blue	-0.05
green	0
red	1

II.b. *The sterile ν mechanism of the 0νββ-decay*
(D-M mass term, V-A,SM int.)
Interpolating formula

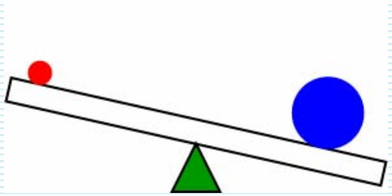
Dirac-Majorana mass term

$$N = \sum_{\alpha=s,e,\mu,\tau} U_{N\alpha}\,\nu_\alpha$$

Mixing of active-sterile neutrinos

small ν masses due to see-saw mechanism

$$\begin{pmatrix} 0 & m_D \\ m_D & m_{LNV} \end{pmatrix}$$



Light ν mass ≈ $(m_D/m_{LNV})\, m_D$
Heavy ν mass ≈ m_{LNV}

Neutrinos masses offer a great opportunity to jump beyond the EW framework via see-saw ...

Different motivations for the LNV scale Λ

Talk of Carlo Giunti

eV **light sterile ν** **10^{-6} GeV**	**keV** **hot DM** **10^{-6} GeV**	**Fermi** **10^{-6} GeV**	**TeV** **LHC** **10^{3} GeV**	**GUT** **10^{16} GeV**	**Planck** **10^{19} GeV**

Left-handed neutrinos: Majorana neutrino mass eigenstate N with arbitrary mass m_N

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\rm A}^4 \left| \sum_{\rm N} \left(U^2_{\rm eN} m_{\rm N}\right) m_{\rm p} \, M'^{\,0\nu}(m_{\rm N}, g_{\rm A}^{\rm eff}) \right|^2$$

General case

light ν exchange

$$M'^{\,0\nu}(m_{\rm N}, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x\, d^3y\, d^3p \times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n\rangle \langle n | J_\mu^\dagger(\mathbf{y}) | 0_I^+\rangle}{\sqrt{p^2 + {m_N}^2}\left(\sqrt{p^2 + {m_N}^2} + E_n - \frac{E_I - E_F}{2}\right)}$$

$$M'^{0\nu}(m_{\rm N} \to 0, g_{\rm A}^{\rm eff}) = \frac{1}{m_{\rm p} m_{\rm e}} M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff})$$

$$M'^{0\nu}(m_{\rm N} \to \infty, g_{\rm A}^{\rm eff}) = \frac{1}{m^2_{\rm N}} M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff})$$

heavy ν exchange

Particular cases

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu} g_{\rm A}^4 \times \left\{ \begin{array}{ll} \left|\frac{\langle m_\nu\rangle}{m_e}\right|^2 \left|M'^{0\nu}_{\nu}(g_{\rm A}^{\rm eff})\right|^2 & \text{for } m_{\rm N} \ll p_{\rm F} \\ \left|\langle\frac{1}{m_{\rm N}}\rangle m_{\rm p}\right|^2 \left|M'^{0\nu}_{\rm N}(g_{\rm A}^{\rm eff})\right|^2 & \text{for } m_{\rm N} \gg p_{\rm F} \end{array} \right.$$

$$\langle m_\nu \rangle = \sum_{\rm N} U^2_{\rm eN} m_{\rm N}$$

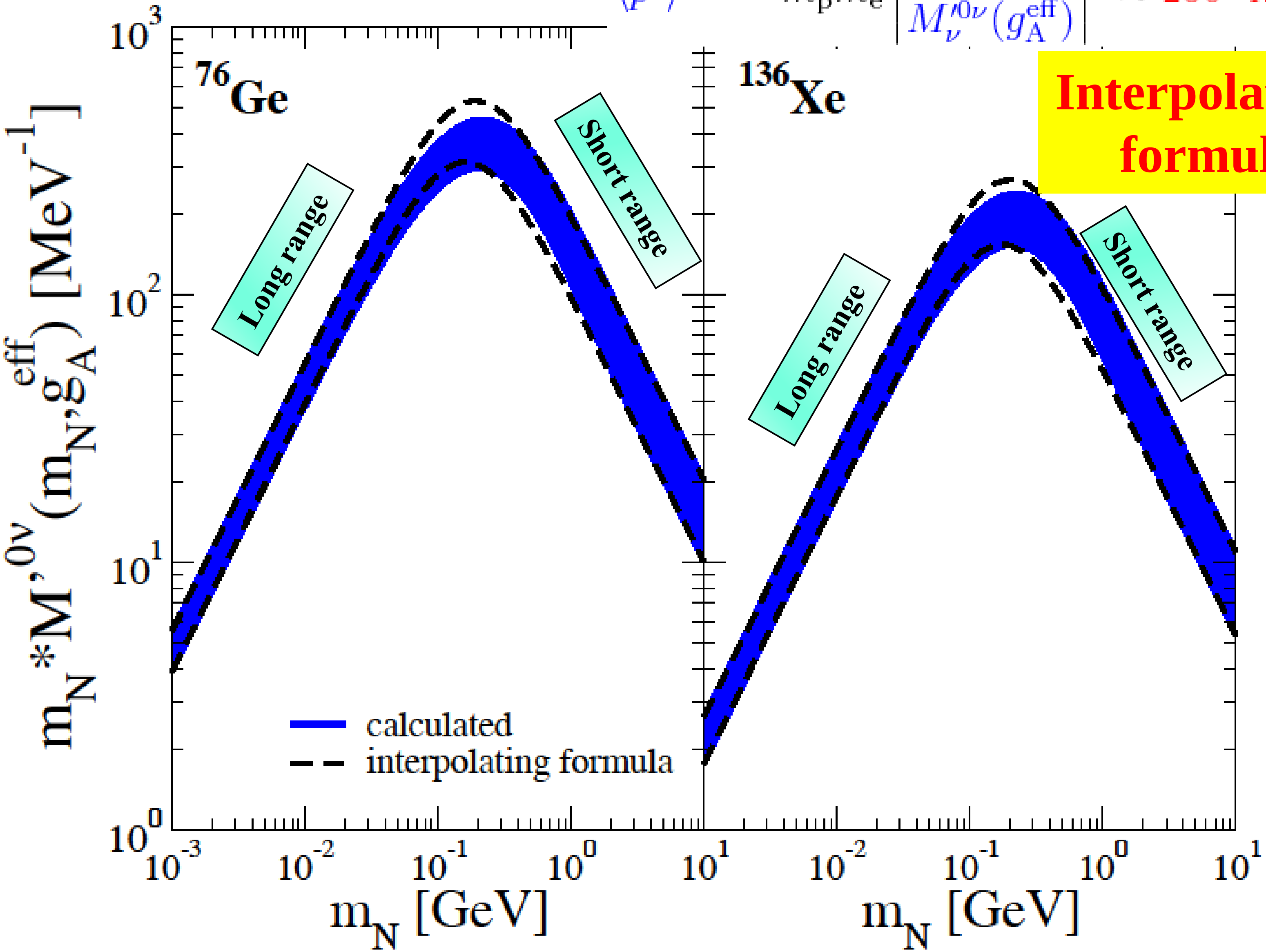
$$\left\langle \frac{1}{m_{\rm N}} \right\rangle = \sum_{\rm N} \frac{U^2_{\rm eN}}{m_{\rm N}}$$

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_{\rm p} \sum_{\rm N} U^2_{e\rm N} \frac{m_{\rm N}}{\langle p^2 \rangle + m^2_{\rm N}} \right|^2,$$

$$\mathcal{A} = G^{0\nu} g^4_{\rm A} \left| M'^{0\nu}_{\rm N}(g^{\rm eff}_{\rm A}) \right|^2,$$

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \left| \frac{M'^{0\nu}_{\rm N}(g^{\rm eff}_{\rm A})}{M'^{0\nu}_{\nu}(g^{\rm eff}_{\rm A})} \right| \approx 200 \ MeV$$

Interpolating formula



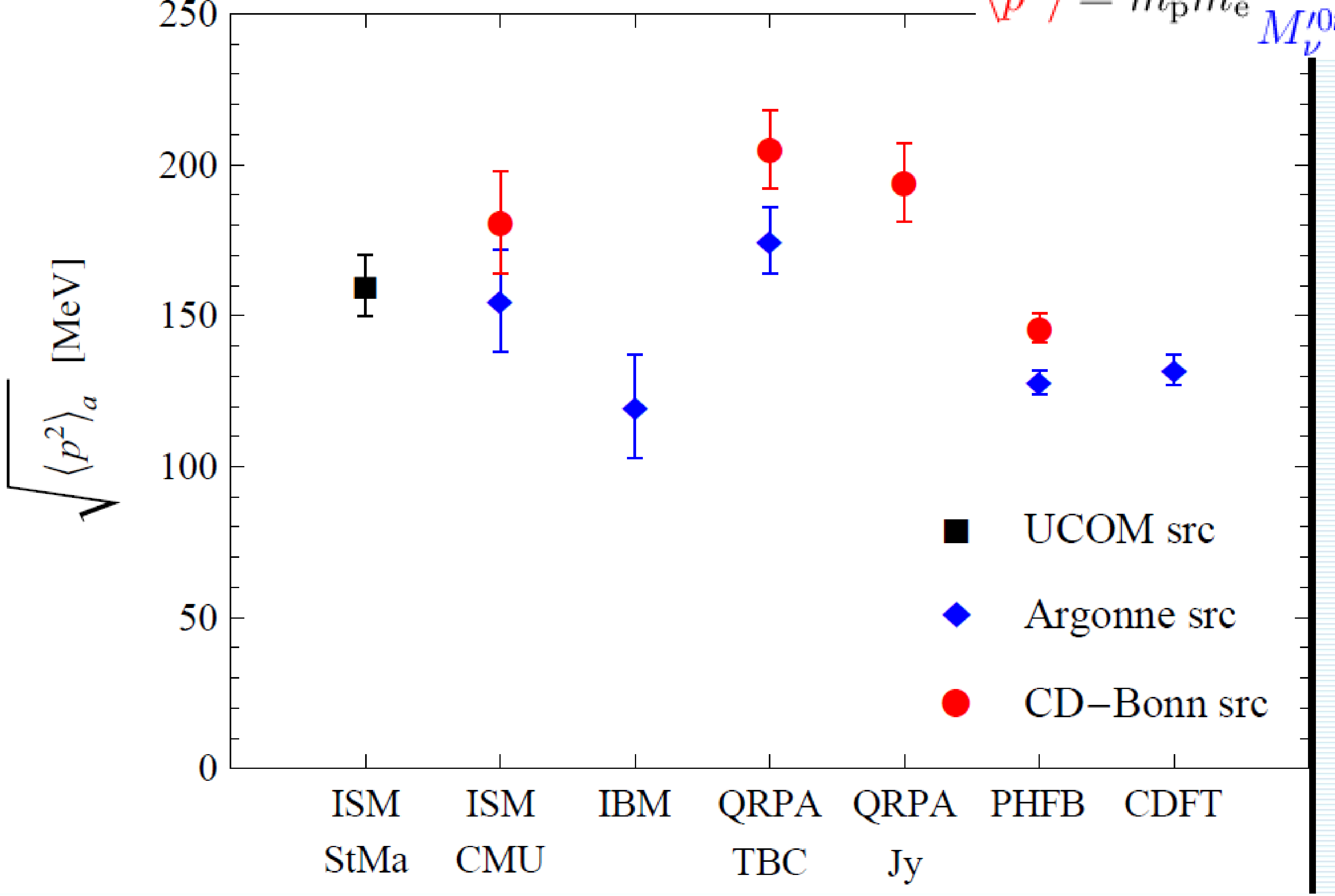
Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

Interpolating formula is justified by practically no dependence <p^2> on A

$$[T^{0\nu}_{1/2}]^{-1} = \mathcal{A} \cdot \left| m_{\rm p} \sum_{\rm N} U^2_{e\rm N} \frac{m_{\rm N}}{\langle p^2 \rangle + m^2_{\rm N}} \right|^2,$$

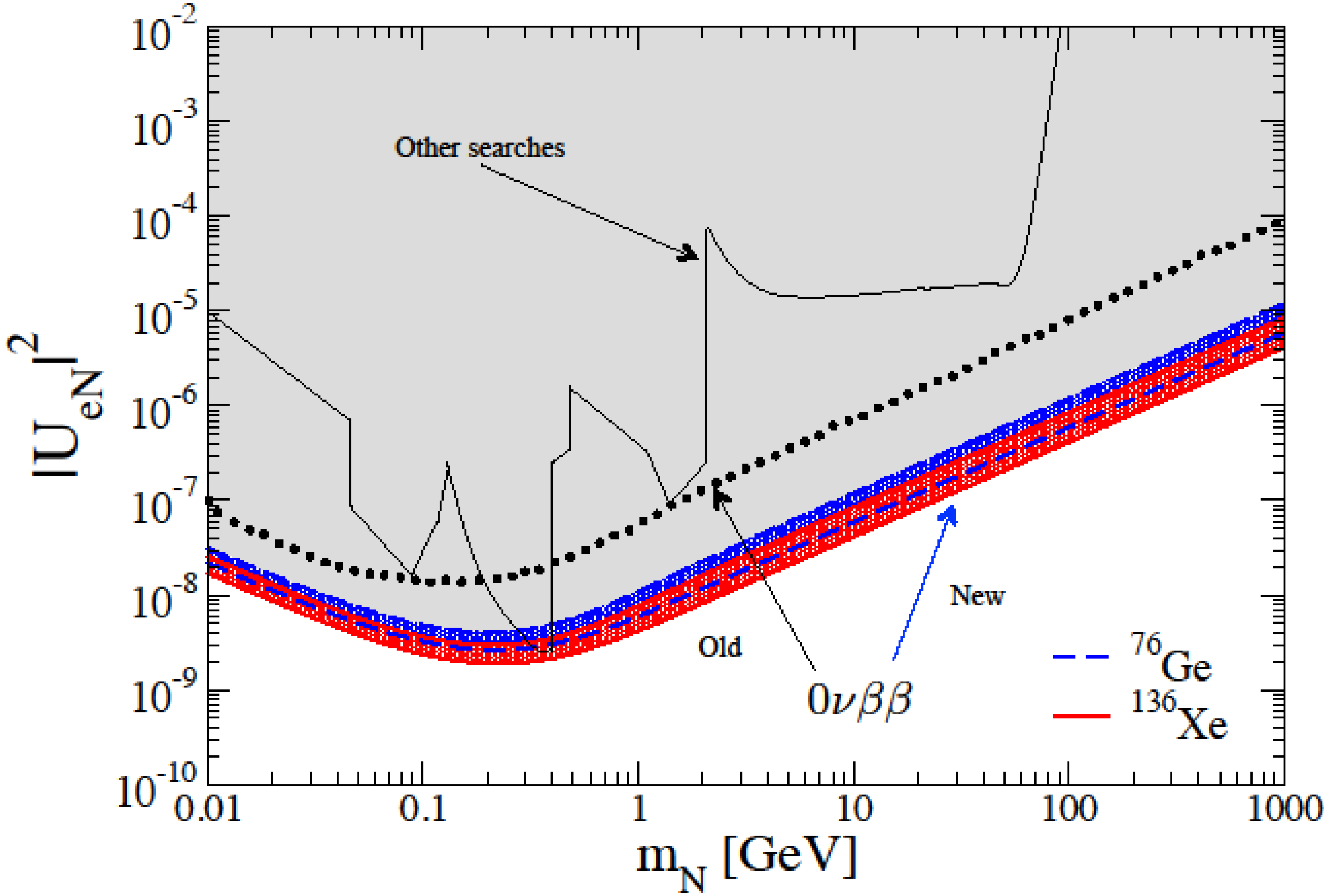
A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$\langle p^2 \rangle = m_{\rm p} m_{\rm e} \frac{M'^{0\nu}_{\rm N}}{M'^{0\nu}_{\nu}}$$



Exclusion plot in $|U_{eN}|^2 - m_N$ plane

$T^{0\nu}_{1/2}(^{76}Ge) \geq 3.0\ 10^{25}$ yr
$T^{0\nu}_{1/2}(^{136}Xe) \geq 3.4\ 10^{25}$ yr



Improvements: i) QRPA (constrained Hamiltonian by 2νββ half-life, self-consistent treatment of src, restoration of isospin symmetry …), ii) More stringent limits on the 0νββ half-life

II.c. The $0\nu\beta\beta$-decay within L-R symmetric theories (interpolating formula)

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$[T^{0\nu}_{1/2}]^{-1} = \eta^2_{\nu N}\ C_{\nu N}$$

$$C_{\nu N} = g^4_{\mathrm{A}}\ \left|M'^{0\nu}_{\nu}\right|^2\ G^{0\nu}$$

Mixing of light and heavy neutrinos

$$\nu_{eL} = \sum_{j=1}^{3}\left(U_{ej}\nu_{jL} + S_{ej}(N_{jR})^C\right),$$

$$\nu_{eR} = \sum_{j=1}^{3}\left(T^*_{ej}(\nu_{jL})^C + V^*_{ej}N_{jR}\right)$$

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

Effective LNV parameter within LRS model

$$\eta^2_{\nu N} = \left|\sum_{j=1}^{3}\left(U^2_{ej}\ \frac{m_j}{m_e}\ +\ S^2_{ej}\ \frac{\langle p^2\rangle_a}{\langle p^2\rangle_a + M^2_j}\ \frac{M_j}{m_e}\right)\right|^2 + \lambda^2\left|\sum_{j=1}^{3}\left(T^2_{ej}\ \frac{m_j}{m_e}\ +\ V^2_{ej}\ \frac{\langle p^2\rangle_a}{\langle p^2\rangle_a + M^2_j}\ \frac{M_j}{m_e}\right)\right|^2$$

$$\langle p^2\rangle = m_{\mathrm{p}} m_{\mathrm{e}} \frac{M'^{0\nu}_{\mathrm{N}}}{M'^{0\nu}_{\nu}}$$

6x6 PMNS see-saw ν-mixing matrix
(the most economical one)

6x6 neutrino mass matrix

Basis

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$(\nu_L, (N_R)^c)^T$$

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

6x6 matrix: **15 angles, 10+5 CP phases**
3x3 matrix: **3 angles, 1+2 CP phases**

3x3 block matrices U, S, T, V are generalization of PMNS matrix

Assumptions:
i) the see-saw structure
ii) mixing between different generations is neglected

$$\mathcal{U}_{\rm PMNS} = \begin{pmatrix} U_{\rm PMNS} & \zeta\, \mathbf{1} \\ -\zeta\, \mathbf{1} & U^\dagger_{\rm PMNS} \end{pmatrix}$$

$$\mathcal{U}_{\rm PMNS}\, \mathcal{U}^\dagger_{\rm PMNS} = \mathcal{U}^\dagger_{\rm PMNS}\, \mathcal{U}_{\rm PMNS} = 1$$

see-saw parameter

$$\zeta = \frac{m_{\rm D}}{m_{\rm LNV}}$$

6x6 matrix: **3 angles, 1+2 CP phases, 1 see-saw par.**

6x6 PMNS see-saw ν-mixing matrix
(the most economical one)

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta\, 1 \\ -\zeta\, 1 & V_0 \end{pmatrix}$$

$$U_0 = U_{\mathrm{PMNS}}$$

A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

$$V_0 = U^\dagger_{\mathrm{PMNS}} =$$

$$\begin{pmatrix} c_{12}\, c_{13}\, e^{-i\alpha_1} & \left(-s_{12}\, c_{23} - c_{12}\, s_{13}\, s_{23}\, e^{-i\delta}\right) e^{-i\alpha_1} & \left(s_{12}\, s_{23} - c_{12}\, s_{13}\, c_{23}\, e^{-i\delta}\right) e^{-i\alpha_1} \\ s_{12}\, c_{13}\, e^{-i\alpha_2} & \left(c_{12}\, c_{23} - s_{12}\, s_{13}\, s_{23}\, e^{-i\delta}\right) e^{-i\alpha_2} & \left(-c_{12}\, s_{23} - s_{12}\, s_{13}\, c_{23}\, e^{-i\delta}\right) e^{-i\alpha_2} \\ s_{13}\, e^{i\delta} & c_{13}\, s_{23} & c_{13}\, c_{23} \end{pmatrix}$$

Assumption about heavy neutrino masses M_i (by assuming see-saw)

Inverse proportional

$$m_i M_i \simeq m_D^2$$

$$M^{\mathrm{R}}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2} \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, m_j\right|$$

Proportional

$$m_i \simeq \zeta^2 M_i$$

$$M^{\mathrm{R}}_{\beta\beta} = \lambda\, \zeta^2 \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, \frac{\langle p^2\rangle_a}{m_j}\right|$$

$M^{R}_{\beta\beta}$ depends on "Dirac" CP phase δ unlike "Majorana" CP phases α_1 and α_2

Heavy Majorana mass $M^{R}_{\beta\beta}$ depends on the "Dirac" CP violating phase δ

