Estimation of nuclear matrix element of neutrinoless double-β decay based on shell model and QRPA

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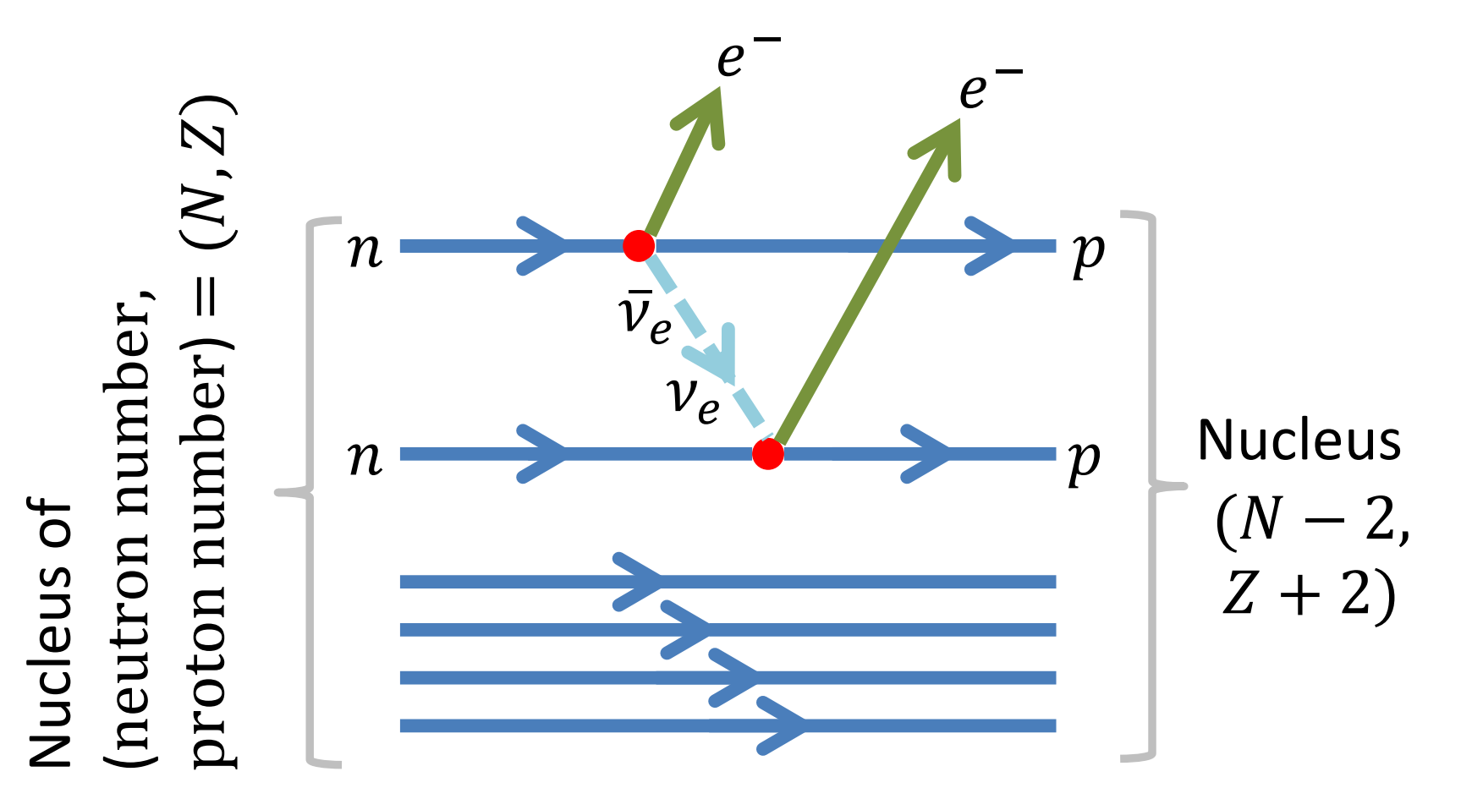
The goal is to determine the effective mass of the neutrino. The double-β decay of nucleus is used for this purpose.

