Axion Cosmology

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1. Axion

- Axion is predicted in PQ mechanism which solves strong CP problem in QCD.

- Axion is the Nambu-Goldstone boson associate with $U(1)_{\text{PQ}}$ breaking and can be identified with the phase of PQ scalar.

$$
\Phi = |\Phi| e^{i\theta} = (\eta + \varphi) e^{i\alpha/\eta} \\
\eta : \text{breaking scale}
$$

- Axion acquires mass through QCD non-perturbative effect.

$$
 m_a \simeq 0.6 \times 10^{-5} \text{eV} \left( \frac{F_a}{10^{12} \text{GeV}} \right)^{-1} \\
F_a = \eta / N_{\text{DW}} \\
N_{\text{DW}} : \text{domain wall number}
$$

- Axion is a good candidate for \textbf{dark matter} of the universe.
1. Axion

- Cosmological evolution of axion (PQ scalar)
  - PQ symmetry breaking after inflation
    - Formation of topological defects
    - Isocurvature perturbations
  - PQ symmetry breaking before inflation
    - Domain wall problem
    - Isocurvature perturbation problem
Today’s Talk

• Introduction

• PQ symmetry breaking after inflation
  ➤ Cosmological evolution of axion
  ➤ Comic axion density
  ➤ Non-topological objects of axions

• PQ symmetry breaking before inflation
  ➤ Isocurvature perturbation problem
  ➤ Suppression of Isocurvature Perturbations

• Conclusion
2. Cosmological Evolution of Axion (PQ after inflation)

- $U_{\text{PQ}}(1)$ symmetry is broken
  - Axion is a phase direction of PQ scalar and massless
    $$\Phi = |\Phi| e^{i\theta} = |\Phi| e^{ia/\eta} \quad m_a = 0$$
  - Formation of Cosmic Strings

- $T \simeq \Lambda_{\text{QCD}}$
  - Axion acquires mass through non-perturbative effect
    - $U_{\text{PQ}}(1)$ is broken to $Z_{N_{\text{DW}}}$
    - Coherent oscillation
    - Formation of Domain Walls
• Domain walls attach to strings

$N_{DW} \geq 2$

$N_{DW} = 1$

Stable and dominate the universe

Domain Wall Problem

Collapse

Axion overproduction

Virenkin Everett (1982)
Barr Choi Kim (1986)
3. Cosmic Axion Density

3.1 Coherent axion oscillation

- Axion field starts to oscillate at $T = T_*$
- Coherent oscillation of axion field gives a significant contribution to the cosmic density ($\Omega_{\text{CDM}} h^2 \simeq 0.12$)

\[ H \simeq m_a(T_*) \]

\[ \Omega_{a,\text{osc}} h^2 \simeq 7 \times 10^{-4} \langle \theta_0^2 \rangle \left( \frac{F_a}{10^{10} \text{GeV}} \right)^{1.19} \]

\[ \theta_* = a_*/F_a : \text{misalignment angle at } T_* \]

\[ \langle \theta_0^2 \rangle \simeq 6 \]

Spatial average

If $F_a \simeq 2 \times 10^{11}$ GeV, then $\Omega_{a,\text{osc}} h^2 \simeq 0.12$
### 3.2 Axions from strings

- Axionic strings are produced when $U(1)_{PQ}$ symmetry is spontaneously broken.

- **Numerical Lattice Simulation**
  
  Hiramatsu, MK, Sekiguchi, Yamaguchi, Yokoyama (2010)  
  MK, Saikawa, Sekiguchi (2014)

- String network obeys scaling solution
  
  $$\rho_{\text{string}} = \xi \frac{\mu}{t^2} \quad (\mu \sim \eta^2 : \text{string tension})$$

  $$\xi = 1.0 \pm 0.5$$

- Scaling solution is established by emitting axions

- **Emitted axion energy** $\rho_{a, \text{str}}$ is estimated from $\rho_{\text{string}}$