Contribution from exchange of heavy neutrino to 0νββ-decay rate might be large

Inverse proportional

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^{\rm R} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \, m_j \right|$$

$$V_0 = U_{PMNS}^\dagger$$

$$M_i = m_D^2/m_i \quad m_D \simeq 5 \text{ MeV}$$

$$\lambda = 7.7 \times 10^{-4}$$

Proportional

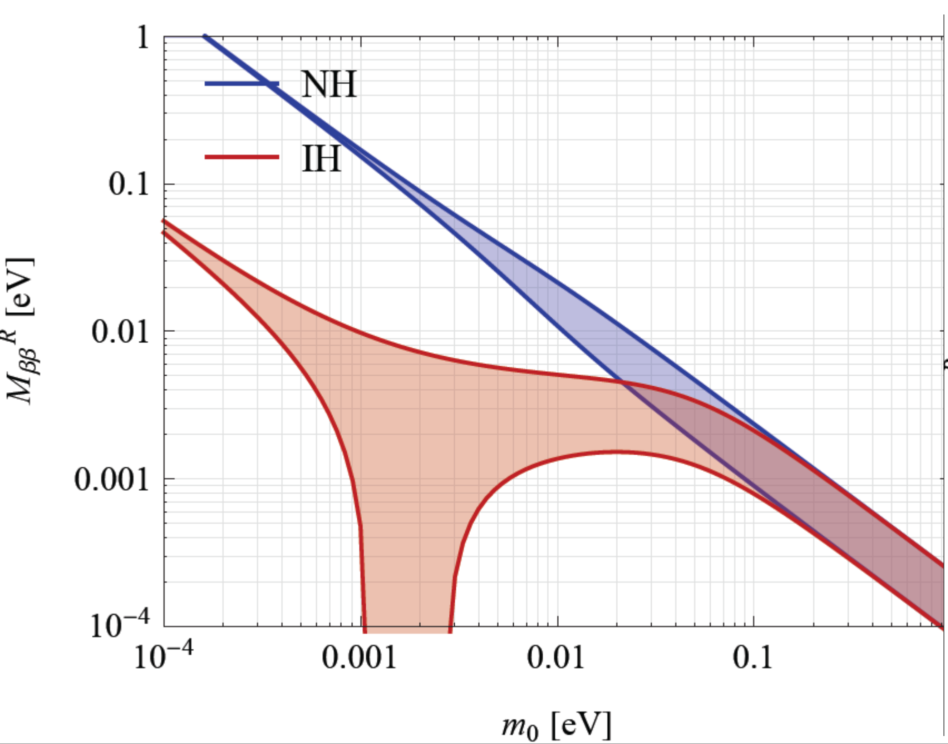
$$m_i \simeq \zeta^2 M_i$$

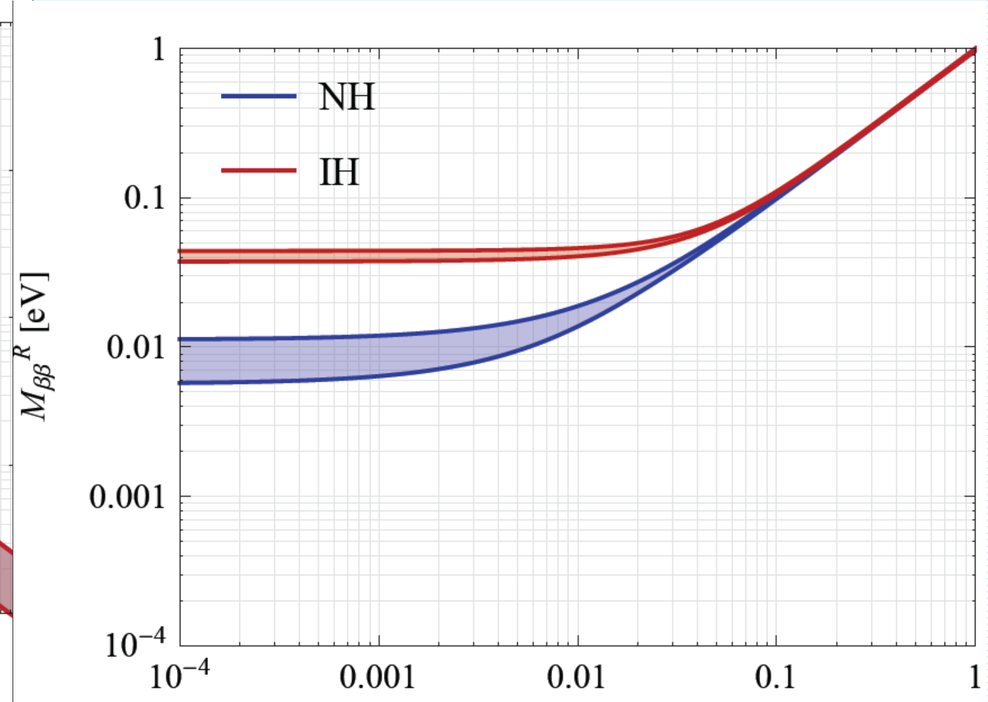
$$M_{\beta\beta}^{\rm R} = \lambda\, \zeta^2 \left| \sum_{j=1}^{3} (U_0^\dagger)_{ej}^2 \frac{\langle p^2 \rangle_a}{m_j} \right|$$

$$V_0 = U_{PMNS}^\dagger$$

$$\zeta = m_i/M_i \quad \zeta^2 \simeq 5 \times 10^{-17}$$

$$\lambda = 7.7 \times 10^{-4}$$





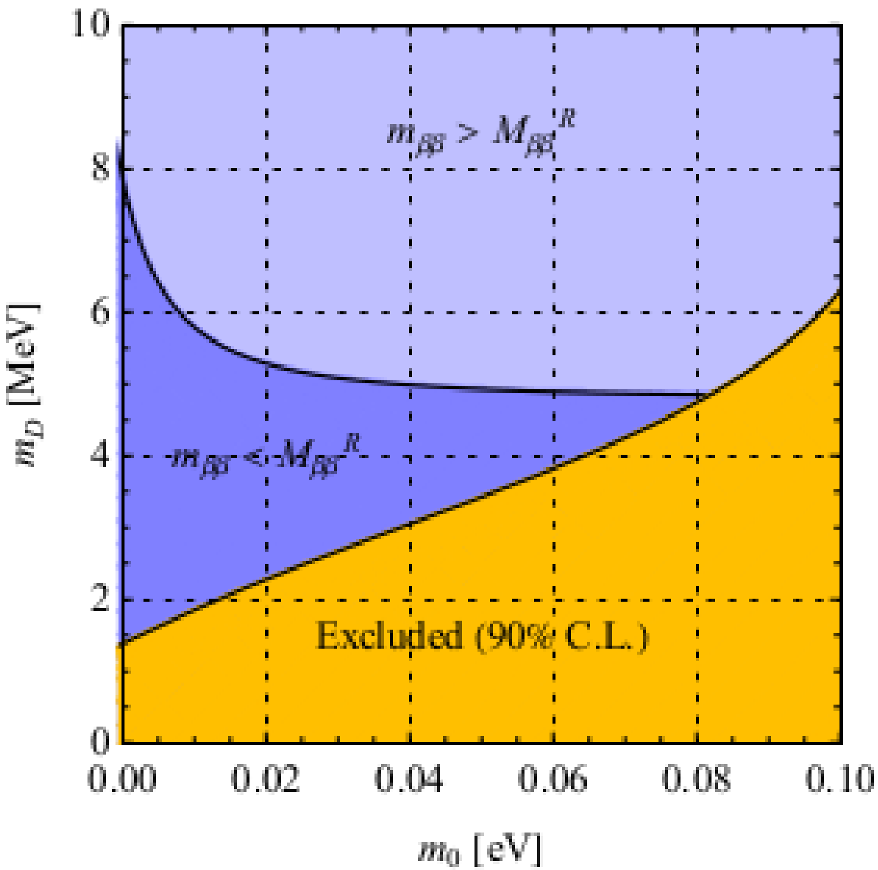
$$\eta^2_{\nu N} = \frac{1}{m_e^2}\left(m^2_{\beta\beta} + (M^R_{\beta\beta})^2\right)$$

See-saw scenario

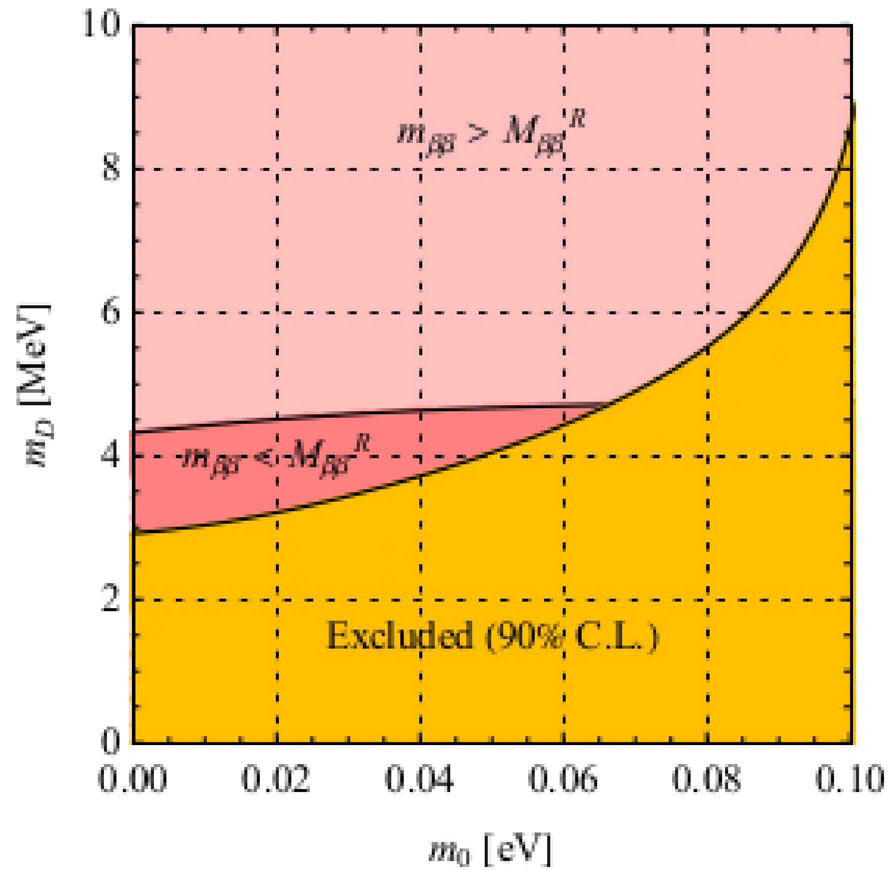
$$m_i M_i \simeq m_D^2$$

$$M^{\mathrm{R}}_{\beta\beta} = \lambda \frac{\langle p^2\rangle_a}{m_D^2} \left|\sum_{j=1}^{3} (U_0^\dagger)^2_{ej}\, m_j\right|$$

Normal spectrum



Inverted spectrum

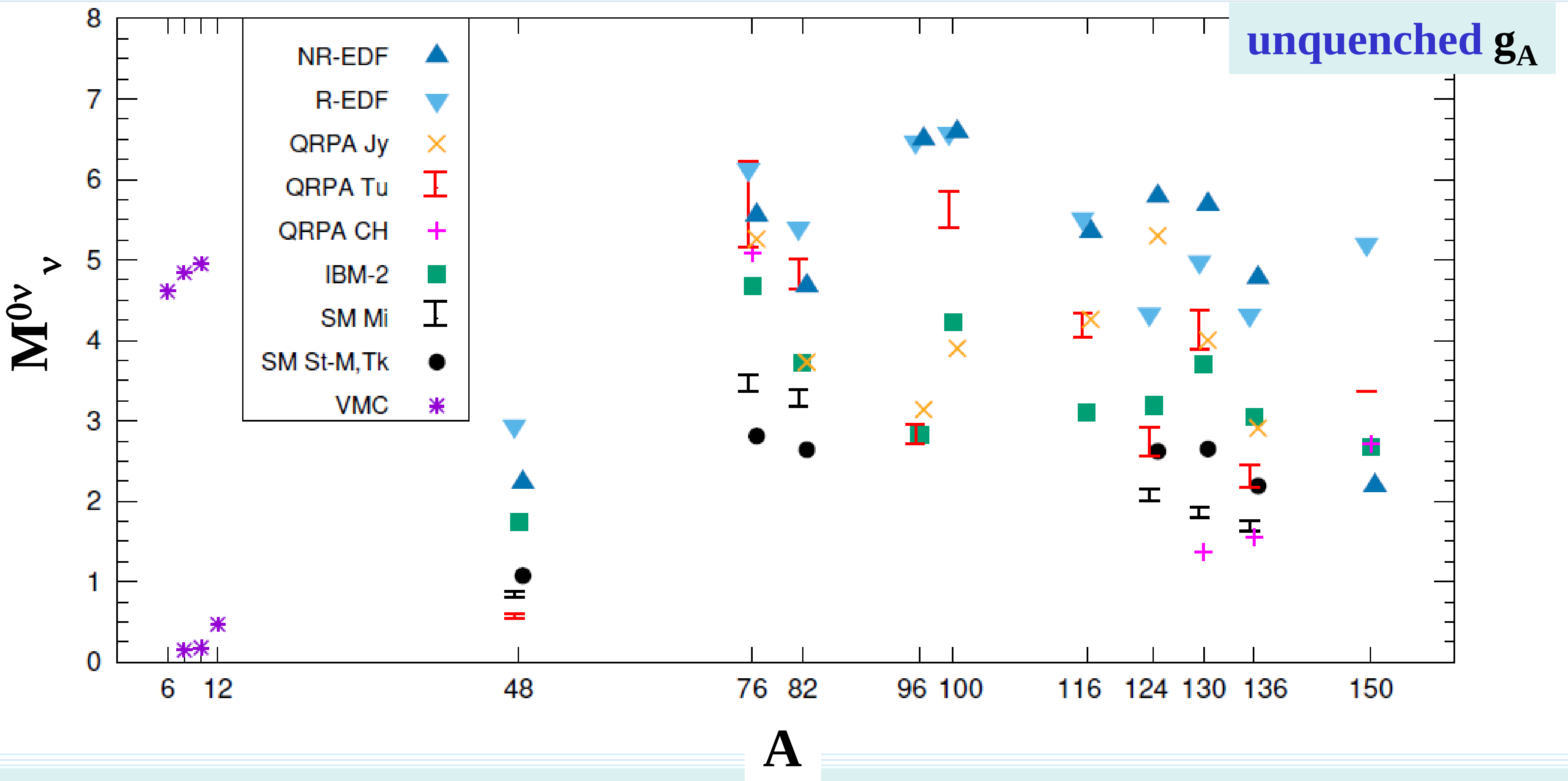


A. Babič, S. Kovalenko, M.I. Krivoruchenko , F.Š., PRD 98, 015003 (2018)

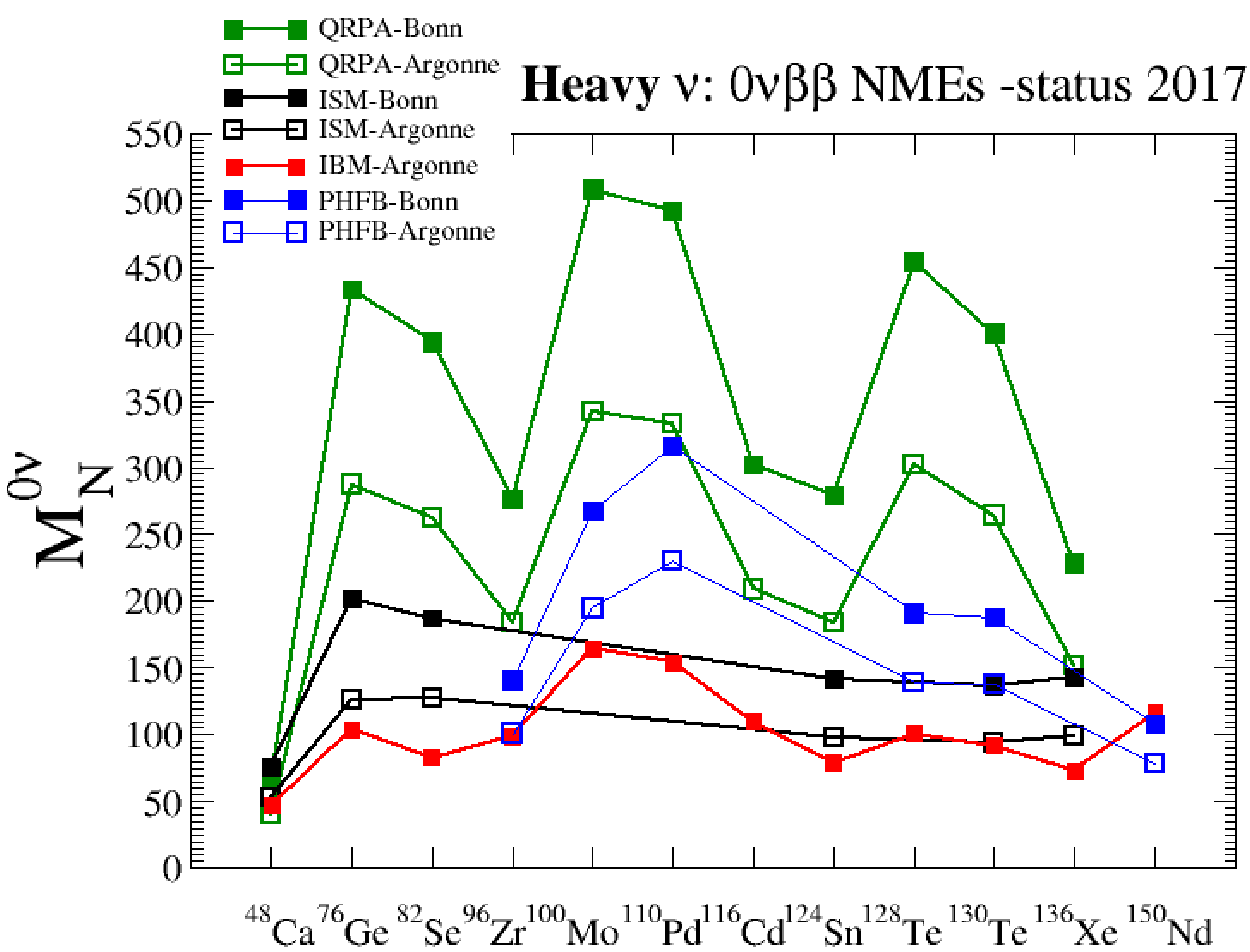
III. $0\nu\beta\beta$ decay NMEs

0νββ-decay NME (light ν mass) – status 2017

J. Engel, J. Menendez, Rept. Prog. Phys. 80, 046301 (2017)

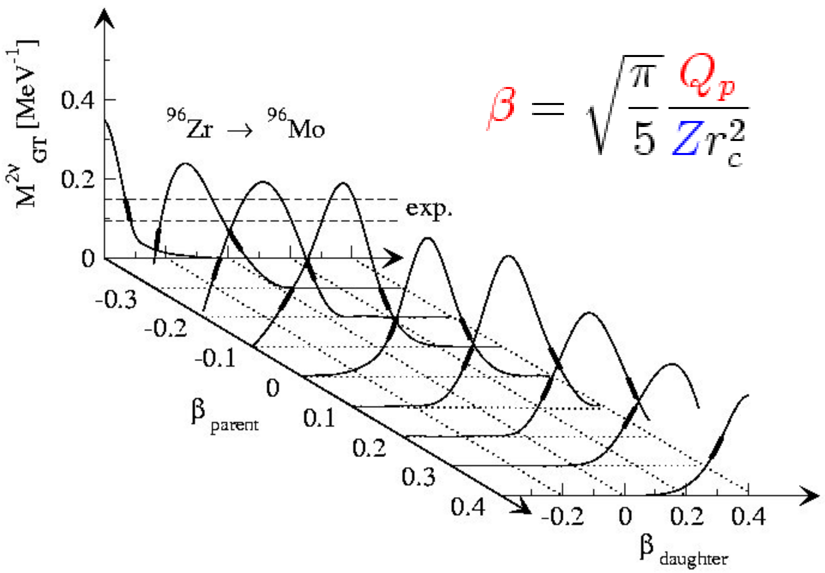


	mean field meth.	ISM	IBM	QRPA
Large model space	yes	no	no	yes
Constr. Interm. States	no	yes	no	yes
Nucl. Correlations	limited	all	restricted	restricted



Heavy ν: 0νββ NMEs -status 2017
QRPA-Bonn
QRPA-Argonne
ISM-Bonn
ISM-Argonne
IBM-Argonne
PHFB-Bonn
PHFB-Argonne
$M_N^{0\nu}$
0
50
100
150
200
250
300
350
400
450
500
550
^{48}Ca ^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{110}Pd ^{116}Cd ^{124}Sn ^{128}Te ^{130}Te ^{136}Xe ^{150}Nd

Suppression of the 0νββ-decay NMEs due to different deformation of initial and final nuclei

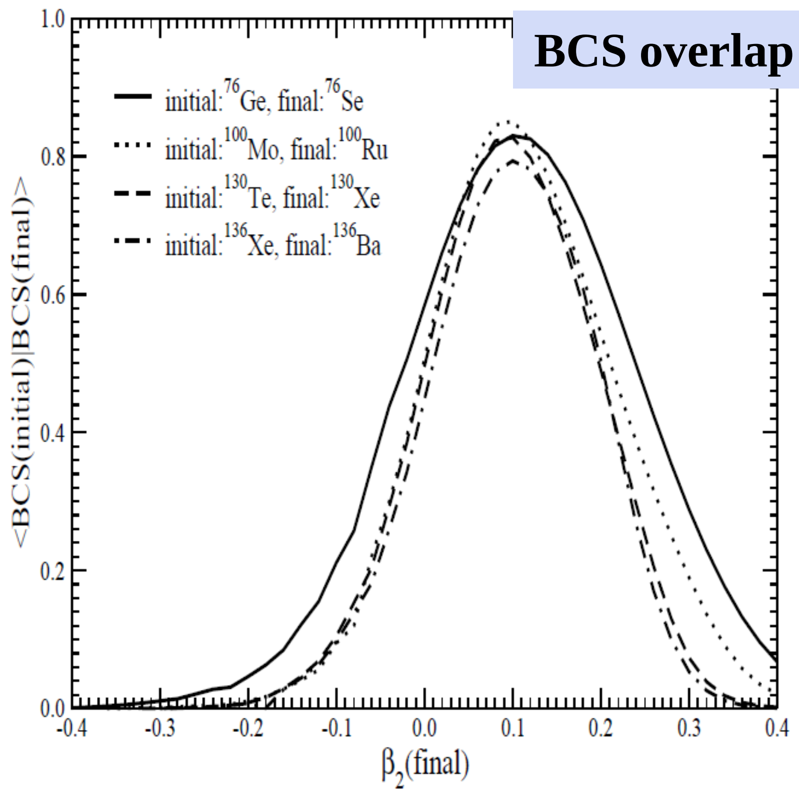


$$\beta = \sqrt{\frac{\pi}{5}}\frac{Q_p}{Zr_c^2}$$

The suppression of the NME depends on the relative deformation of initial and final nuclei

