擬南部-ゴールドストン暗黒物質の起源と大統一理論

Pseudo-Nambu-Goldstone dark matter model inspired by grand unification

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Based on the collaboration with

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Ref. arXiv:2104.13523 [hep-ph]

Messages

What is the origin of (pNGB) dark matter ?

Does the existence of DM (+ QG assumption?) imply the grand unification?

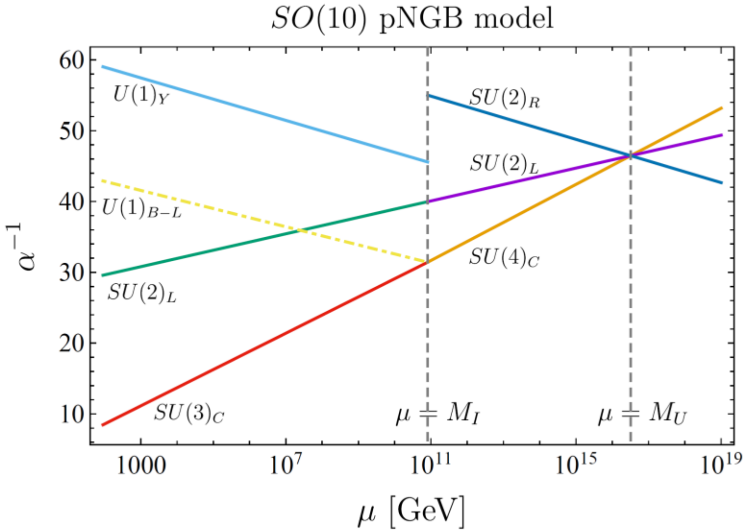
- Pseudo-Nambu-Goldstone DM model + no global symmetry

$+$ $\Rightarrow$ *gauged* $U(1)_{B-L}$ model [YA-Toma-Tsumura '20, Okada *et al.* '20]

$$\langle\Phi\rangle \sim 10^{13}\ \text{GeV}, \quad m_\chi \sim 0.5\ \text{TeV}$$

- $SO(10)$ grand unified theory

[Georgi '75, Fritzsch-Minkowski '75]

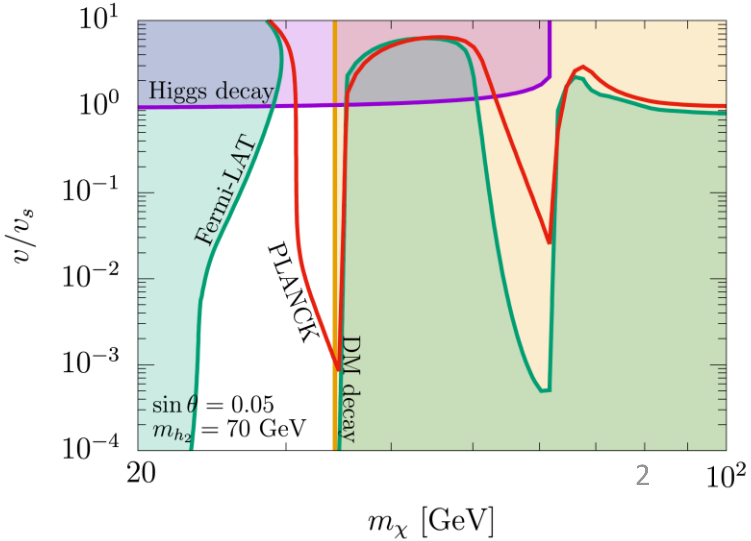


$\Rightarrow$
- Large kinetic mixing
- Intermediate scale $\cancel{U(1)_{B-L}}$ and unification scale

$$\boxed{M_I \approx \langle\Phi\rangle \approx 10^{11}\ \text{GeV} \qquad M_U \approx 10^{16}\ \text{GeV}}$$