- If we know average energy $\bar{\omega}_a$, we can estimate the present axion density as
  
  $$\rho_a = m_a (\rho_{a, \text{str}} / \bar{\omega}_a)$$
Density of Axions from Strings

- Energy Spectrum
  - peak at low $k \sim (\text{horizon scale})^{-1} \sim 1/t$
  - suppressed at higher $k$
- Average energy parameter
  \[ \bar{\omega}_a = \epsilon \frac{2\pi}{t} \]
  \[ \epsilon = 4.02 \pm 0.70 \]
- Cosmic density of produced axion
  \[ \Omega_{a,\text{string}} h^2 = (7.3 \pm 3.9) \times 10^{-3} N_{DW}^2 \left( \frac{F_a}{10^{10}\text{GeV}} \right)^{1.19} \]
  \[ \Omega_{a,\text{osc}} h^2 \approx 4 \times 10^{-3} \left( \frac{F_a}{10^{10}\text{GeV}} \right)^{1.19} \]

MK, Saikawa, Sekiguchi (2014)
3.3 Axion from Domain Walls ($N_{\text{DW}} = 1$)

- Axion energy density from collapsing domain walls can be estimated in the same way as strings.
- Simulation of string-wall network
  - Lattice simulation with $N(\text{grid}) = (512)^3$
  - Scaling property
  - Average energy
- Axions from collapsed domain walls

$$\Omega_{a,\text{wall}} h^2 = (5.4 \pm 2.1) \times 10^{-3} \times \left( \frac{F_a}{10^{10} \text{GeV}} \right)^{1.19}$$

Hiramatsu, MK, Saikawa, Sekiguchi (2012)
Cosmic Axion Density ($N_{DW} = 1$)

- Total cosmic axion density

\[
\Omega_{a,\text{tot}} h^2 = \Omega_{a,\text{osc}} h^2 + \Omega_{a,\text{string}} h^2 + \Omega_{a,\text{wall}} h^2
\]

\[
= (1.7 \pm 0.4) \times 10^{-2} \left( \frac{F_a}{10^{10} \text{GeV}} \right)^{1.19}
\]

- Constraint on $F_a$

\[
F_a \lesssim (4.2 - 6.5) \times 10^{10} \text{ GeV}
\]

\[
m_a \gtrsim (0.9 - 1.4) \times 10^{-4} \text{ eV}
\]
3.4 Axion from Walls ($N_{DW} \geq 2$)

- Wall-string networks are stable and soon dominate the universe
  
  Domain Wall Problem

- The problem can be avoided by introducing a “bias” term which explicitly breaks PQ symmetry.

  \[ V_{bias} = -\Xi \eta^3 (\Phi e^{-i\delta} + h.c.) \]

  \( \Xi \): bias parameter  \( \delta \): phase of bias term

- Bias term lifts degenerated vacua and leads to DW annihilation
  
  large bias is favored

- Bias term shifts the minimum of the potential \( \langle \theta \rangle \neq 0 \) and spoils the original idea of Peccei and Quinn
  
  small bias is favored

  More stringent constraint on \( F_a \)

Axion can be dark matter for smaller \( F_a \)
3.5 Summary: case of symmetry breaking after inflation

- Axion can be dark matter of the universe for $F_a \sim 10^9$ GeV or $\sim 5 \times 10^{10}$ GeV and can be probed by the next generation experiments

MK, Saikawa, Sekiguchi (2014)
3.6 Recent Progress  

- Axion emission from defects heavily depends on scaling behavior
- scaling behavior on longer time scale?
- We have updated our simulations (from $N_{\text{grid}}=512^3$ to $4096^3$)
- scaling parameter $\xi$ increases logarithmically in time $\xi \propto \log(t/d)$
- consistent with another recent simulation  Gorghetto, Hardy, Villardoro (2018)
3.6 Recent Progress

- Energy spectrum has a peak at low $k$
- Power law $d\rho_a/d\ln k \propto k^{-1}$
- Average energy $\epsilon \approx 2 \sim 4$

- However, Gorghetto et al. obtained

$$d\dot{\rho}_a/d\ln k \propto k^{-q} \quad (q < 0)$$

Large uncertainties in the previous estimation?