F.Š., Pacearescu, Faessler, NPA 733 (2004) 321

BCS overlap

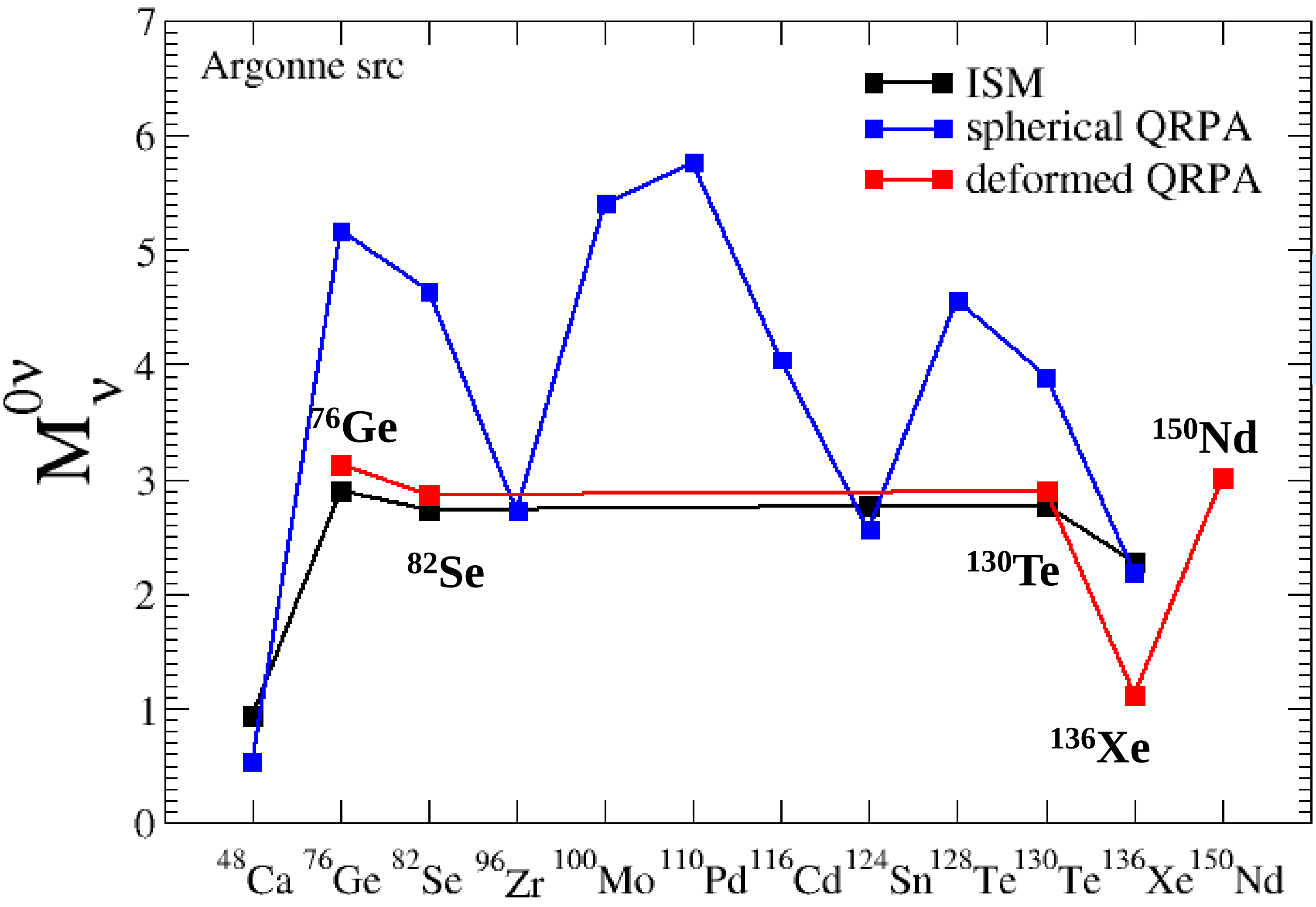


Systematic study of the deformation effect on the 2νββ-decay NME within deformed QRPA

Alvarez, Sarriguren, Moya, Pacearescu, Faessler, F.Š., Phys. Rev. C 70 (2004) 321

0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (light neutrino exchange)

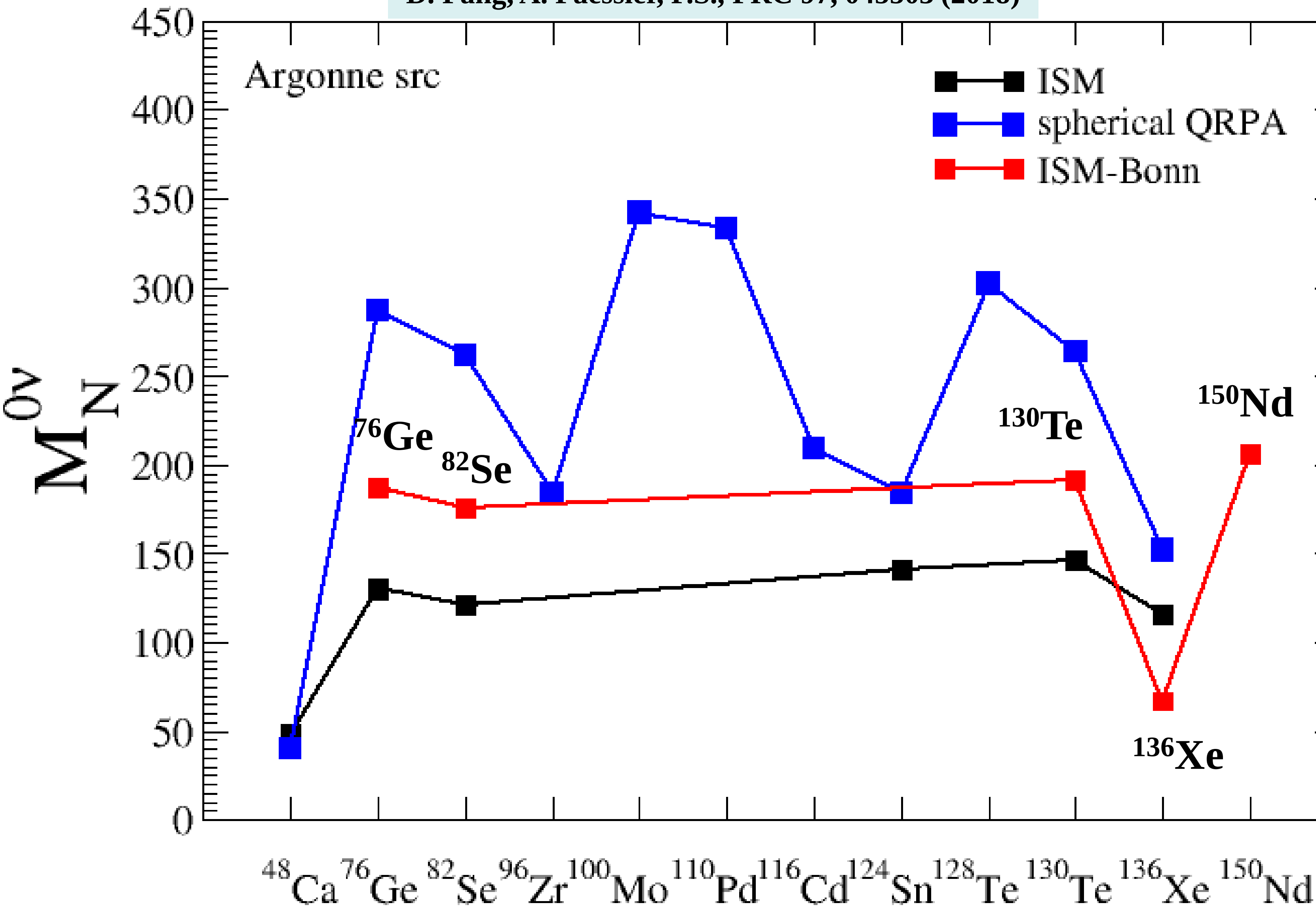
D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)



Agreement by a chance?

0νββ-decay NMEs within deformed QRPA with partial restoration of isospin symmetry (heavy neutrino exchange, Argonne src)

D. Fang, A. Faessler, F.Š., PRC 97, 045503 (2018)



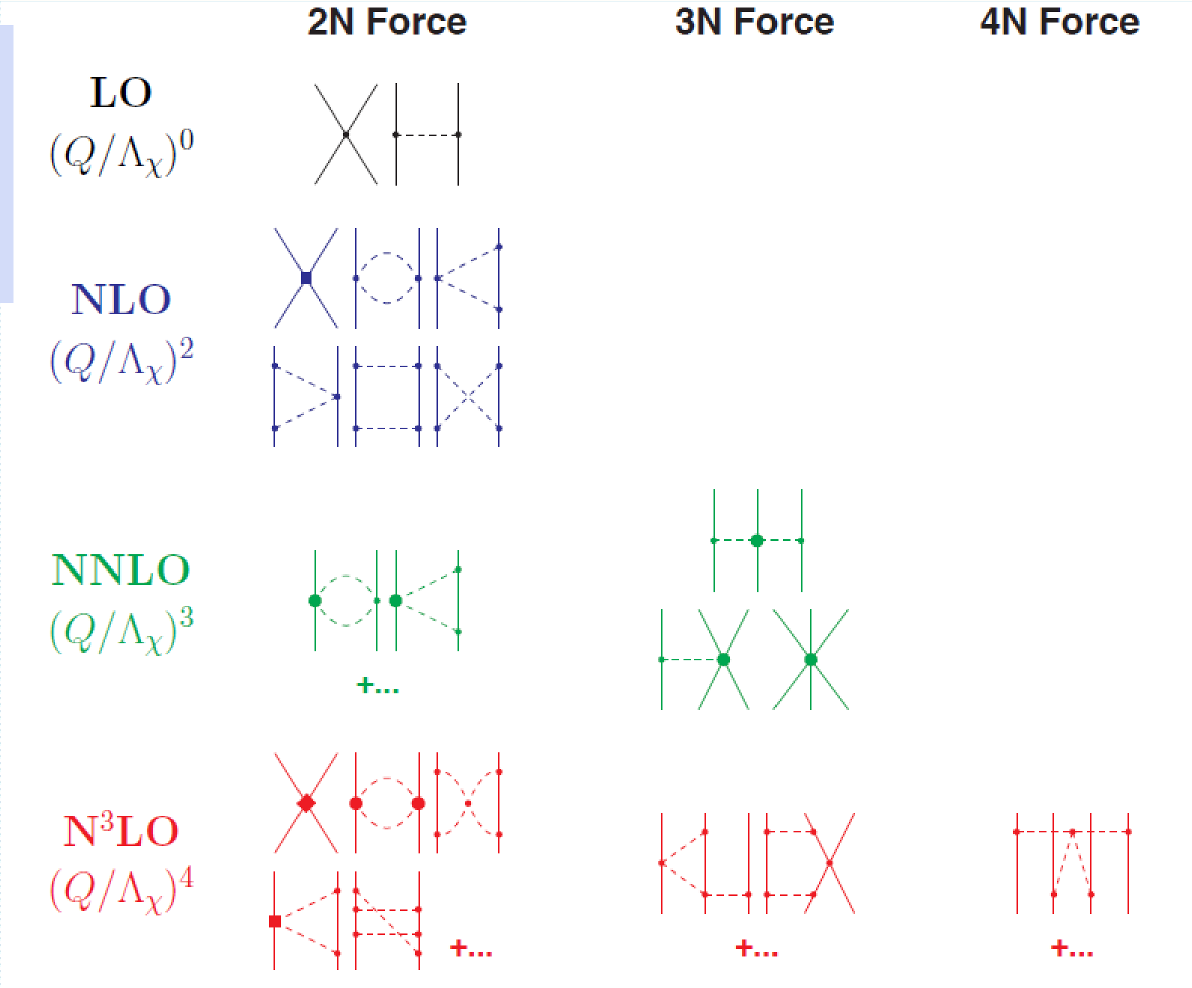
Ab Initio Nuclear Structure
(Often starts with chiral effective-field theory)

Nucleons, pions sufficient below chiral symmetry breaking scale.
Expansion of operators in power of Q/Λ_χ. $Q=m_\pi$ or typical nucleon momentum.

A. Schwenk (Darmstadt U.)
P. Navratil (TRIUMPH)
J. Engel (North Caroline U.)
J. Menendez (Tokyo U.)

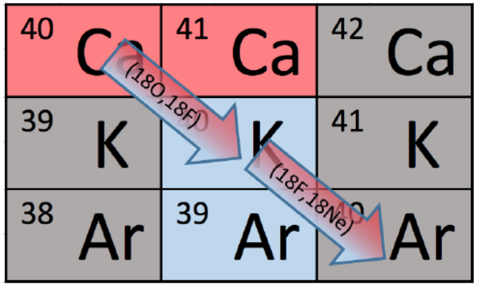
Calculation for the hypothetical $0\nu\beta\beta$ decay of ^{10}He:
$^{10}He \to {}^{10}Be + e^- + e$-

masses, spectra

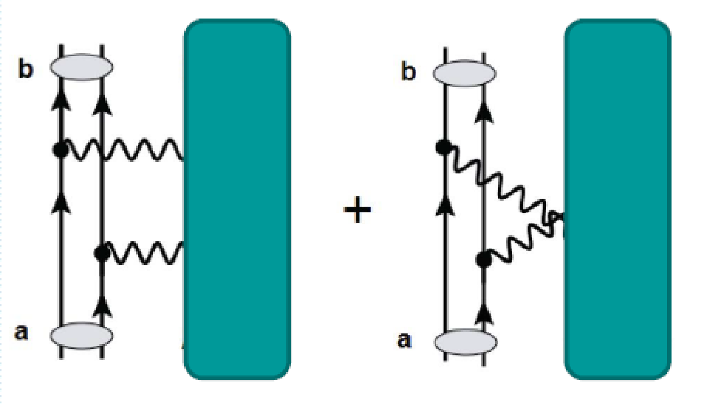


Supporting nuclear physics experiments

(2νββ-decay ChER, pion and heavy ion DCX, nucleon transfer reactions etc)

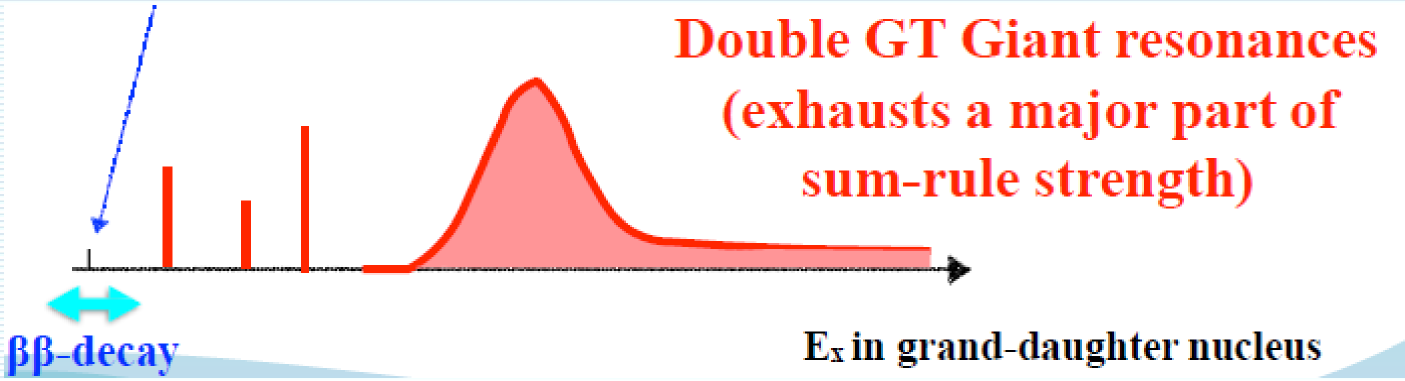


$$^{18}O + {}^{40}Ca \rightarrow {}^{18}F + {}^{40}K \rightarrow {}^{18}Ne + {}^{40}Ar$$



H. Lenske group
Theory of heavy ion DCX and Connection to DBD NMEs

Heavy ion DCX:
NUMEN (LNC-INFN),
HIDCX (RCNP/RIKEN)



IV. Is there a proportionality between 0νββ- and 2νββ-decay NMEs?

Understanding of the 2νββ-decay NMEs is of crucial importance for correct evaluation of the 0νββ-decay NMEs

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\overline{\nu}_e$$

Both 2νββ and 0νββ operators connect the same states. Both change two neutrons into two protons.

Explaining 2νββ-decay is necessary but not sufficient

There is no reliable calculation of the *2νββ-decay NMEs*

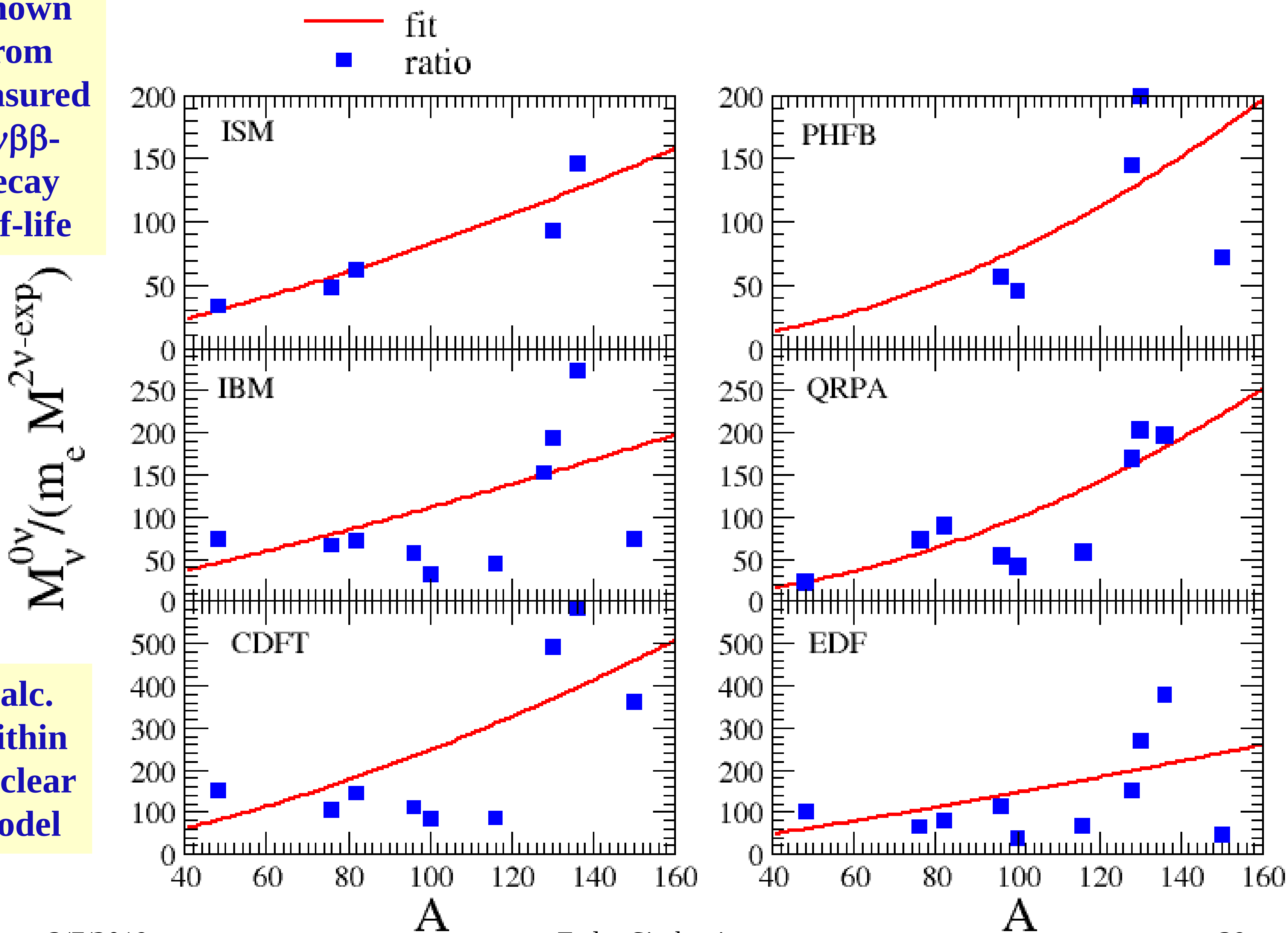
Calculation via intermediate nuclear states: QRPA (sensitivity to pp-int.) ISM (quenching, truncation of model space, spin-orbit partners)