Abbreviations List
ν: neutrino
NME: nuclear matrix element
0νββ: neutrinoless double-β
2νββ: two-neutrino double-β
QRPA: quasiparticle random-phase approximation
GT: Gamow-Teller
GT 0νββ NME: GT component of 0νββ-decay NME

Introduction



Possible change of two neutrons to two protons in a nucleus emitting two electrons with neutrino exchange (0νββ decay). If ν is a Majorana particle ($\nu = \bar{\nu}$), this decay occurs, and the effective neutrino mass can be determined, see the equations below. Determination of the effective neutrino mass is one of the most important subjects in modern physics.

Why nuclei?

Because Q value > 0 is necessary. $\approx E$(initial state) $- E$(final state)

Other conditions for the nuclei used in the experiments:
- Single beta decay is suppressed.
- The energy spectrum of two electrons in $2\nu\beta\beta$ decay can be distinguished well from that of $0\nu\beta\beta$.
- Large Q value.
- The parent nuclei can be produced massively with high purity.

List of nuclei used in the experiments

^{76}Ge→^{76}Se	^{130}Te→^{130}Xe	^{136}Xe→^{136}Ba
^{150}Nd→^{150}Sm	^{48}Ca→^{48}Ti	^{82}Se→^{82}Kr
^{96}Zr→^{96}Mo	^{100}Mo→^{100}Ru	^{110}Pd→^{110}Cd
^{116}Cd→^{116}Sn	^{124}Sn→^{124}Te	and more

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

U: Pontecorvo–Maki–Nakagawa–Sakata matrix
m_i: eigen mass (i=1,2,3)

$$\underbrace{1/T_{0\nu}(\mathrm{g.s.} \to \mathrm{g.s.})}_{\text{Half-life}} = \left|M^{(0\nu)}\right|^2 G_{0\nu} g_A^4 \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Half-life ← Experimental measurement
NME, Phase-space factor: Theoretical calculation
g_A: Axial-vector current coupling
$\langle m_\nu \rangle$: Effective ν mass
m_e: electron mass

Phase-space factor ← Wave functions of emitted electrons

NME ← Nuclear wave functions

Approximation is indispensable.

Accurate calculation more difficult than the phase-space factor

0νββ NME

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp'|V(r_{12},E_b)|nn'\rangle \langle 0_\mathrm{f}^+|c_{p'}^\dagger c_{n'}|b\rangle \langle b|c_p^\dagger c_n|0_\mathrm{i}^+\rangle$$

$\langle 0_\mathrm{f}^+|$: Final state, ground state of nucleus (N–2,Z+2)
$|b\rangle$: intermediate state, nucleus (N–1,Z+1)
$|0_\mathrm{i}^+\rangle$: Initial state, ground state of nucleus (N,Z)

The transition operator used in my calculation is

$$V(r_{12},E_b) \cong h_+(r_{12})\{-\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2) + g_V^2/g_A^2\}\,\tau^-(1)\tau^-(2)$$

h_+: Neutrino potential
Double-GT + Double-Fermi
g_V: vector current coupling = 1

Status

The calculated NMEs by various approximation methods and groups are distributed typically in a range of factor of 2–3. The NME cannot be obtained by experiment. Thus, examination and improvement of the calculation are essential.

Performance of shell model and QRPA

Physical feature	Shell model	QRPA
Many-particle many-hole correlations	OK	Not as much as shell model has*
Convergence of the 0νββ NME w.r.t. valence single-particle space	One major valence shell not OK	OK

* The importance of many-particle many-hole correlations depends on nucleus.