- Mass region of DM

$$\boxed{m_\chi \lesssim \mathcal{O}(100)\ \text{GeV}}$$

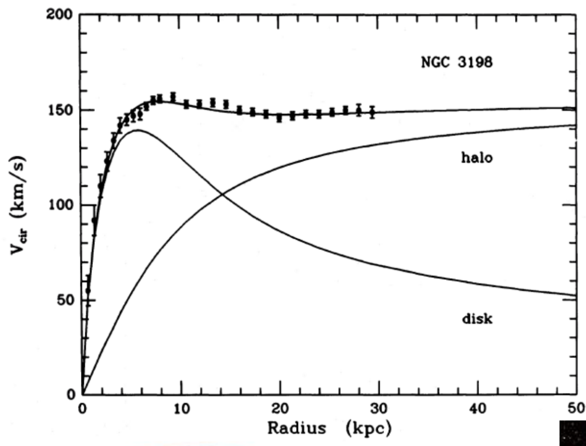


Dark Matter

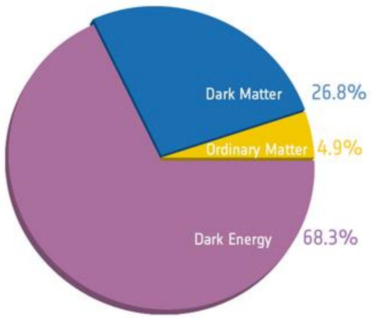
- Dark matter (DM)
  - The existence of dark matter is inferred from various observations.
  - The nature of dark matter is still unknown.
  - Identification of dark matter ⇒ big probe for BSM

$$\boxed{\Omega_{\rm DM} h^2 = 0.120 \pm 0.001}$$

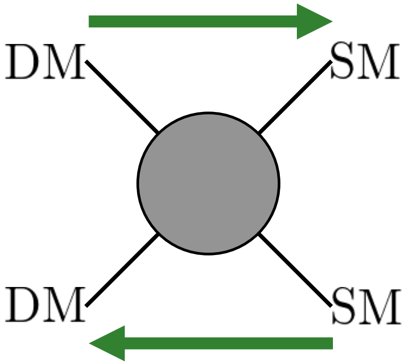
[PLANCK collaboration arXiv:1807.06209]

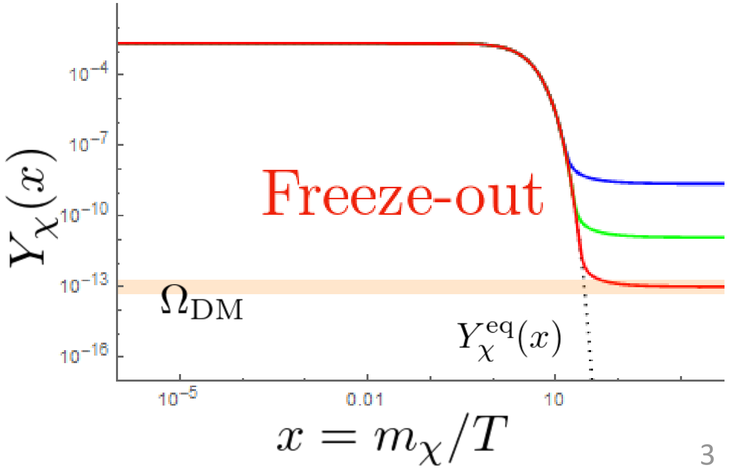


[Albada *et al.* ’85]



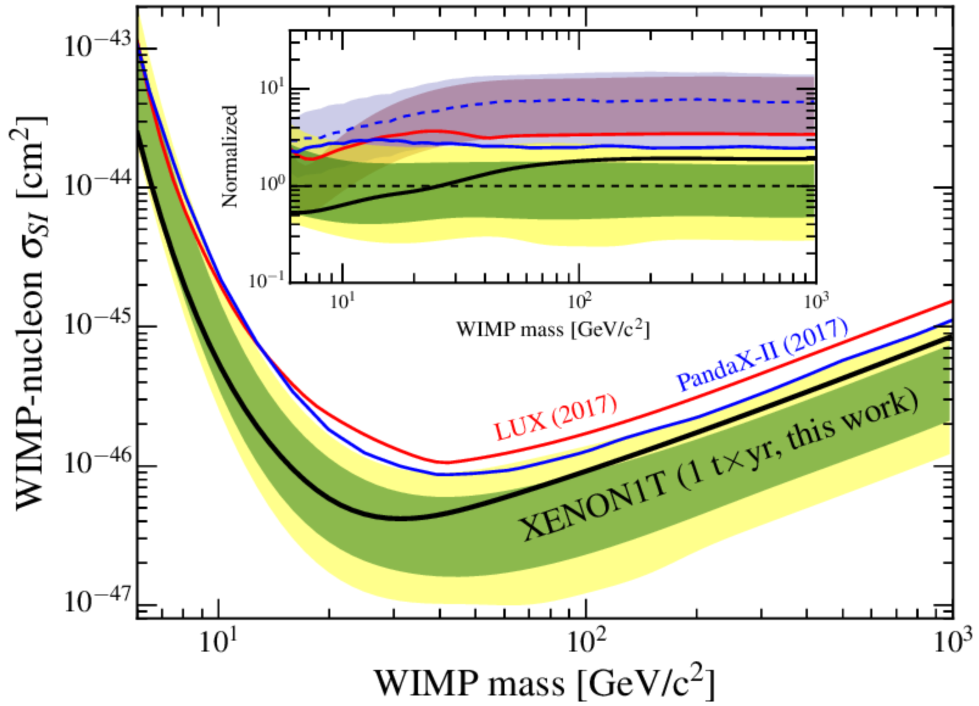
- WIMP DM
  - Dark matter relic abundance is realized as the thermal relic