Calculation via closure NME: IBM, PHFB

No calculation: EDF

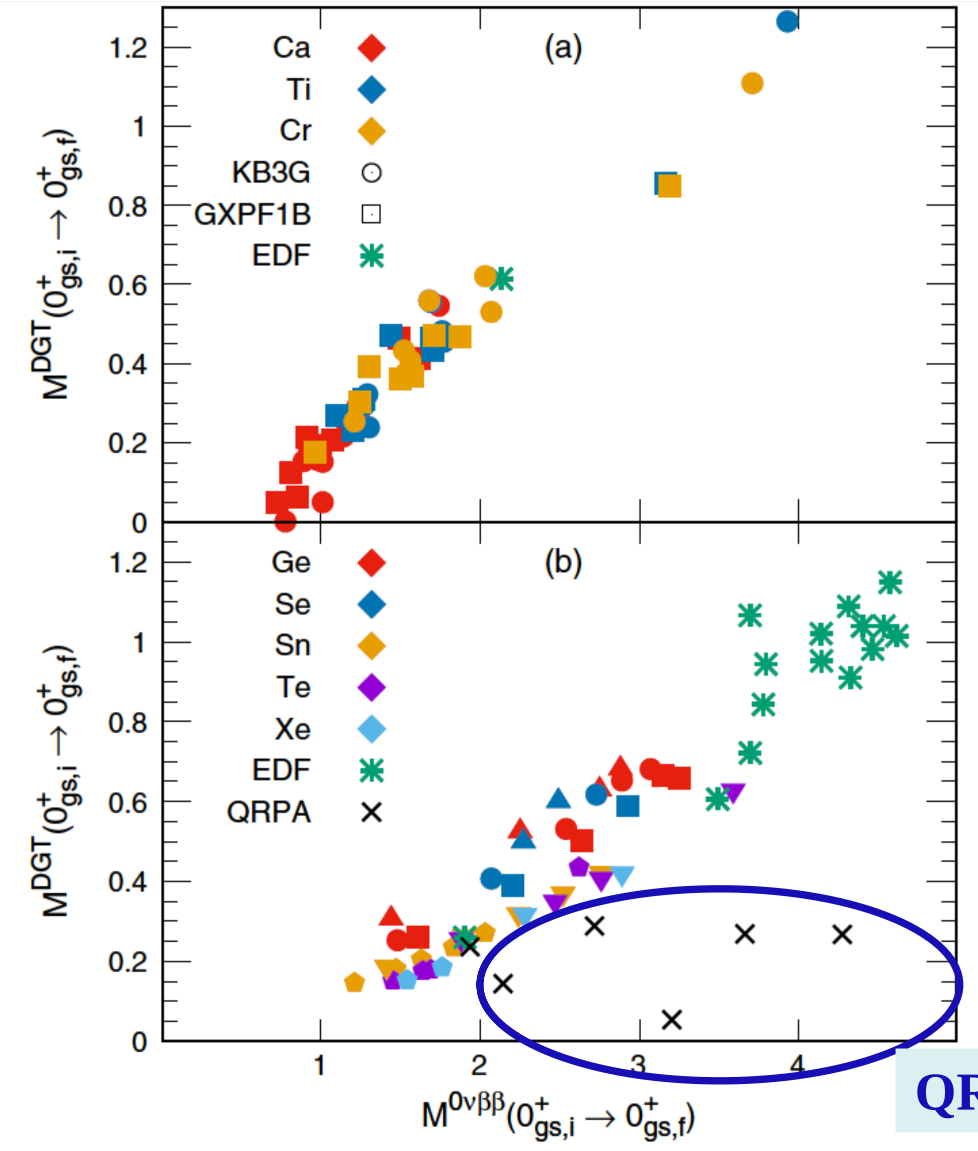
Is there a proportionality between 0νββ- and 2νββ-decay NMEs?

Known krom measured 2νββ-decay half-life



Calc. within nuclear model

$M^{0\nu} \propto M^{2\nu}_{GT\text{-}cl}$: ISM, EDF



QRPA?

$M^{DGT}=M^{2\nu}{}_{GT}$

	SSD	ChER
^{48}Ca		0.22
^{76}Ge		0.52
^{96}Zr		0.22
^{100}Mo	0.35	
^{116}Cd	0.35	0.30
^{128}Te	0.41	

EDF: $0.6 \rightarrow 1.2$
ISM: $0.1 \rightarrow 0.7$
IBM: $1.6 \rightarrow 4.4$
QRPA: $|0.1| \rightarrow |0.7|$

IBM: J. Barea, J. Kotila, F. Iachello, PRC 91, 034304 (2015)

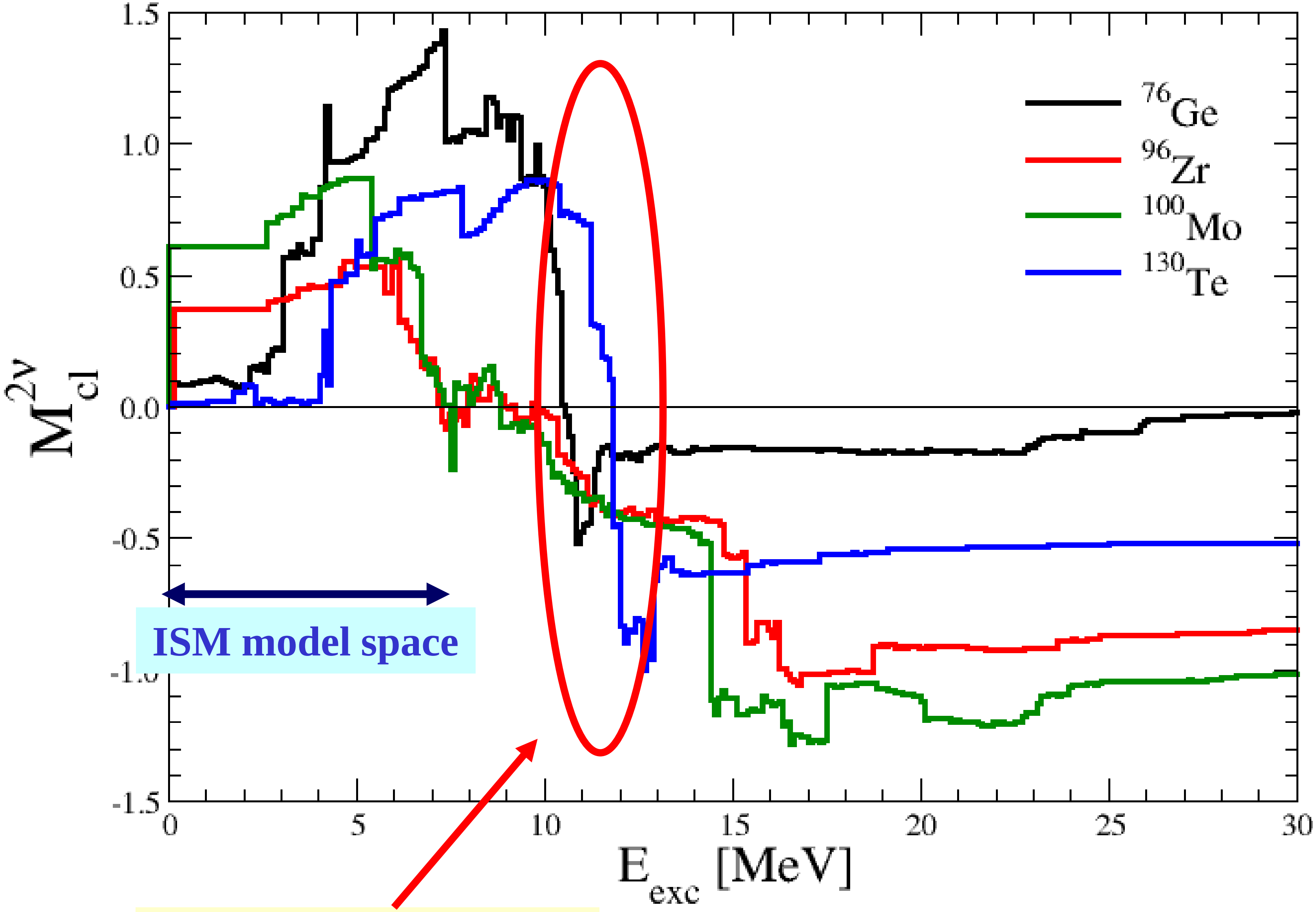
QRPA: F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

ISM: N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

M^{DGT} – only 1^+
$M^{0\nu}$ - contribution from many J^π (!)

QRPA: There is no proportionality between 0νββ-decay and 2νββ-decay NMEs

F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)



Region of GT resonance

Going to relative coordinates:

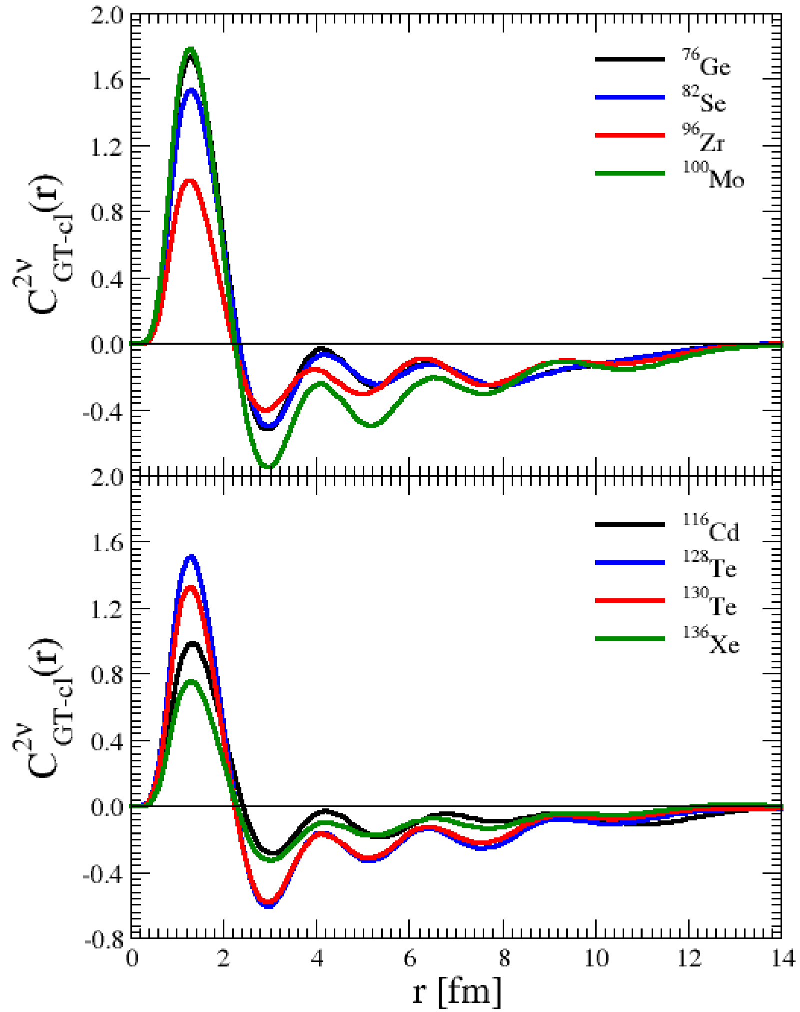
A connection between closure $2\nu\beta\beta$ and $0\nu\beta\beta$ GT NMEs

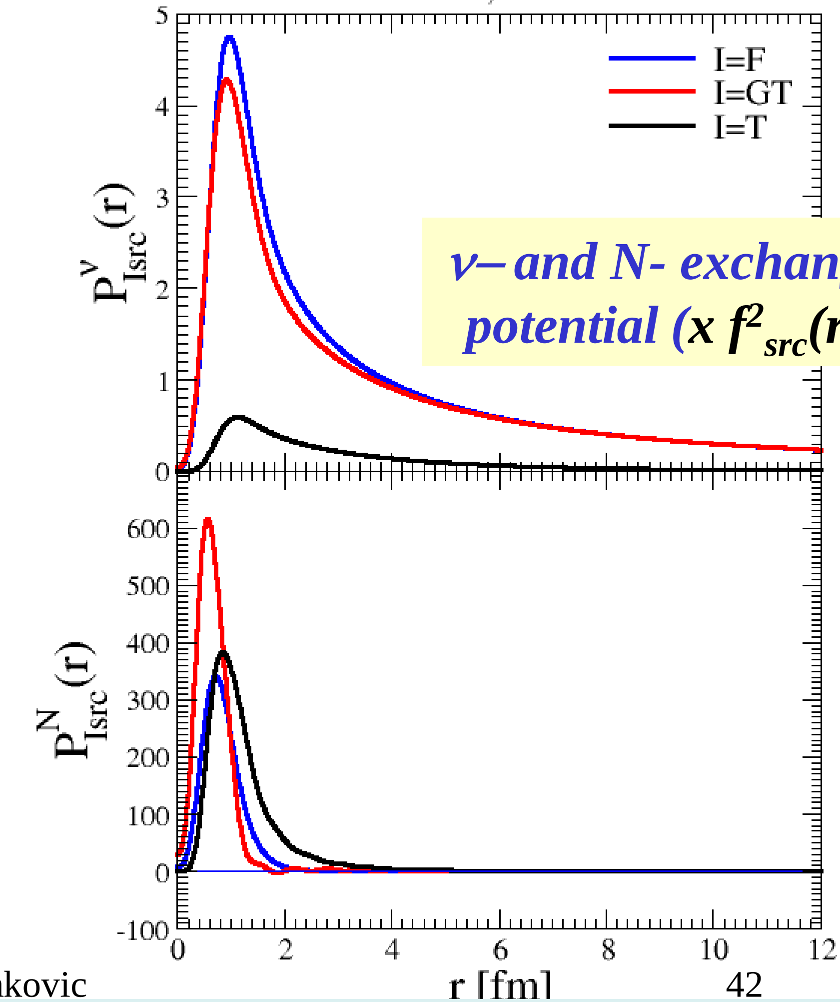
F.Š., R. Hodák, A. Faessler, P. Vogel, PRC 83, 015502 (2011)

$$M^{2\nu}_{GT-cl} = \int_0^\infty C^{2\nu}_{GT-cl}(r) dr$$

$$M^{0\nu}_{\nu,N-I} = \int_0^\infty P^{\nu,N}_{I-src}(r)\ C^{2\nu}_{I-cl}(r)\ dr = \int_0^\infty f^2_{src}(r)\ P^{\nu,N}_{I}(r)\ C^{2\nu}_{I-cl}(r)\ dr$$

$$I = F,\ GT \text{ and } T$$





ν– and N- exchange potential (x $f^2_{src}(r)$)

r- relative distance of two decaying nucleons

Neutrino potential prefers short distances

Closure $2\nu\beta\beta$ GT NME

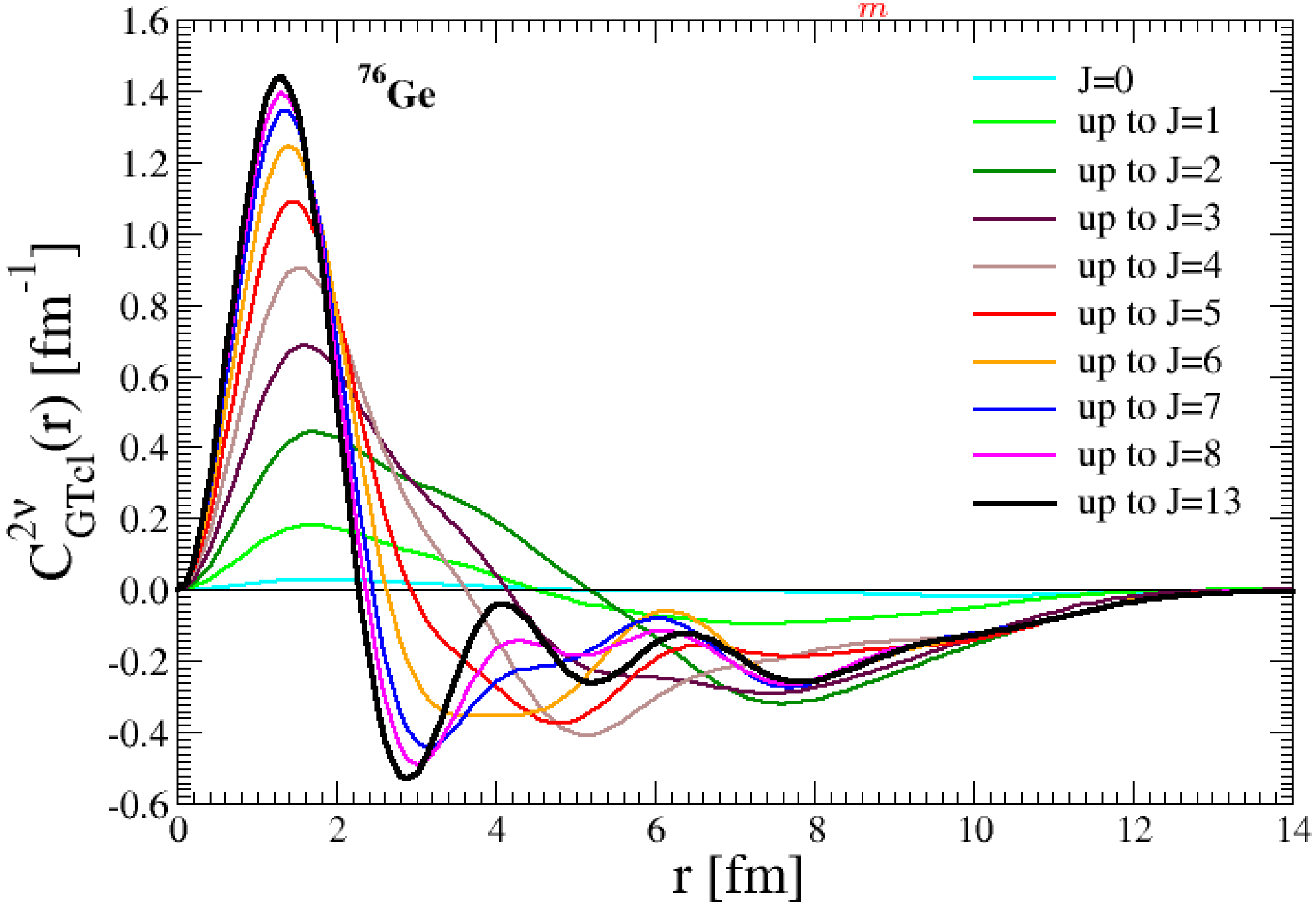
The only non-zero contribution from $J^\pi=1^+$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi} \int_0^\infty C^{2\nu}_{GT-J^\pi}(r)dr$$

$$M^{2\nu}_{GT-cl} = \sum_{J^\pi,m} < 0^+_f|\tau^+\vec{\sigma}|J^\pi, m > \cdot < J^\pi, m|\tau^+\vec{\sigma}|0^+_i >$$

$$\sum_m < 0^+_f|\tau^+\vec{\sigma}|1^+, m > \cdot < 1^+, m|\tau^+\vec{\sigma}|0^+_i >$$

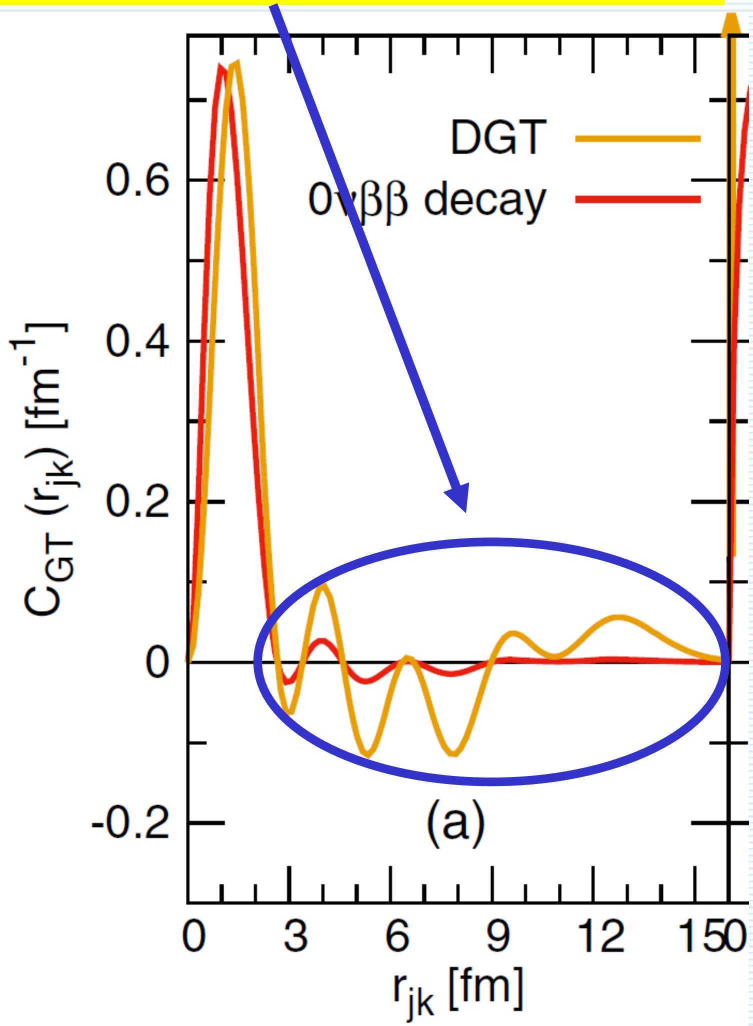
=



Many multipole contributions not included within the ISM due to truncation of the model space