Purpose of this study

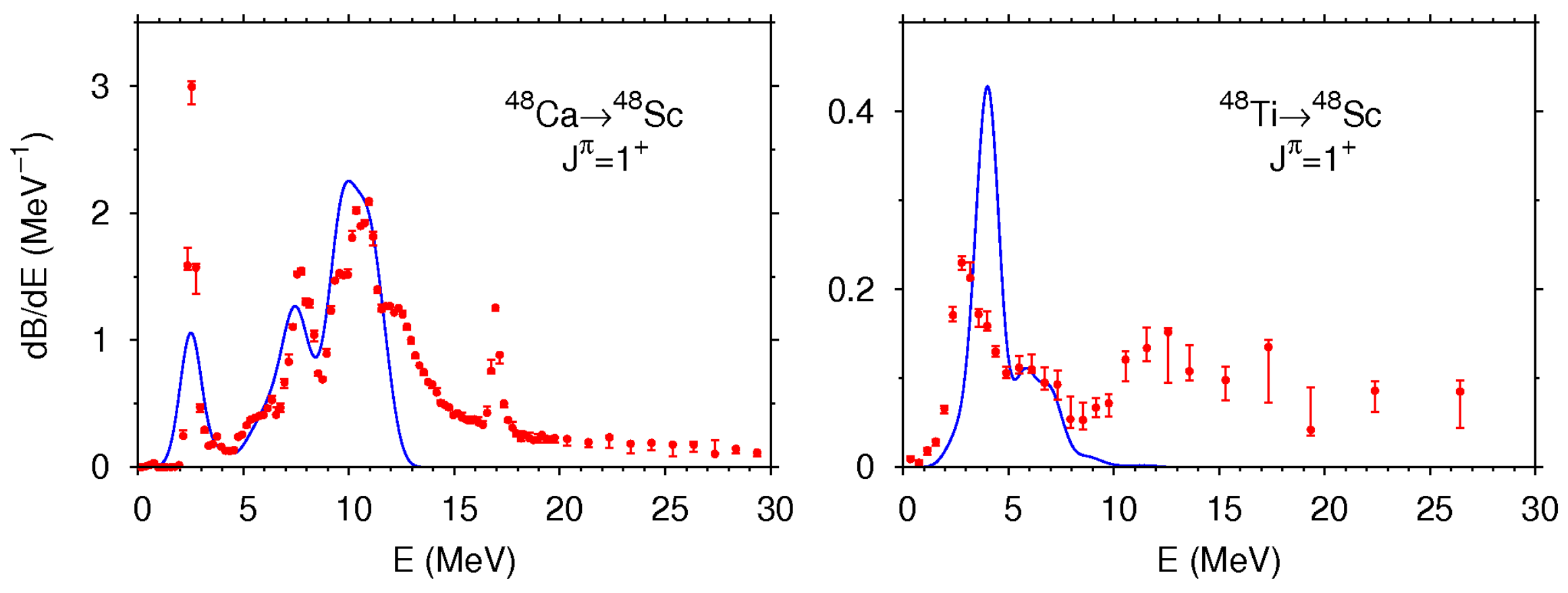
To estimate the 0νββ NME by compensating the insufficient points of the two methods phenomenologically.

1. Extrapolation of the shell-model NME toward a very large valence single-particle space by using the intermediate-state dependence of the QRPA NME
2. Multiplication of modification factors to the QRPA NME enhancing the many-particle many-hole effects. These factors are obtained by comparing the GT strength functions of the exp., the shell model and the QRPA.

Modification of shell-model GT 0νββ NME of ^{48}Ca

Step 1. Identification of the reliable energy region of the shell model

Hint a) GT strength functions of the shell model and exp.



[Y. Iwata et al., Phys. Rev. Lett. **116**, 112502 (2016)]
Blue curves: shell model with the *pf* valence shell
Red bars: exp. data [K. Yako et al, Phys. Rev. Lett., **103**, 012503 (2009)]
Abscissa: excitation energy of ^{48}Sc.

For the shell model, GT$^-$ (GT$^+$) operator with a quenching factor 0.77 is used for the left (right) figure.

For the exp. data, the possibility is pointed out [the above Yako et al.] that the isovector spin monopole operator is involved in addition to the GT operator.