- We need to understand more about underlying physics
4. Non-topological objects of axions

- Axion fluctuations can form scalar lumps like
  - Axiton (= I-ball/Oscillon)
    - I-balls/oscillons are non-topological soliton solutions existing for scalar potential flatter than $\phi^2$
  - Axion minicluster
    - dense axion dark matter halo
  - Axion star
    - star made of axions
- They could affect cosmological evolution of axion field and enhance or suppress detectability
4.1 Axiton and axion minicluster

- Axion field has large fluctuations at QCD phase transition
  \[ \frac{\delta a}{a} \sim \mathcal{O}(1) \]

- Axion potential has an I-ball/oscillon solutions (= axitons)
  - Fluctuations form axitons
  - Axitons decay into axions
  - Seeds for large density perturbations?

- Large over-density regions
  \[ L_1 \sim 0.036 \text{pc} \left( \frac{50 \mu\text{eV}}{m_a} \right)^{0.167} \]
  comoving horizon at \( H(T_1) = m_a(T_1) \)
  mini halos (= axion minicluster)
  \[ Z \sim Z_{eq} \]

Kolb, Tkachev (1994)

Vaquero, Redondo, Stadler (2018)
4.1 Axiton and axion minicluster

- **Axion minicluster**
  - density \( \rho_c = 140\delta^3(1+\delta)\rho_a(1+z_{eq})^3 \)
  - size \( L \sim 0.1L_1 \sim 0.01\text{pc} \)
  - mass \( M \sim 10^{-13}M_\odot \)

- **Axion miniclusters could be detected by microlensing**
  - \( f_{MC} = (\text{minicluster fraction of axion density}) \)
  - If \( f_{MC} \sim 1 \) direct detection is difficult since encounters with minicluster are very rare

Fairbairn et al. (2018)
4.2 Axion star

- If gravitational interaction is included, non-relativistic axion field has a stable spherical clump solution = axion star

\[ R = 388 \text{ km} \tilde{R} \]

\[ M = 3.5 \times 10^{21} \text{ g} \left( \frac{F_a}{10^{12} \text{ GeV}} \right)^2 \tilde{N} \]

- Recent simulation implies axion stars are produced in DM halos and miniclusters

\[ \tau \sim 10^9 \text{ yr} \left( \frac{m_a}{10^{-5} \text{ eV}} \right)^3 \left( \frac{v}{\text{ km/s}} \right)^6 \left( \frac{10^{20} \text{ GeV/cm}^3}{\rho} \right)^2 \]

Schiappacasse, Hertzberg (2017)
5. Axion in the Inflationary Universe (PQ before inflation)

- If PQ symmetry is broken during or before inflation
  - Strings and domain walls are diluted away by inflation
    - No domain wall problem
  - Only coherent oscillation gives a significant contribution to the cosmic density

\[
\Omega_{a,\text{osc}} \simeq 0.19 \, \theta_*^2 \left( \frac{F_a}{10^{12} \text{GeV}} \right)^{1.19}
\]

- \( \theta_* \) becomes almost homogeneous by inflation (\( \theta_* \) is a kind of free parameter)
- Isocurvature perturbation problem
5.1 Axion Isocurvature Fluctuations

- Axion acquires fluctuations during inflation

\[ \delta a = F_a \delta \theta_a \simeq \frac{H_{\text{inf}}}{2\pi} \]

- After axion obtains mass, axion fluctuations produce density perturbations

\[ \Rightarrow \frac{\delta \rho_a}{\rho_a} \simeq 2 \frac{\delta \theta_a}{\theta_*} \]

- Axion fluctuations contribute to CDM isocurvature density perturbations

\[ S = \frac{\delta \rho_{\text{CDM}}}{\rho_{\text{CDM}}} - 3 \frac{\delta \rho_\gamma}{\rho_\gamma} = \frac{\Omega_a}{\Omega_{\text{CDM}}} \frac{\delta \rho_a}{\rho_a} \]
5.1 Axion Isocurvature Fluctuations