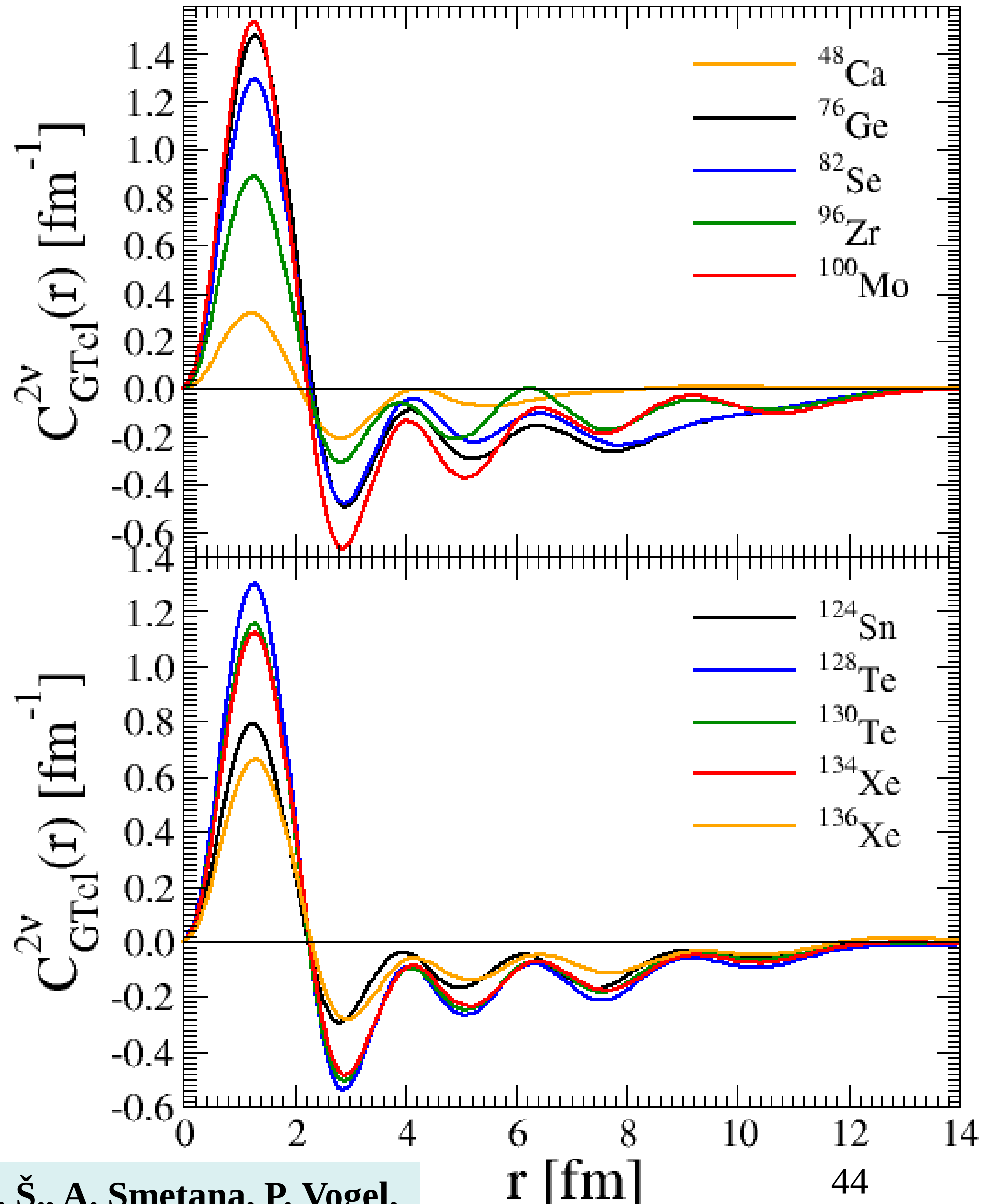
QRPA: Bump ≈ - Tail => $M^{2\nu}_{cl} \approx 0$

ISM: Tail ≈ 0 (?!) => $M^{2\nu}_{cl} \gg 0$



N. Shimizu, J. Menendez, K. Yako, PRL 120, 142502 (2018)

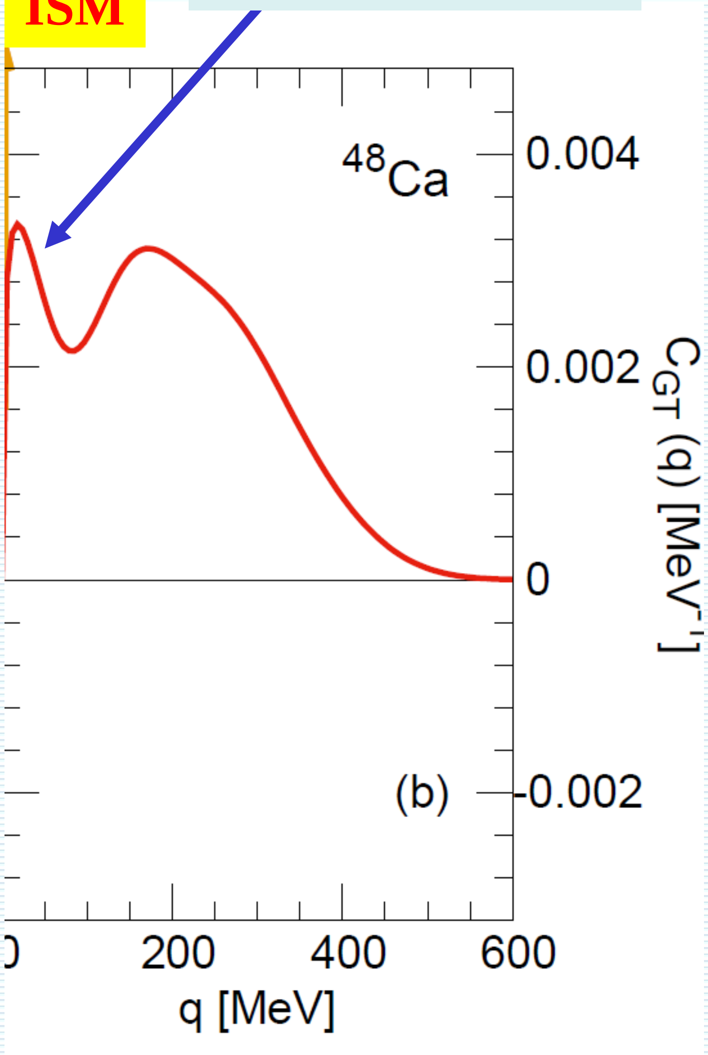
Close to restoration of the SU(4) symmetry of residual Hamiltonian



F. Š., A. Smetana, P. Vogel, PRC 98, 064325 (2018)

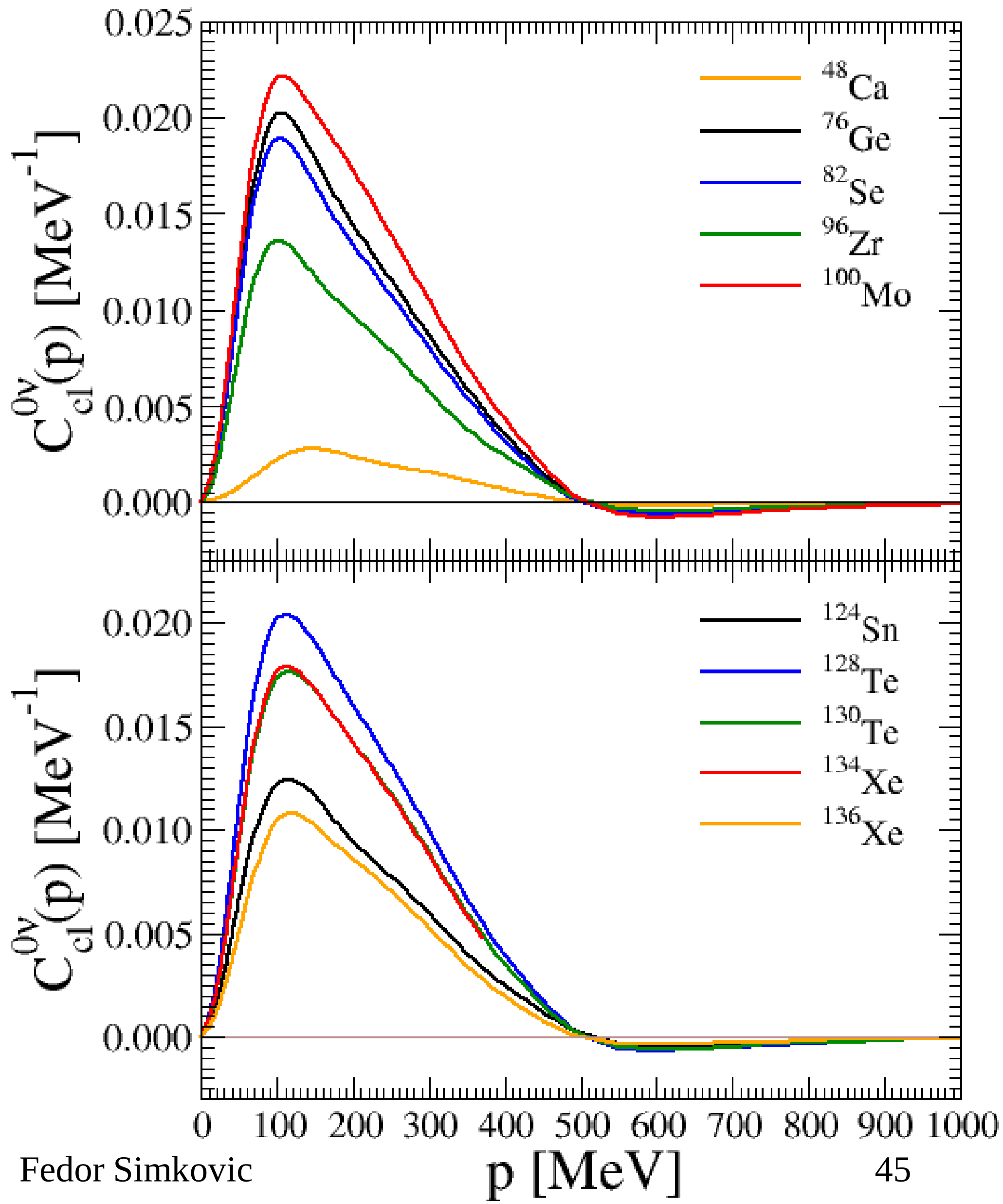
What is the origin of this peak?

ISM



QRPA

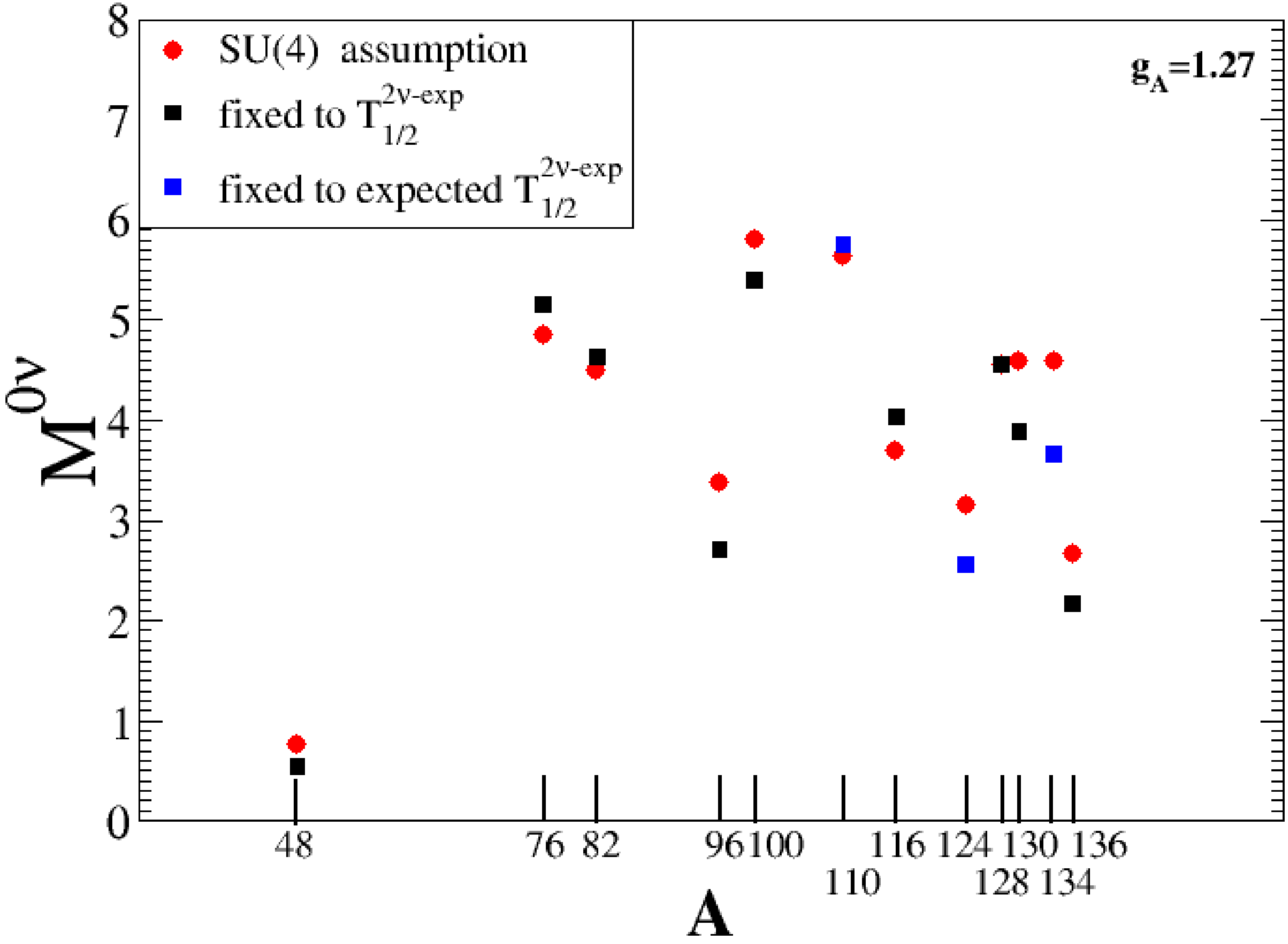
F. Š., A. Smetana, P. Vogel, PRC 98, 064325 (2018)



N. Shimizu, J. Menendez, K. Yako, Phys. Rev. Lett. 120, 14502 (2018)

QRPA – SU(4) parametrization

F. Š., A. Smetana, P. Vogel,
PRC 98, 064325 (2018)



2νββ–decay within the QRPA

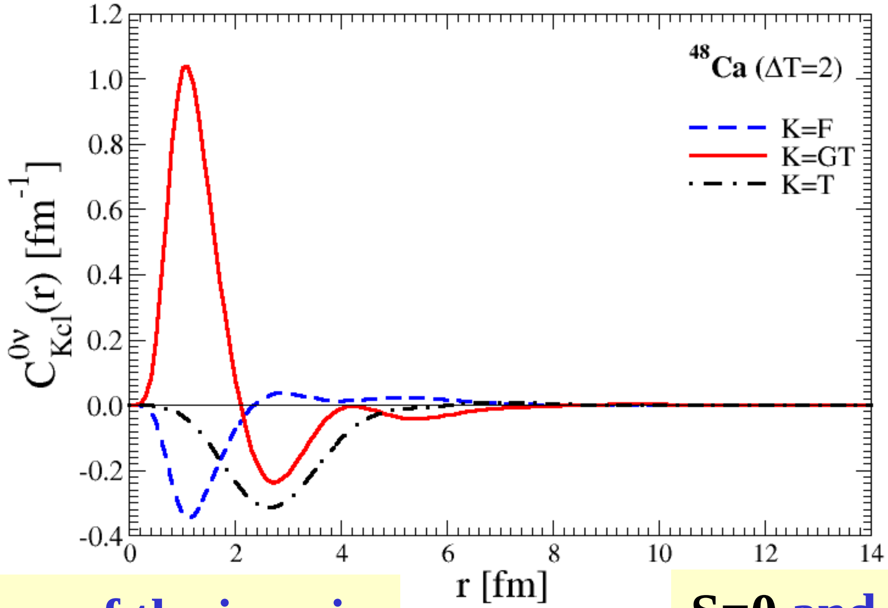
(restoration of the SU(4) symmetry – $M^{2\nu}_{cl}$ =0)

$$
\begin{aligned}
g_A^{\rm eff} &= q \times g_A^{\rm free} = 0.901 \\
g_A^{\rm free} &= 1.269, \qquad q = 0.710
\end{aligned}
$$

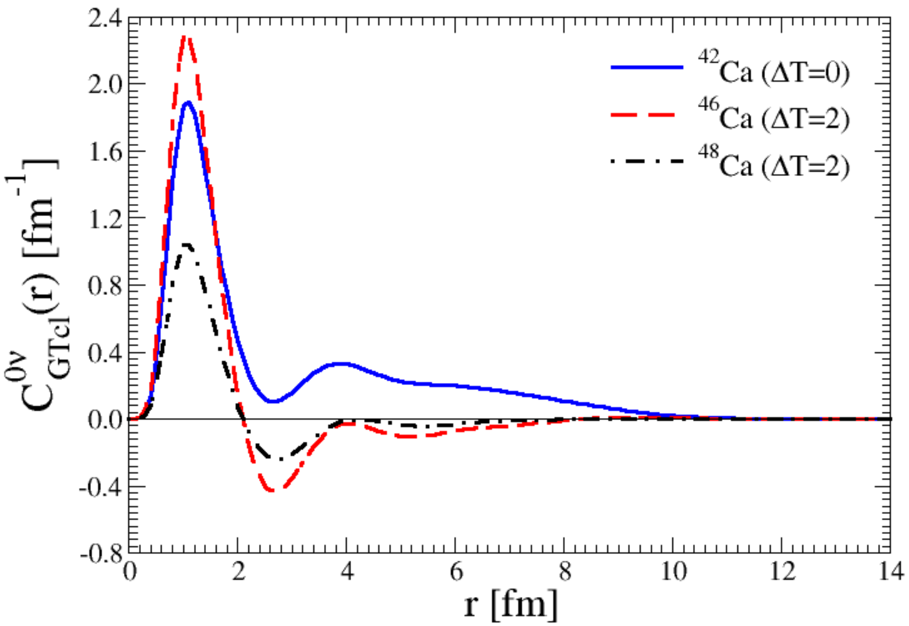
Nucleus	d^i_{pp}	d^f_{pp}	d^i_{nn}	d^f_{nn}	$g^{T=1}_{pp}$	$g^{T=0}_{pp}$	$M^{2\nu}_F$ [MeV^{-1}]	$M^{2\nu}_{GT} \times q^2$ [MeV^{-1}]	$M^{2\nu}_{exp}$ [MeV^{-1}]
^{48}Ca	-	1.069	-	0.982	1.028	0.745	-0.003	0.037	0.046
^{76}Ge	0.922	0.960	1.053	1.085	1.021	0.733	0.003	0.076	0.136
^{82}Se	0.861	0.921	1.063	1.108	1.016	0.737	0.001	0.070	0.100
^{96}Zr	0.910	0.984	0.752	0.938	0.961	0.739	0.001	0.161	0.097
^{100}Mo	1.000	1.021	0.926	0.953	0.985	0.799	-0.001	0.304	0.251
^{116}Cd	0.998	-	0.934	0.890	0.892	0.877	-0.000	0.059	0.136
^{128}Te	0.816	0.857	0.889	0.918	0.965	0.741	0.017	0.075	0.052
^{130}Te	0.847	0.922	0.971	1.011	0.963	0.737	0.016	0.064	0.037
^{136}Xe	0.782	0.885	-	0.926	0.910	0.685	0.014	0.039	0.022

QRPA

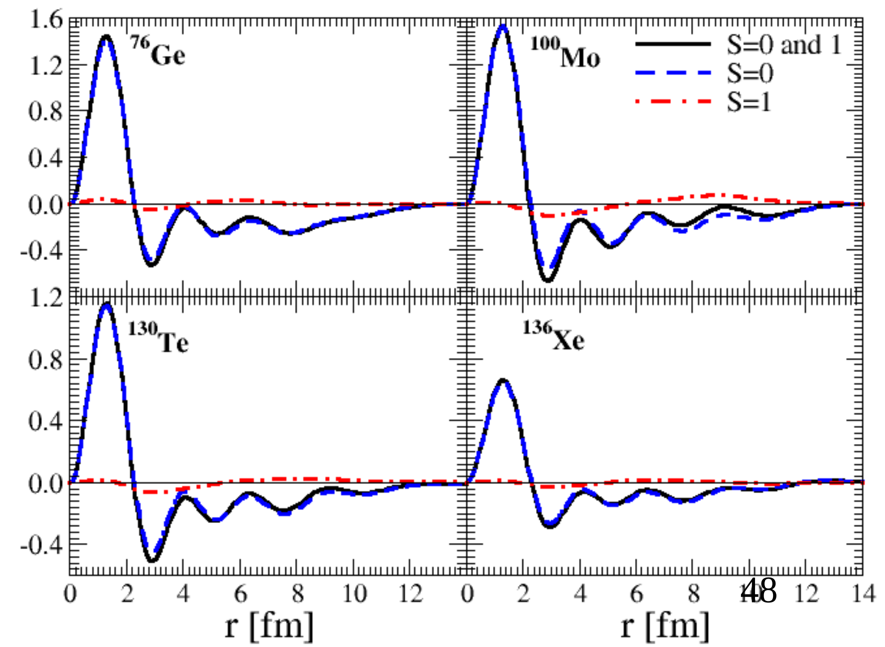
Fermi, Gamow-Teller and tensor



Role of the change of the isospin



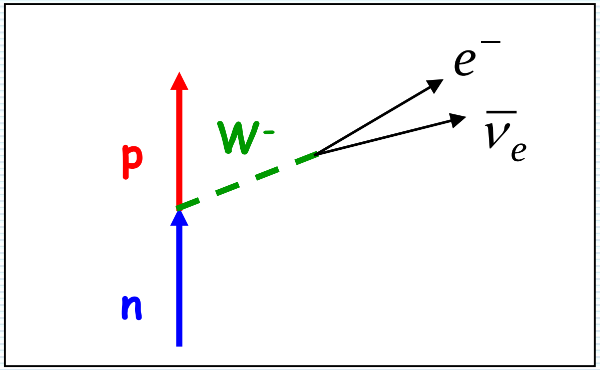
S=0 and S=1 contributions

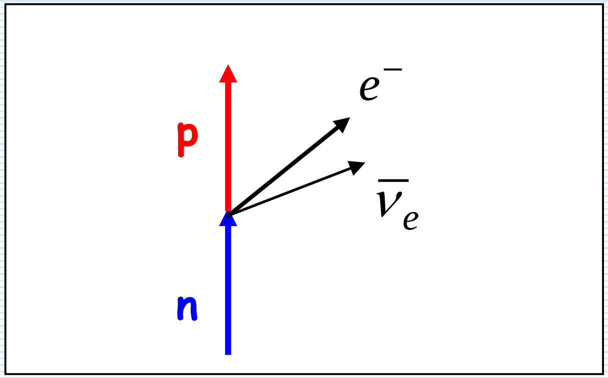


V. Quenching of g_A

Quenching in nuclear matter: $g^{eff}_A = q\, g^{free}_A$

(from theory: $T_{1/2}^{0\nu}$ up 50 x larger)