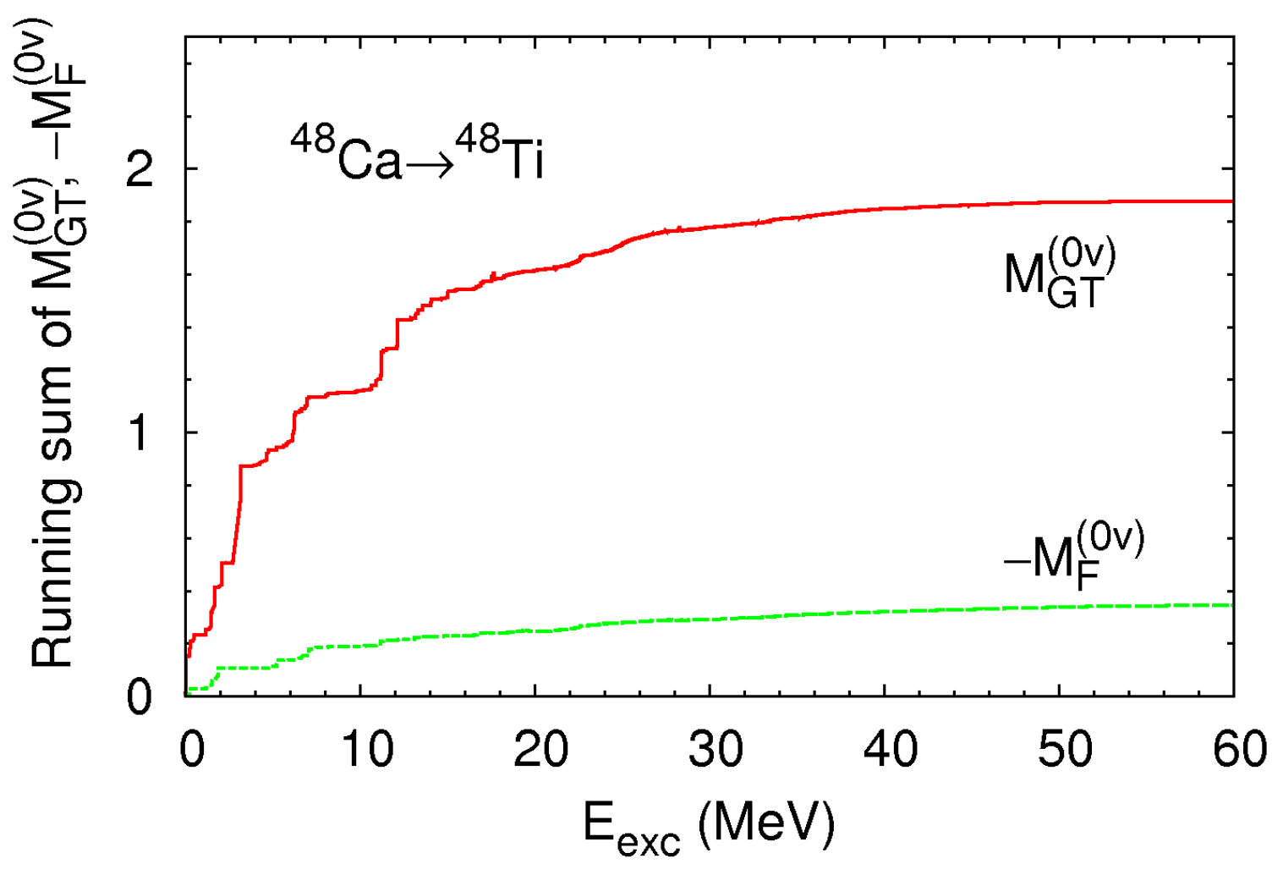
The GT strength function ⇨ The reliable energy region of the shell model (*pf*) for the 0νββ NME is < **7.5** MeV.

Because $\langle 0_\mathrm{f}^+|c_{p'}^\dagger c_{n'}|b\rangle$ and $\langle b|c_p^\dagger c_n|0_\mathrm{i}^+\rangle$ in the 0νββ NME are shared by the GT strength functions.

Hint b) Maximum particle-hole energy in the neutron *pf* and *sdpf* shell.

The max energy = **8.8** MeV (*pf*), **14.4** MeV (*sdpf*), from a Woods-Saxon spectrum.

Step 2. Referring to running sum of GT 0νββ NME of QRPA



Running sums of the GT and Fermi components of the 0νββ NME of QRPA as functions of the excitation energy of the intermediate states (^{48}Sc). No quenching factor is used.

Shell	GT comp. of 0νββ NME (shell model*)	Reliable max energy (shell model)	GT comp. of 0νββ NME (QRPA**)
pf	0.77	7.5	1.14
		8.8	1.15
sdpf	1.00	14.4	1.51
Very large			1.88

* Y. Iwata et al, previously cited. No quenching factor is used.
** J. T. Phys. Rev. C, **97**, 034304 (2018)