- Isocurvature perturbations lead to CMB angular power spectrum which is different from adiabatic one

- Stringent constraint on amplitude of isocurvature perturbation

\[
\beta_{\text{iso}} \equiv \frac{P_S(k_0)}{P_\zeta(k_0) + P_S(k_0)}
\]

\[k_0 = 0.002 \text{ Mpc}^{-1}\]

- **PLANCK 2015**

\[
\beta_{\text{iso}} < 0.033 \text{ (95\% CL)}
\]
Axion isocurvature fluctuations

- Stringent constraints from CMB

If axion is dark matter

\[
H_{\text{inf}} < 2.2 \times 10^7 \text{GeV} \left( \frac{F_a}{10^{12} \text{GeV}} \right)^{0.41}
\]

Constraint from power spectrum is updated including Planck data

- Only low energy scale inflation models are allowed
  High scale inflation (\( H_{\text{inf}} > 10^{13} \text{GeV} \)) inconsistent with axion

Hikage, MK, Sekiguchi, T.Takahashi (2012)
5.2 Suppressing Isocurvature Perturbations

- Observationally high scale inflation is favored because it is testable by observing B-mode polarization of CMB

  **Tensor mode (gravitational wave) produced during inflation**

  \[ r : \text{tensor-to-scalar ratio} \]

  \[ H_{\text{inf}} = 8.6 \times 10^{13} \text{ GeV} \ (r/0.1)^{1/2} \]

  \[ r \sim 0.01 \text{ by experiments on the earth} \]

  \[ r \sim 0.001 \text{ by satellite experiments} \]

- Can we suppress isocurvature perturbations?

  ➤ **PQ scalar has a large field value during inflation**

    **Isocurvature perturbations suppressed by** \( \frac{\eta}{|\Phi|} \)

Linde (1991)
• Assuming PQ field has a large field value \( |\Phi| \sim M_p \) during inflation and axion is dark matter

\[ \Omega_a < \Omega_{DM} \]

SN1987A

\( N_{DW} = 1 \)

- Dark matter axion is consistent with high scale inflation whose tensor mode is detectable in future

MK Sonomoto Yanagida (2018)
• Are there concrete models which make PQ field value large during inflation?

• Successful models exist

  ➤ **Sextet potential**  
  Moroi, Mukaida, Nakayama, Takimoto (2014)  
  Ibe, Harigaya, MK, Yanagida (2015)

  \[
  V(\Phi) = -m_{\Phi}^2 |\Phi|^2 + \lambda_4^2 |\Phi|^4 + \frac{\lambda_6^2}{M_p^2} |\Phi|^6 - c_{H} H^2 |\Phi|^2
  \]

  ➤ **SUSY axion model**  
  MK Sonomoto (2017)
Axion-like particle

- Physics beyond the standard model like string theory predicts many axion-like particles (particles similar to axion)

- Mass $m_a$ and axion scale $F_a$ are independent theoretical parameters

\[
V \sim m_a^2 F_a^2 [1 - \cos(a/F_a)]
\]

- Many of cosmological implications of QCD axion apply to axion-like particles changing mass and axion scale
6. Conclusion

- If PQ symmetry is broken after inflation, topological defects are formed and axions from them give a significant contribution to the CDM density.

- However, recent simulations imply larger uncertainties in estimation of the present axion density.

- Fluctuations of axions leads to formation of axitons, axion miniclusters and axion stars.

- If PQ symmetry is broken before or during inflation, axion has isocurvature density perturbations which are stringently constrained by CMB observations.

- Isocurvature perturbations are suppressed if PQ scalar has a large field value during inflation.
“A decades long search for WIMPs in direct detection experiments and colliders in the most obvious regime of parameter space has so far been unsuccessful (although interesting parameter space remains available). While the most highly motivated regime of the QCD axion’s parameter space has yet to be fully probed experimentally”.

— E. D. Schiappacasse and M. P. Hertzberg (2018)
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