$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}}\left[\overline{u}\gamma^\alpha(1-\gamma^5)d\right]\left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}}\left[\overline{p}\gamma^\alpha(g_V - g_A\gamma^5)n\right]\left[\overline{e}\gamma^\alpha(1-\gamma^5)\nu_e\right]$$

CVC hypothesis

g_V =1 at the quark level

g_V =1 at the nucleon level

g_V =1 inside nuclei

Quenching of g_A

g_A =1 at the quark level

g^{free}_A =1.27 at the nucleon level

g^{eff}_A = ? inside nuclei

ISM: $(g^{eff}_A)^4 \simeq 0.66$ (^{48}Ca), 0.66 (^{76}Ge), 0.30 (^{76}Se), 0.20 (^{130}Te) and 0.11 (^{136}Xe)

IBM: $(g^{eff}_A)^4 \simeq (1.269\, A^{-0.18})^4 = 0.063$

QRPA: $(g^{eff}_A)^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd

$g_A{}^4 = (1.269)^4 = 2.6$

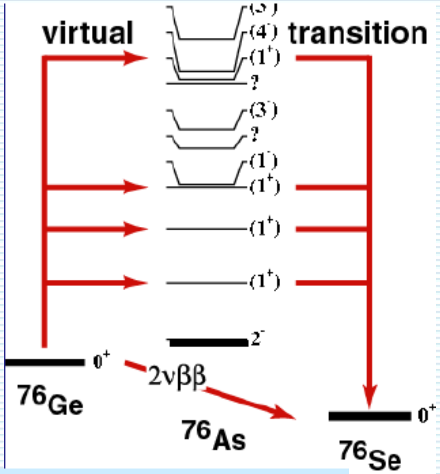
Quenching of g_A (from exp.: $T_{1/2}^{0\nu}$ up 2.5 x larger)

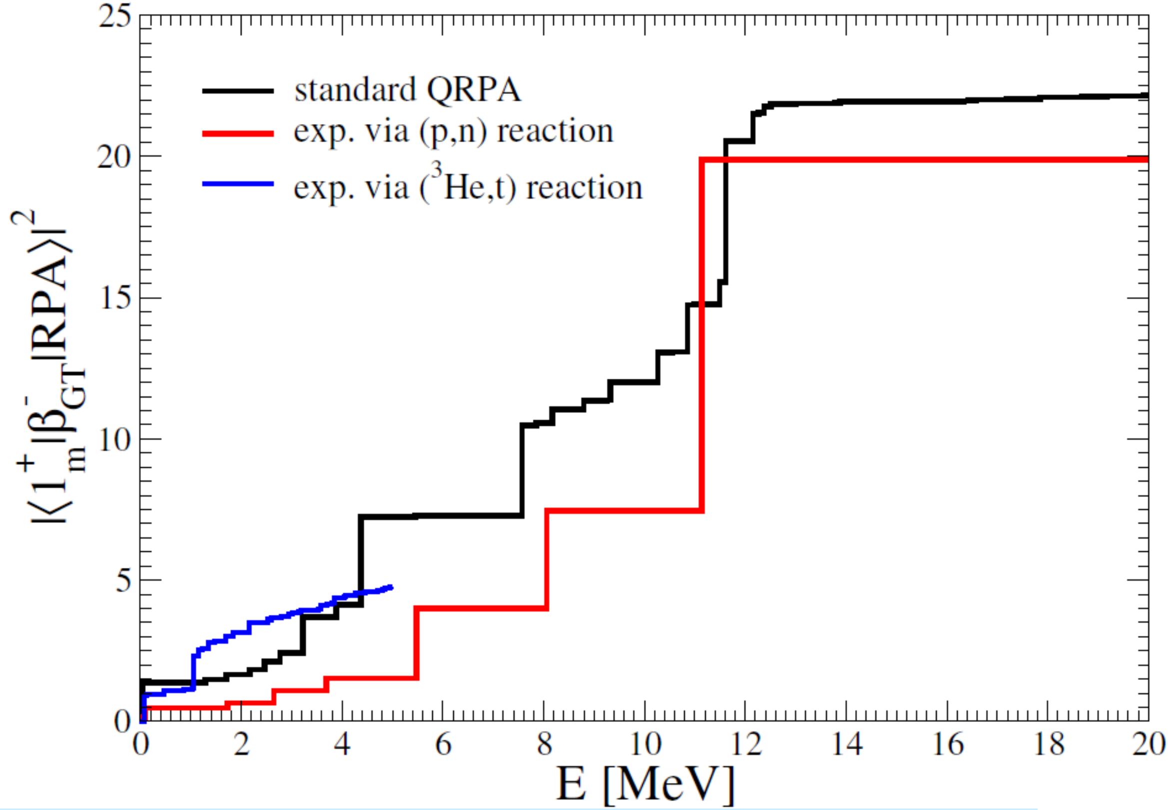
$(g^{eff}{}_A)^4 = 1.0$

Strength of GT trans. (approx. given by Ikeda sum rule =3(N-Z)) has to be quenched to reproduce experiment

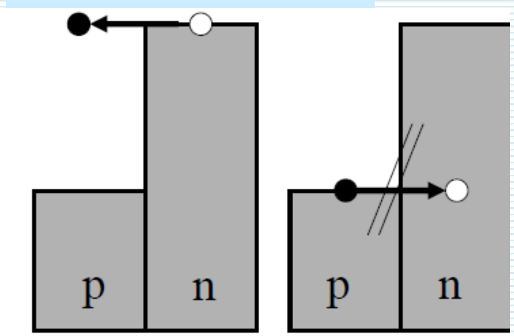
$^{76}_{32}Ge_{44} \Rightarrow$

$S_{\beta}{}^{-} - S_{\beta}{}^{+} = 3(N-Z) = 36$





Pauli blocking



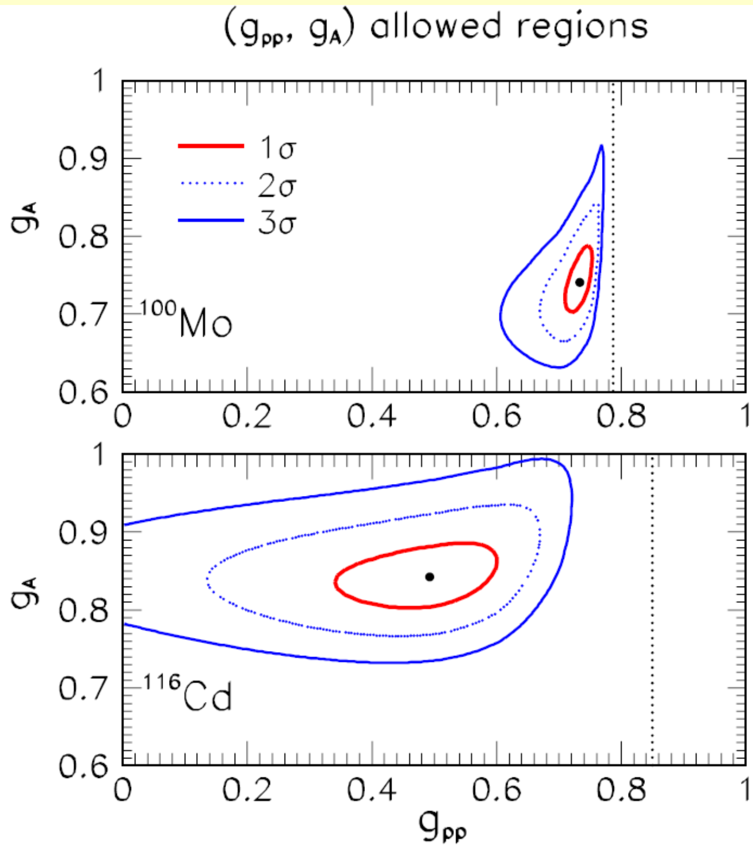
Cross-section for charge exchange reaction:

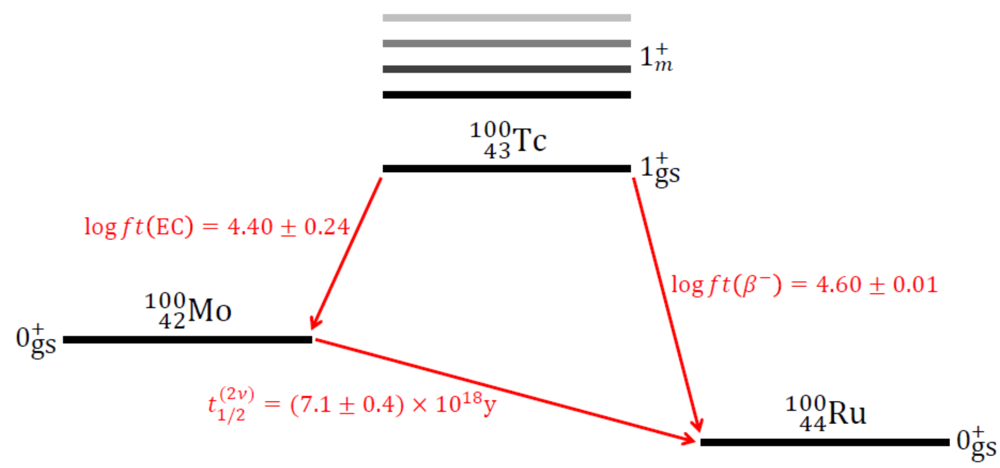
$$\left[\frac{d\sigma}{d\Omega}\right] = \left[\frac{\mu}{\pi\hbar}\right]^2 \frac{k_f}{k_i} N_d \underbrace{|V_{\sigma\tau}|^2}_{\text{largest at 100 - 200 MeV/A}} |\langle f|\sigma\tau|i\rangle|^2$$

$q = 0!!$

Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š, J. Phys. G 35, 075104 (2008).

$(g^{eff}_A)^4 = 0.30$ and 0.50 for ^{100}Mo and ^{116}Cd, respectively (The QRPA prediction).
g^{eff}_A was treated as a completely free parameter alongside g_{pp} (used to renormalize particl-particle interaction) by performing calculations within the QRPA and RQRPA. It was found that a least-squares fit of g^{eff}_A and g_{pp}, where possible, to the β-decay rate and β+/EC rate of the J = 1$^+$ ground state in the intermediate nuclei involved in double-beta decay in addition to the 2νββ rates of the initial nuclei, leads to an effective g^{eff}_A of about 0.7 or 0.8.





Extended calculation also for neighbor isotopes performed by

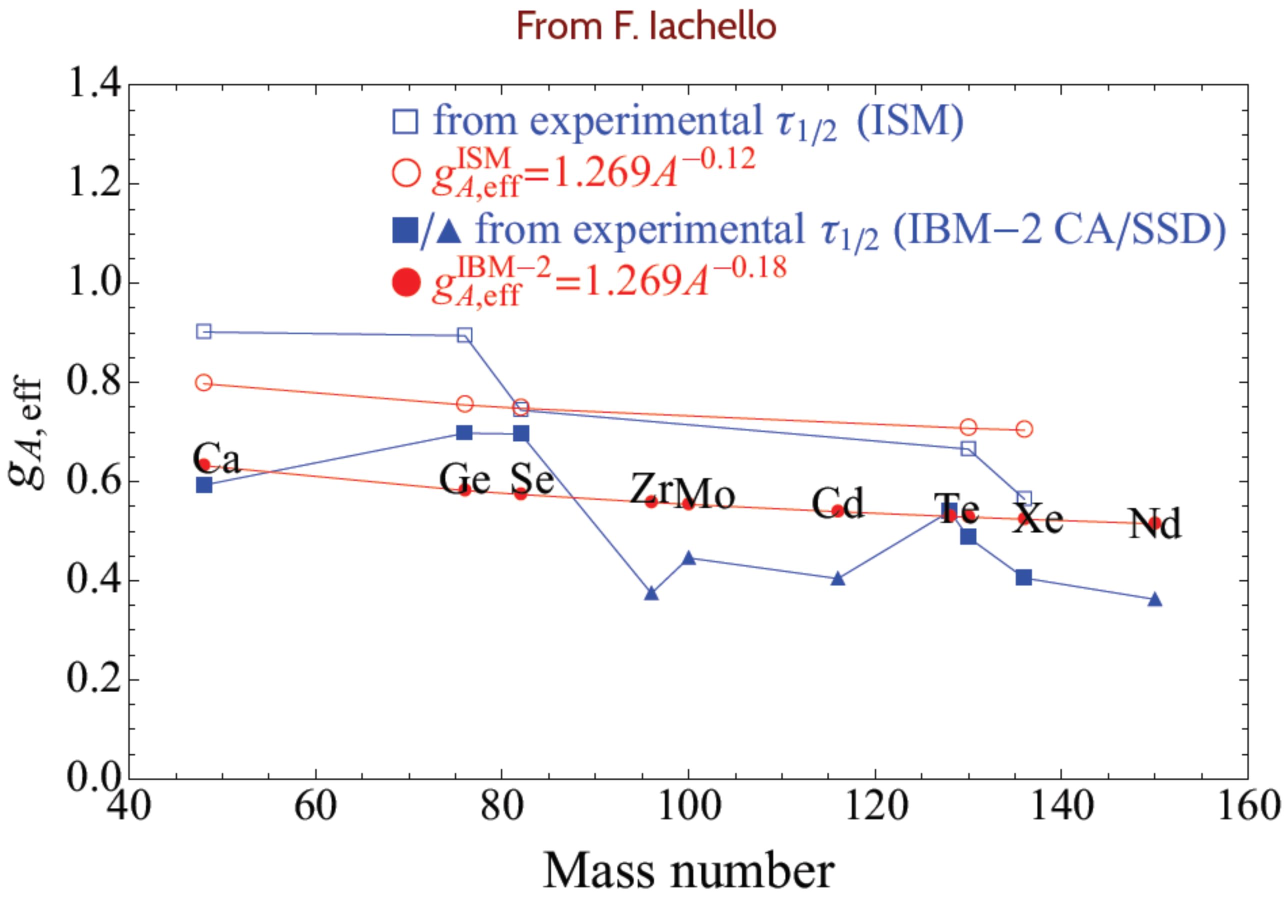
F.F. Depisch and J. Suhonen, PRC 94, 055501 (2016)

Dependence of g^{eff}_A on A was not established.

Quenching of g_A -IBM *($T_{1/2}^{0\nu}$ suppressed up to factor 50)*

$(g^{eff}_A)^4 \simeq (1.269\ A^{-0.18})^4 = 0.063$ (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the 2νββ-decay half-lives, which were based on within closure approximation calculated Corresponding NMEs, with the measured half-lives.



3/7/2019

J. Barea, J. Kotila, F. Iachello, PRC 87, 014315 (2013).

Improved description of the 0νββ–decay rate (and novel approach of fixing g_A^{eff})

F. Š, R. Dvornický, D. Štefánik, A. Faessler, PRC 97, 034315 (2018).

Let perform Taylor expansion

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i+E_f)/2]^2 - \varepsilon_{K,L}^2}$$

$$\frac{\varepsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\begin{aligned} \epsilon_K &= (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2 \\ \epsilon_L &= (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2 \end{aligned}$$

$$\epsilon_{K,L} \in (-\frac{Q}{2}, \frac{Q}{2})$$

We get

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\mathrm{eff}}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{13}^{2\nu}\right|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

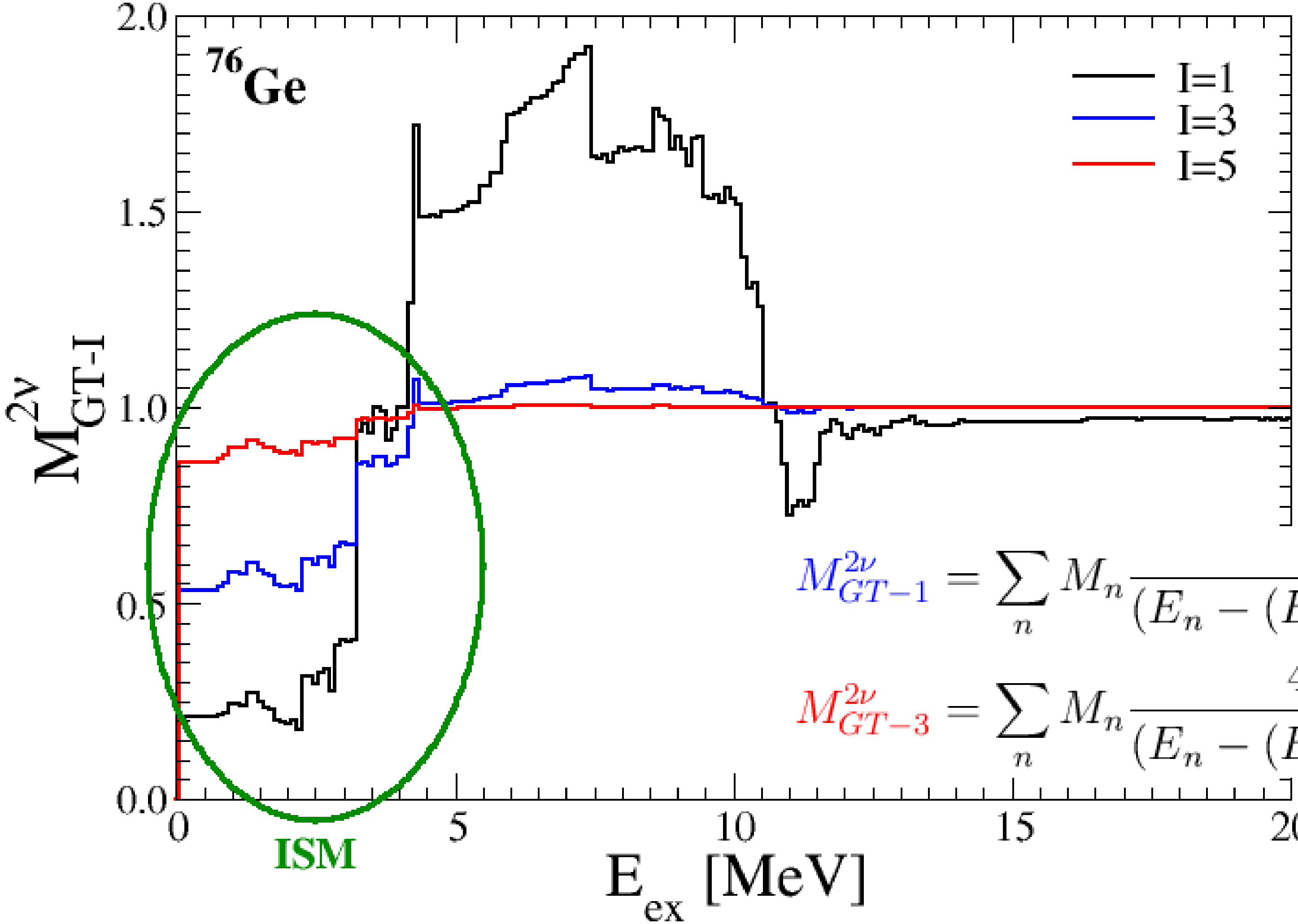
$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4\, m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM?)