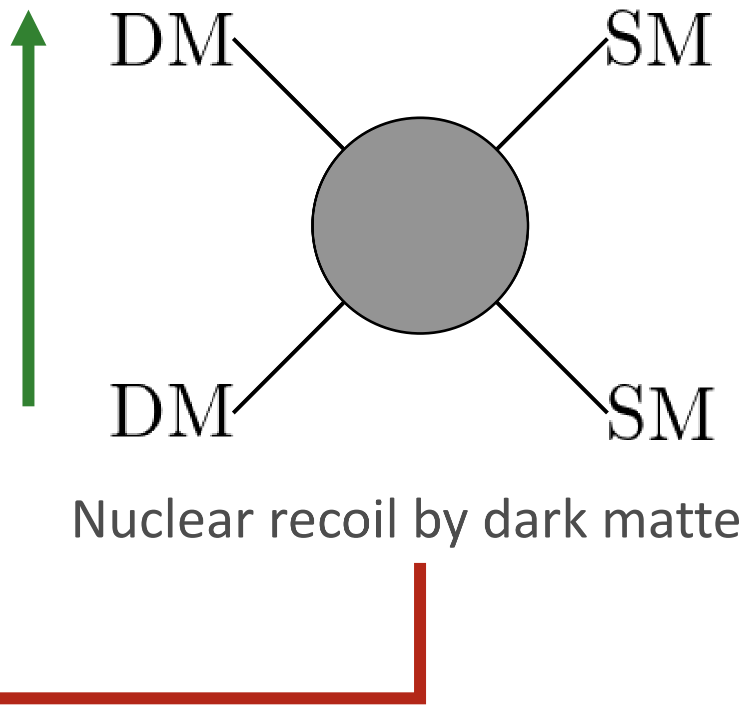


Direct detection experiments

- Direct detection experiments
  LUX, PandaX-II, XENON
  ⇒ **Severe constraints** on the WIMP-nucleon cross section



[XENON collaboration ’18]



Nuclear recoil by dark matter

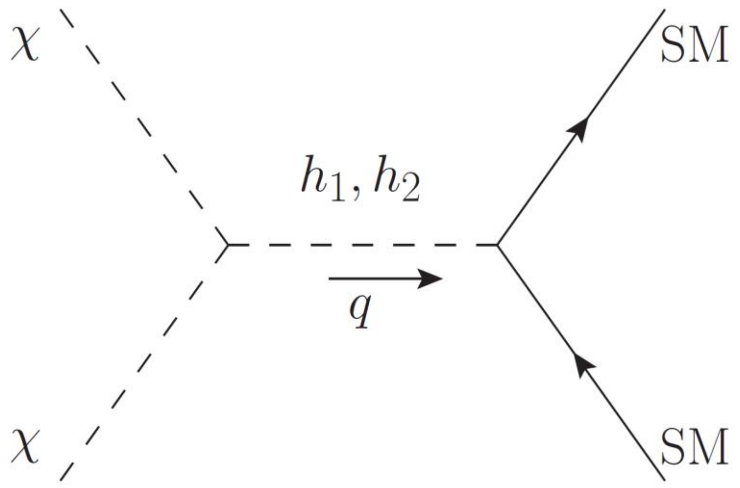
pNGB DM

- Pseudo-Nambu-Goldstone boson (pNGB) DM [Gross-Lebedev-Toma '17]
  - SM + SM singlet scalar S w/ global $U(1)_S$ softly broken to $\mathbb{Z}_2$ by $\boldsymbol{m}^2$

$$V(H,S) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2 - \frac{\boldsymbol{m^2}}{4}(S^2 + S^{*2})$$