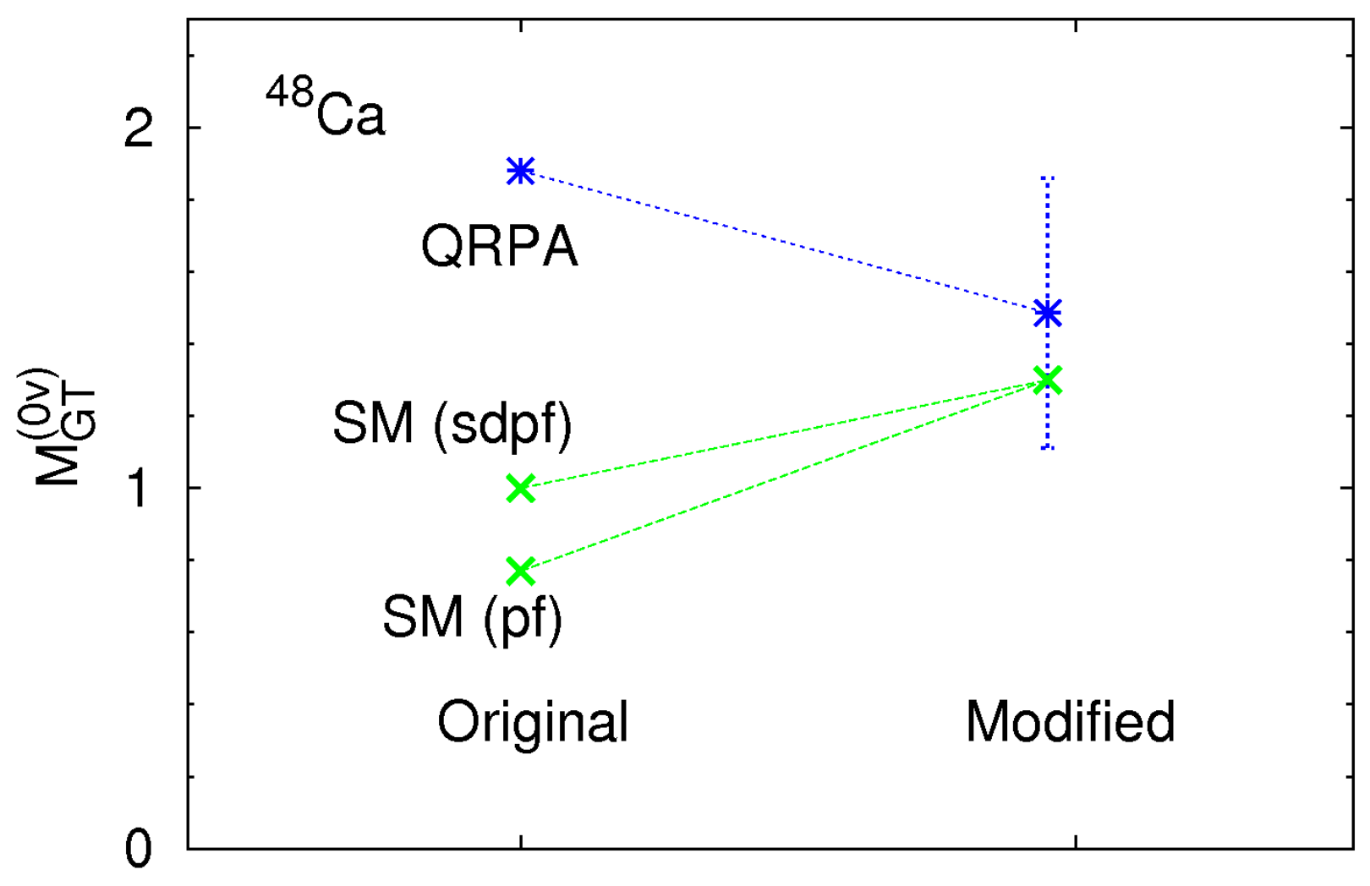
Step 3. Linear extrapolation of GT 0νββ NME of shell model

0.77 · (1.88/1.14) = 1.27,
0.77 · (1.88/1.15) = 1.26,
1.00 · (1.88/1.51) = 1.25

The average of **1.26** is the estimate of converged shell-model value w.r.t. the valence single-particle space.

For the Fermi component, the modification to the QRPA, and the 2νββ NME, see [J. T. and Y. Iwata arXiv: 2104.08250 (2021)].

Modified GT 0νββ NME



Modified 0νββ NME

The estimated 0νββ NME of ^{48}Ca by our modification method is

1.5–1.7

from the modified shell-model and QRPA values. Used are g_A=1.276 and the mean value of the GT 0νββ NME of the QRPA (see the figure).

The 0νββ NME of ^{48}Ca by various methods are in the range of

0.6–3.0.

Conclusion

It is possible to obtain the consistent values of the 0νββ NME of ^{48}Ca from the shell model and the QRPA by including the large valence space and the many-particle many-hole effects.