The running sum of the 2νββ–decay NMEs (QRPA)

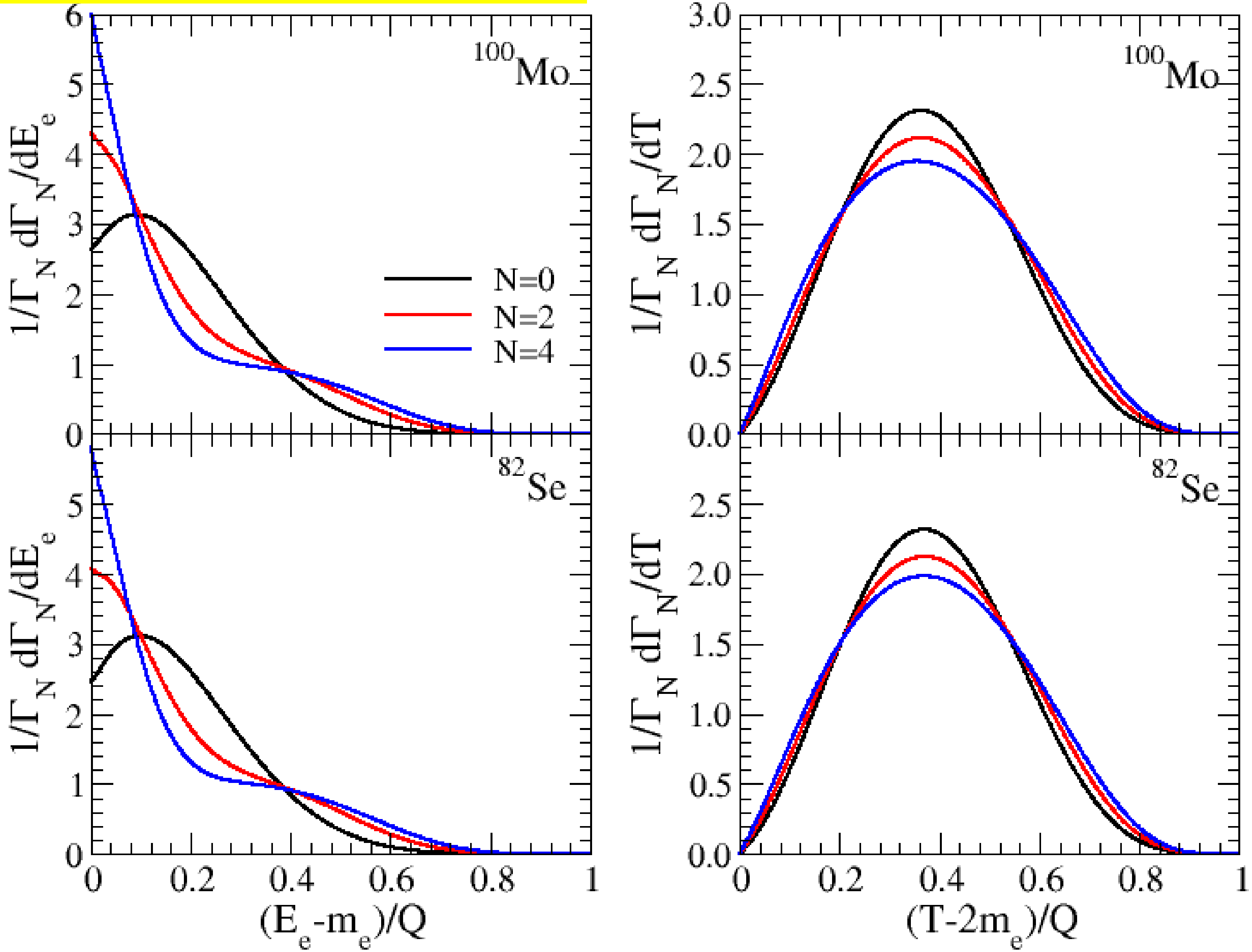


$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2}$$

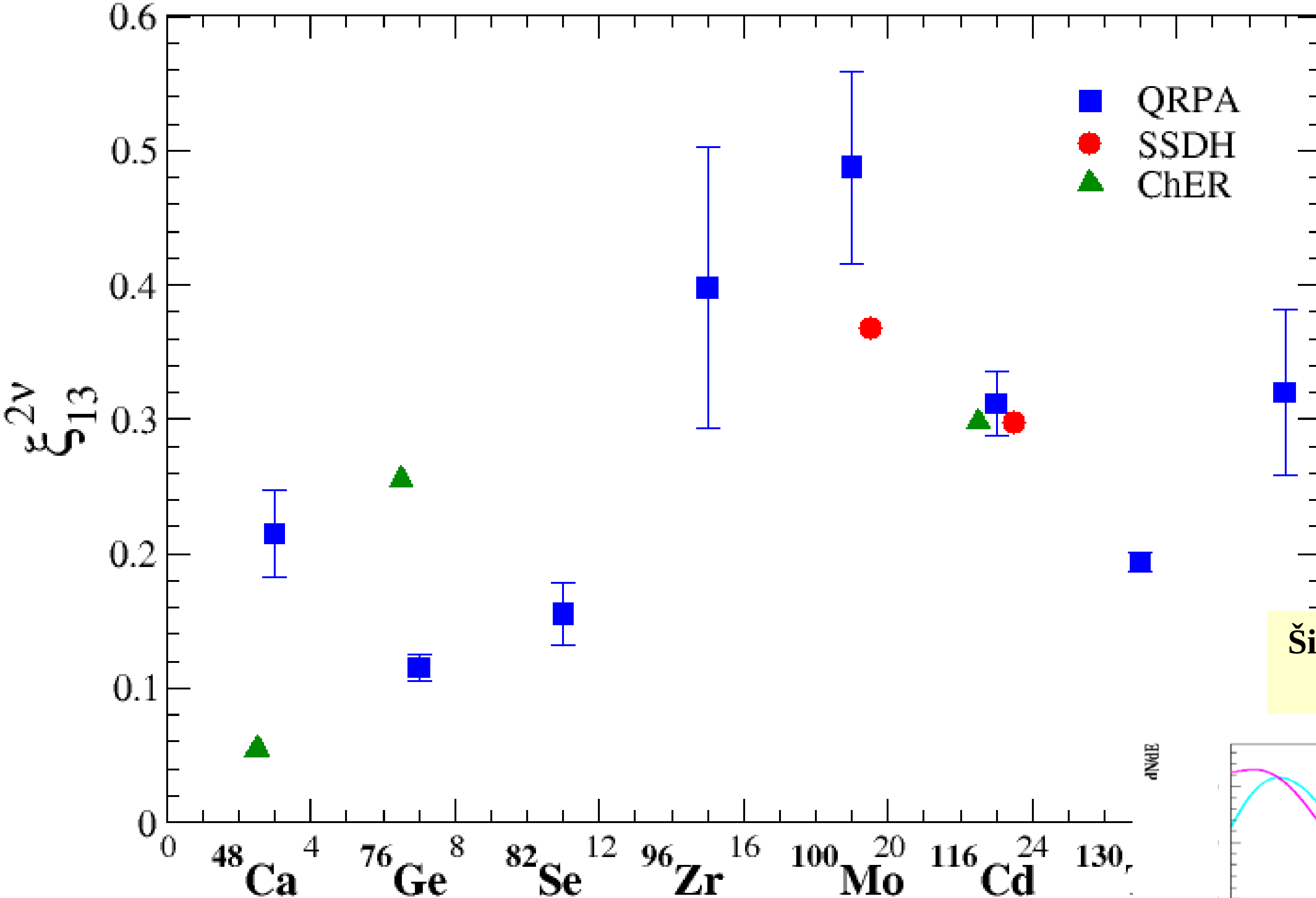
$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

Normalized to unity
different partial energy distributions

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \equiv \frac{\Gamma^{2\nu}}{\ln(2)} \simeq \frac{\Gamma_0^{2\nu} + \Gamma_2^{2\nu} + \Gamma_4^{2\nu}}{\ln(2)}$$

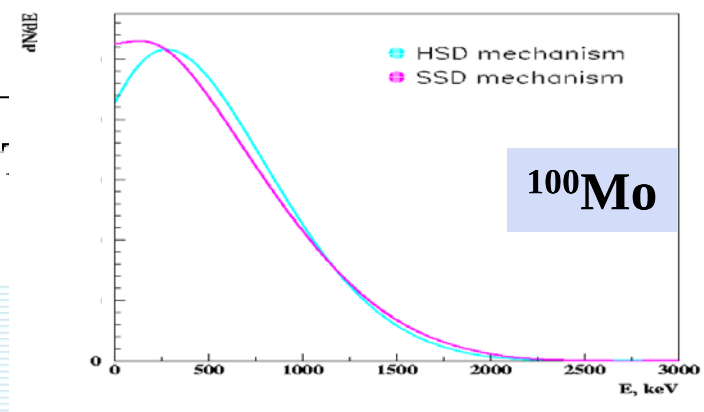


ξ_{13} tell us about importance of higher lying states of int. nucl.



HSD: ξ_{13}=0

Šimkovic, Šmotlák, Semenov
J. Phys. G, 27, 2233, 2001



ξ_{13} can be determined phenomenologically from the shape of energy distributions of emitted electrons

The g_A^{eff} can be deterimed with measured half-life and ratio of NMEs ξ_{31} and calculated NME dominated by transitions through low lying states of the intermediate nucleus.

M_{GT-3} have to be calculated by nuclear theory - ISM

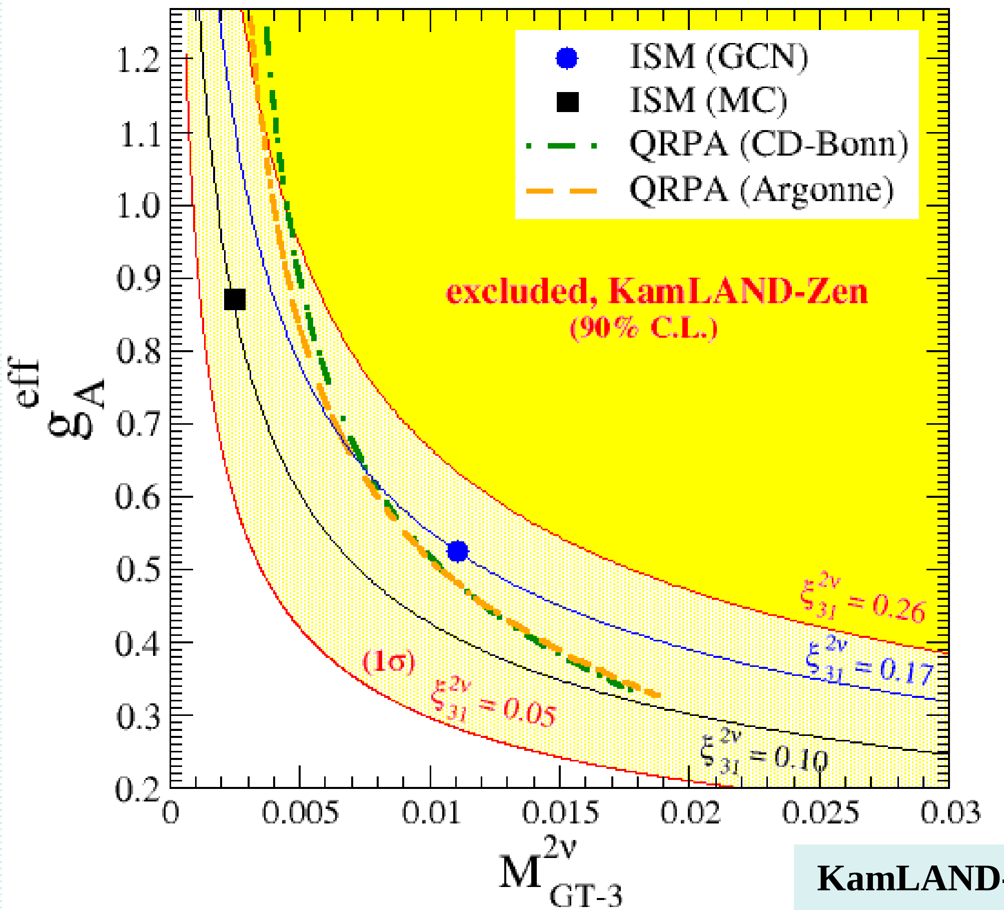
$$\left(g_A^{\rm eff}\right)^2 = \frac{1}{\left|M^{2\nu}_{GT-3}\right|}\frac{\left|\xi^{2\nu}_{13}\right|}{\sqrt{T^{2\nu-exp}_{1/2}\left(G^{2\nu}_0 + \xi^{2\nu}_{13} G^{2\nu}_2\right)}}$$

$$M^{2\nu}_{GT-1} = \sum_n M_n \frac{1}{(E_n - (E_i+E_f)/2}$$

$$M^{2\nu}_{GT-3} = \sum_n M_n \frac{4\ m_e^3}{(E_n - (E_i+E_f)/2)^3}$$

$$\xi^{2\nu}_{13} = \frac{M^{2\nu}_{GT-3}}{M^{2\nu}_{GT-1}}$$

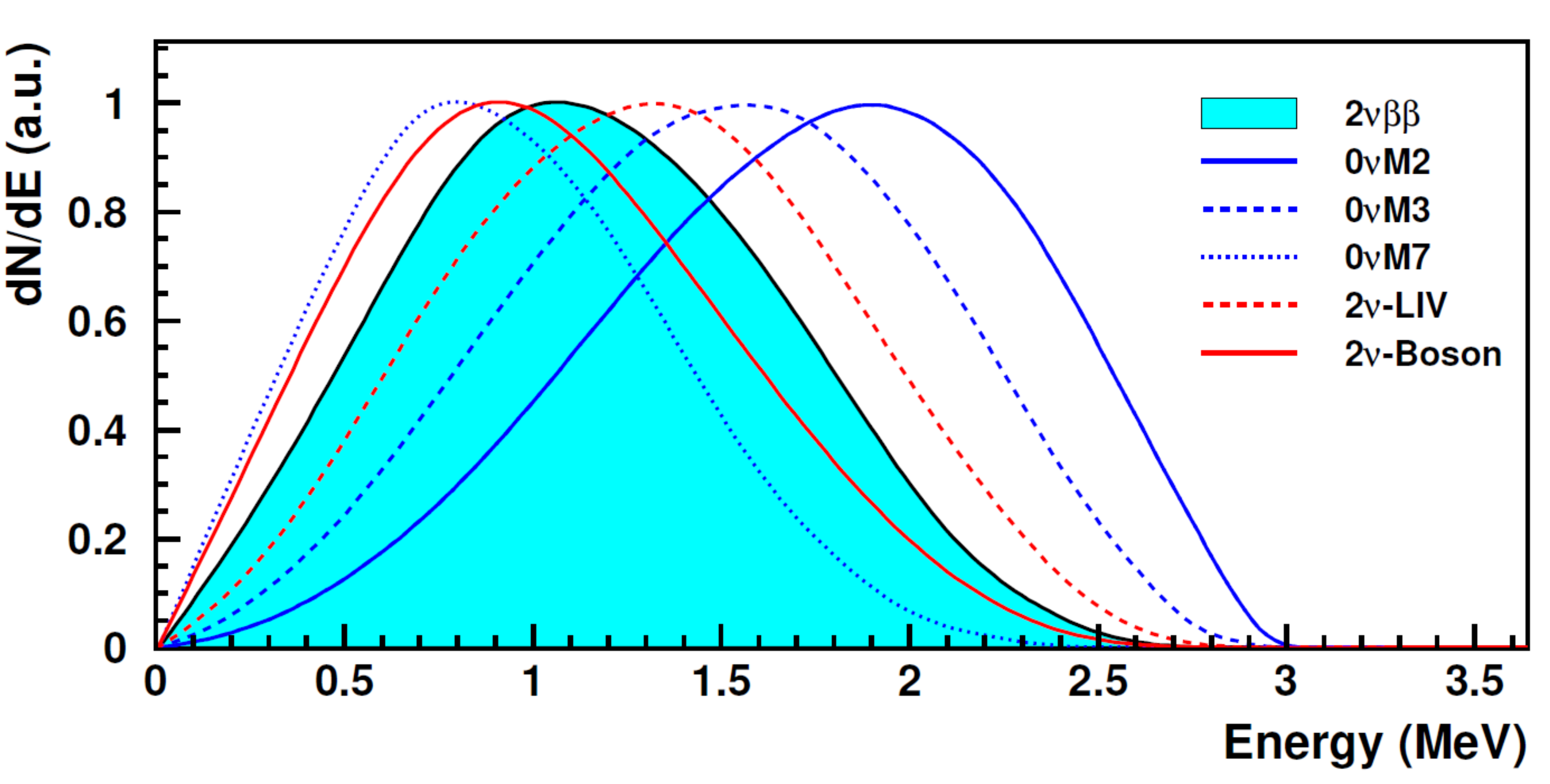
$$\xi^{2\nu}_{15} = \frac{M^{2\nu}_{GT-5}}{M^{2\nu}_{GT-1}}$$



F. Š., R. Dvornický, D. Štefánik, A. Faessler, PRC 97, 034315 (2018)

KamLAND-Zen Coll. (+J. Menendez, F.Š.), arXiv: 1901.03871 [hep-ex]

Looking for a new physics with differential characteristics



Spectral index n

$$\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = C(Q - \varepsilon_1 - \varepsilon_2)^n \; [p_1\varepsilon_1 F(\varepsilon_1)] \; [p_2\varepsilon_2 F(\varepsilon_2)]$$

VI. New modes of the double beta decay

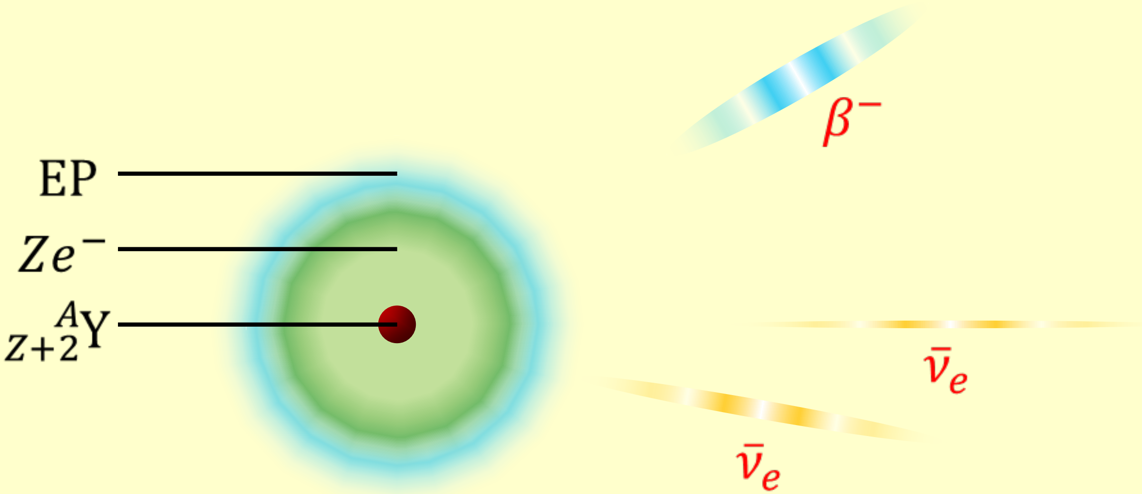
Double Beta Decay with emission of a single electron

A. Babič, M.I. Krivoruchenko, F.Š., PRC 98, 065501 (2018)

[Jung *et al.* (GSI), 1992] observed beta decay of $^{163}_{66}\mathrm{Dy}^{66+}$ ions with Electron Production (EP) in K or L shells: $T_{1/2}^{\mathrm{EP}} = 47\ \mathrm{d}$

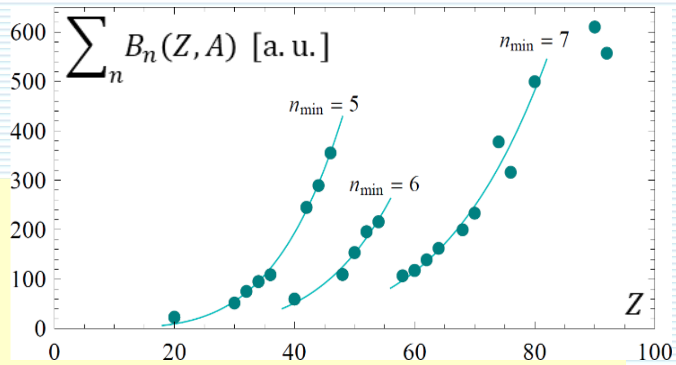
Bound-state double-beta decay $0\nu\mathrm{EP}\beta^-$ $(2\nu\mathrm{EP}\beta^-)$ **with EP in available** $\mathrm{s}_{1/2}$ **or** $\mathrm{p}_{1/2}$ **subshell of daughter 2+ ion:**

$$^{A}_{Z}\mathrm{X} \longrightarrow {}^{A}_{Z+2}\mathrm{Y} + e_{\mathrm{b}}^{-} + e^{-} + (\bar{\nu}_e + \bar{\nu}_e)$$



Search for possible manifestation in single-electron spectra…

Phase space factors



$0\nu EP\beta^-$ and $2\nu EP\beta^-$ phase-space factors:

$$G^{0\nu \mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4\, m_e^2}{32\pi^4 R^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A)\, F(Z+2,E)\, E\, p$$

$$G^{2\nu \mathrm{EP}\beta}(Z,Q) = \frac{G_\beta^4}{8\pi^6 m_e^2 \ln 2} \sum_{n=n_{\min}}^{\infty} B_n(Z,A) \int_{m_e}^{m_e+Q} \mathrm{d}E\, F(Z+2,E)\, E\, p \int_0^{m_e+Q-E} \mathrm{d}\omega_1\, \omega_1^2\, \omega_2^2$$

Single-electron spectra for ^{82}Se ($Q = 2.998\ MeV$):

Bound- and free-electron Fermi functions:

$$B_n(Z,A) = f_{n,-1}^2(R) + g_{n,+1}^2(R)$$
$$F(Z,E) = f_{-1}^2(R,E) + g_{+1}^2(R,E)$$

Relativistic electron wave functions in central field:

$$\psi_{\kappa\mu}(\vec{r}) = \begin{pmatrix} f_\kappa(r)\,\Omega_{\kappa\mu}(\hat{r}) \\ i g_\kappa(r)\,\Omega_{-\kappa\mu}(\hat{r}) \end{pmatrix}$$

$\kappa = (l-j)(2j+1) = \pm 1, \pm 2, \ldots$

$j = |l \pm 1/2|$ $\quad \mu = -j, \ldots, +j$

CALCULATION: GRASP2K

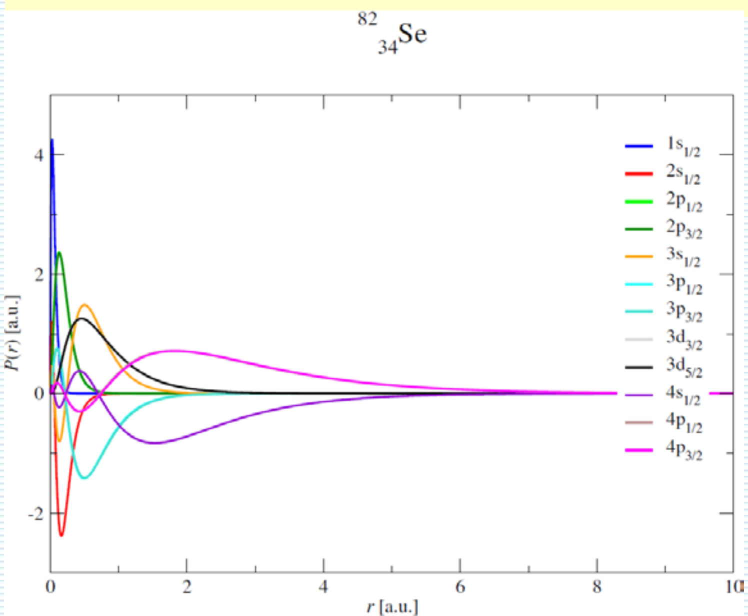
Stationary N-particle Dirac eq. with separable central atomic Hamiltonian $[a.u.]$:

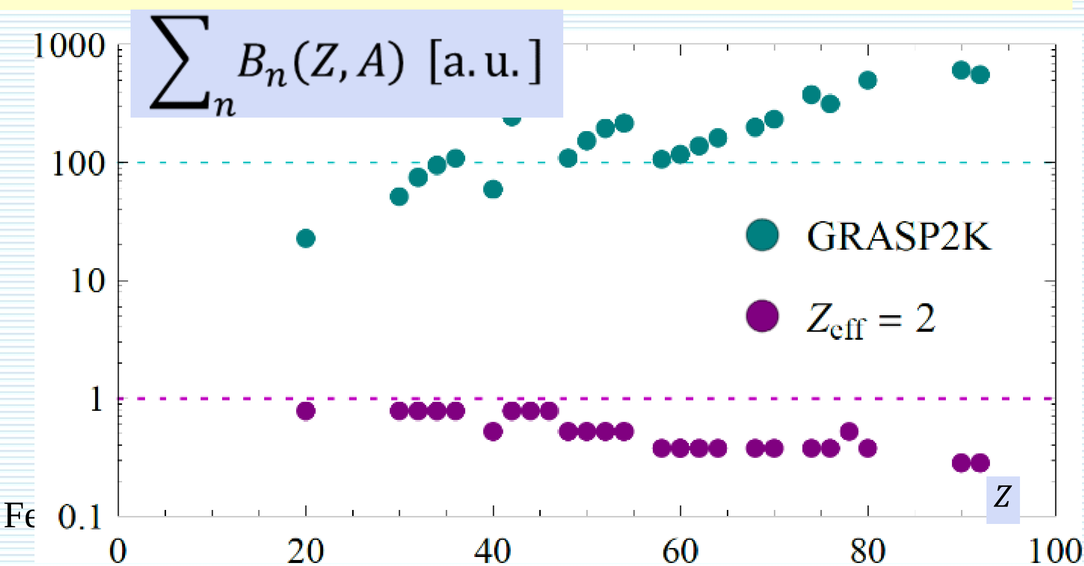
$$\left[\sum_{i=1}^{N} -i\nabla_i \cdot \vec{\alpha} c + \beta c^2 - \frac{Z}{r_i} + V(r_i)\right]\Psi = E\,\Psi$$

$$\Psi = \frac{1}{\sqrt{N!}}\begin{vmatrix} \psi_1(\vec{r}_1) & \cdots & \psi_1(\vec{r}_N) \\ \vdots & \ddots & \vdots \\ \psi_N(\vec{r}_1) & \cdots & \psi_N(\vec{r}_N) \end{vmatrix}$$

Multiconfiguration Dirac–Hartree–Fock package GRASP2K:

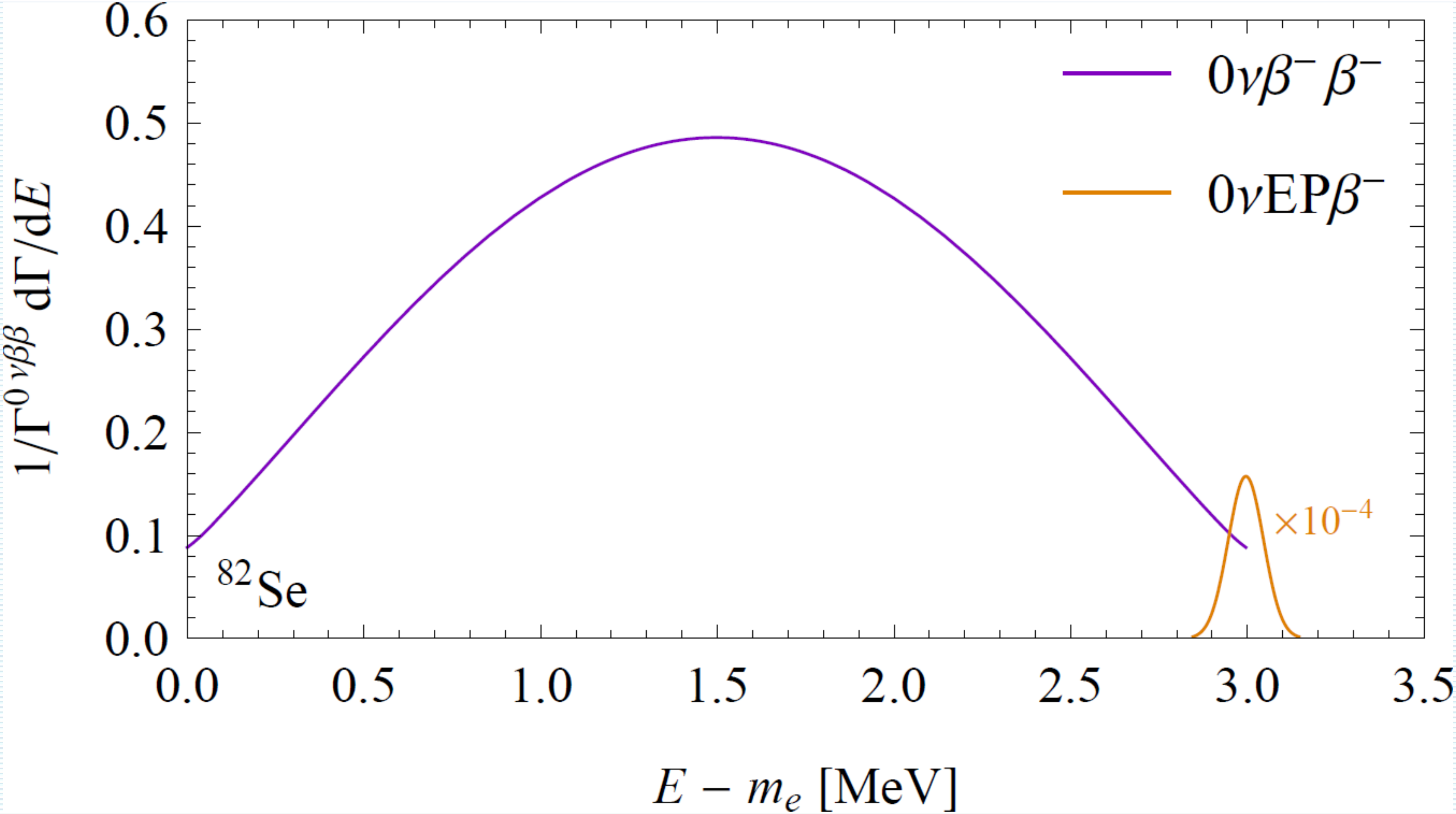
- **Fit of non-convergent orbitals:** $f^2_{n,-1}, g^2_{n,+1}(R) \approx aZ^b$
- **Fit of orbitals beyond $n = 9$:** $f^2_{n,-1}, g^2_{n,+1}(R) \approx cn^d$





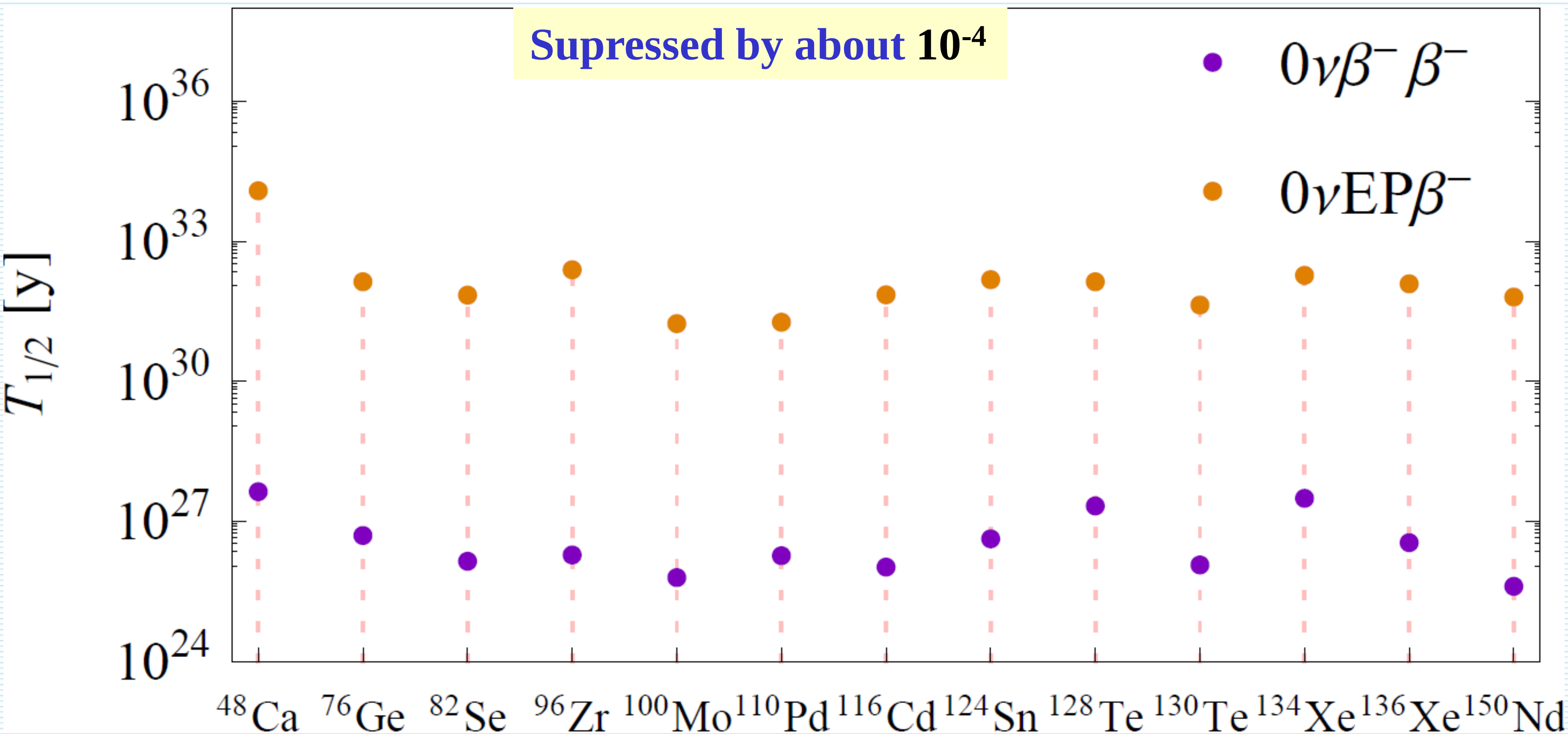
$0\nu EP\beta^-$ Single-Electron Spectrum (^{82}Se)

$0\nu\beta^-\beta^-$ **and** $0\nu EP\beta^-$ **single-electron spectra** $1/\Gamma^{0\nu\beta\beta}\, d\Gamma/dE$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** ^{82}Se ($Q = 2.996$ MeV)



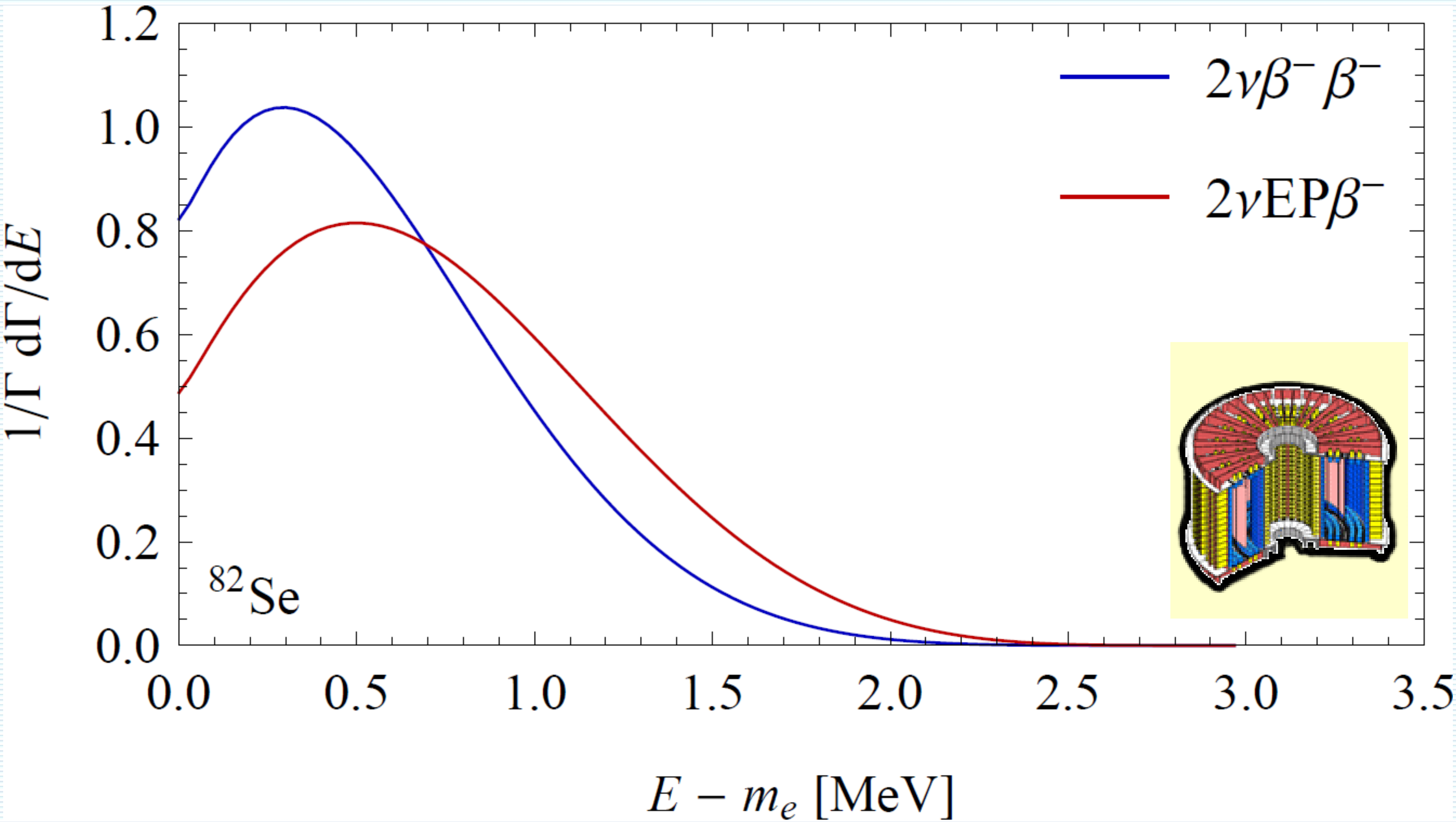
$0\nu\text{EP}\beta^-$ Half-Lives

$0\nu\beta^-\beta^-$ **and** $0\nu\text{EP}\beta^-$ **half-lives** $T_{1/2}^{0\nu\beta\beta}$ **and** $T_{1/2}^{0\nu\text{EP}\beta}$ **estimated for** $\beta^-\beta^-$ **isotopes with known NME** $|M^{0\nu\beta\beta}|$, **assuming unquenched** $g_A = 1.269$ **and** $|m_{\beta\beta}| = 50\ \text{meV}$



$2\nu\mathrm{EP}\beta^-$ Single-Electron Spectrum (^{82}Se)

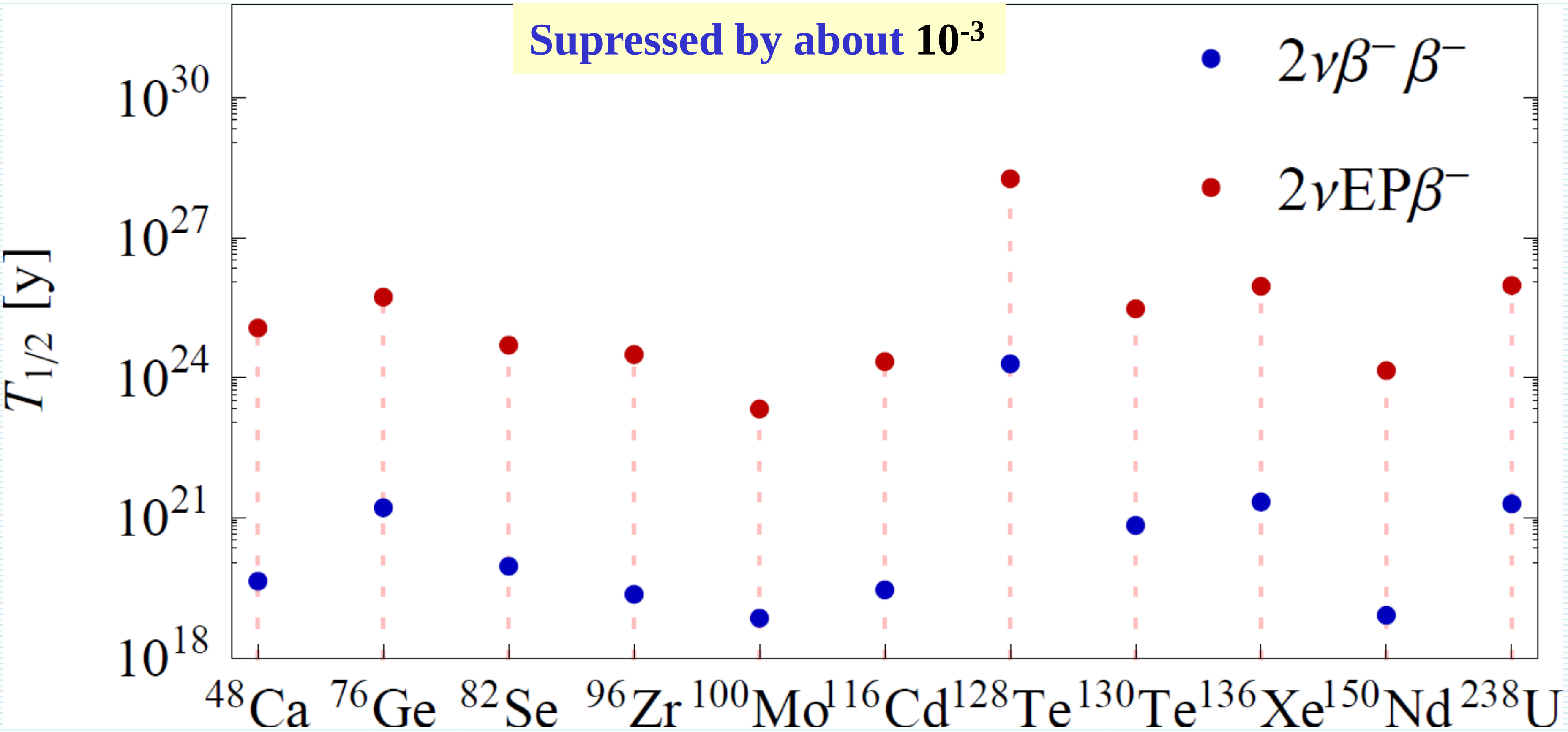
$2\nu\beta^-\beta^-$ **and** $2\nu\mathrm{EP}\beta^-$ **single-electron spectra** $1/\Gamma\, \mathrm{d}\Gamma/\mathrm{d}E$ ***vs.*** **electron kinetic energy** $E - m_e$ **for** ^{82}**Se** ($Q = 2.996$ MeV)



$2\nu EP\beta^-$ Half-Lives predictions

(independent on g_A and value of NME)

$2\nu\beta^-\beta^-$ **and** $2\nu EP\beta^-$ **half-lives** $T_{1/2}^{2\nu\beta\beta}$ **and** $T_{1/2}^{2\nu EP\beta}$ **calculated for** $\beta^-\beta^-$ **isotopes observed experimentally, assuming unquenched** $g_A = 1.269$



Instead of Conclusions

LHC physics

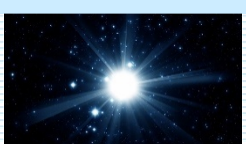
$$\frac{1}{\Lambda^2}\sum_i c_i^{(6)}\mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

Neutrino physics

$$\frac{1}{\Lambda}\sum_i c_i^{(5)}\mathcal{O}_i^{(5)}$$

Progress in nuclear structure calculations is highly required

We are at the beginning of the Beyond Standard Model Road…



The future of neutrino physics is bright