  - Breaking of $U(1)_S$ ⇒ Nambu-Goldstone boson arise $\mathrm{Im}\, S$
  - This is an attractive WIMP candidate escaping the direct detection constraint naturally

- Scattering between pNGB DM and SM fermions



$$\boxed{i\mathcal{M}_{\chi\mathrm{SM}\to\chi\mathrm{SM}} \sim \frac{\sin\theta\cos\theta(m_{h_1}^2 - m_{h_2}^2)}{v_s m_{h_1}^2 m_{h_2}^2} q^2 \xrightarrow[q^2\to 0]{} 0}$$

Cf. soft pion theorem

pNGB DM from *gauged* $U(1)_{B-L}$

[YA-Toma-Tsumura ’20, Okada *et al.* ’20]

Origin of pNGB DM

- What is the origin of pNGB DM?
  - Where does the soft breaking term come?
  - What kind of symmetry exists behind?

$$V_{\text{soft}}(S) = -\frac{m^2}{4}(S^2 + S^{*2})$$

- Our proposal:

[YA-Toma-Tsumura ’20, Okada *et al.* ’20]

$\boxed{\text{pNGB DM from } \textit{gauged } U(1)_{B-L} \text{ model}}$ is a UV completion of the simple pNGB DM model.

The symmetry of the UV physics (consistent w/ QG) maybe *gauge symmetry*
(discrete symmetry should be ***gauged***)

[Bansk-Seiberg ’10, Vafa ’05]

	Q_L	u_R^c	d_R^c	L	e_R^c	H	ν_R^c	S	Φ
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$+1/6$	$-2/3$	$+1/3$	$-1/2$	$+1$	$+1/2$	0	0	0
$U(1)_{B-L}$	$+1/3$	$-1/3$	$-1/3$	-1	$+1$	0	$+1$	$+1$	$+2$

$$V(H,S,\Phi) \supset -\frac{\mu_c}{\sqrt{2}}(\Phi^* S^2 + \text{h.c.}) \to -\underbrace{\frac{\mu_c}{\sqrt{2}}\langle\Phi^*\rangle}_{\sim m^2} S^2 + \text{h.c.})$$

Gauged $U(1)_{B-L}$ model

[YA-Toma-Tsumura '20]

- Lagrangian

$$\mathcal{L} = |D_\mu S|^2 + |D_\mu \Phi|^2 + i\overline{\nu_{Ri}}\not{D}\nu_{Ri} - \frac{1}{4}X_{\mu\nu}^2 \boxed{- \frac{\sin\epsilon}{2}X_{\mu\nu}B^{\mu\nu}} - y_{ij}^{\nu}\tilde{H}^\dagger\overline{\nu_{Ri}}L_j - \frac{y_{ij}^{\Phi}}{2}\Phi\overline{\nu_{Ri}^c}\nu_{Rj} + \text{h.c.} + \mathcal{L}_{\text{SM}} - V(H,S,\Phi)$$

Gauge kinetic mixing of $U(1)_Y$ and $U(1)_{B-L}$

- Intuitive story of our gauged $U(1)_{B-L}$ model

Energy scale

$|d\Phi + 2ig_{B-L}X|^2$

$v_\phi \sim 10^{13}$ GeV: $U(1)_{B-L}$ is broken by v_ϕ

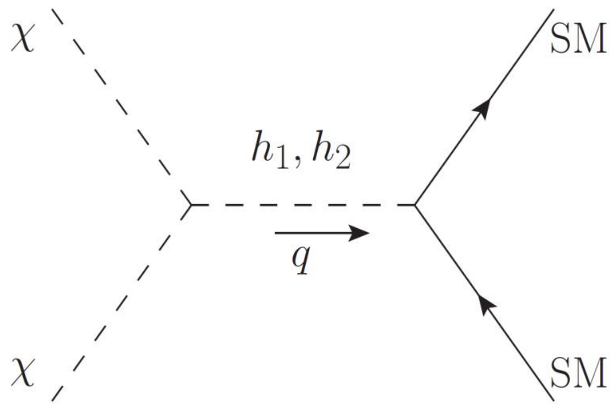
$\tilde{\chi}$ is eaten by X_μ ⇒ X_μ becomes massive $m_X \sim v_\phi$

Large VEV hierarchy ~ heavy particles decouple X_μ, ϕ

$v_s \sim$ TeV SM + singlet scalar S with S^2 term

⇒ pNGB dark matter χ + second Higgs h_2

$v \sim 246$ GeV

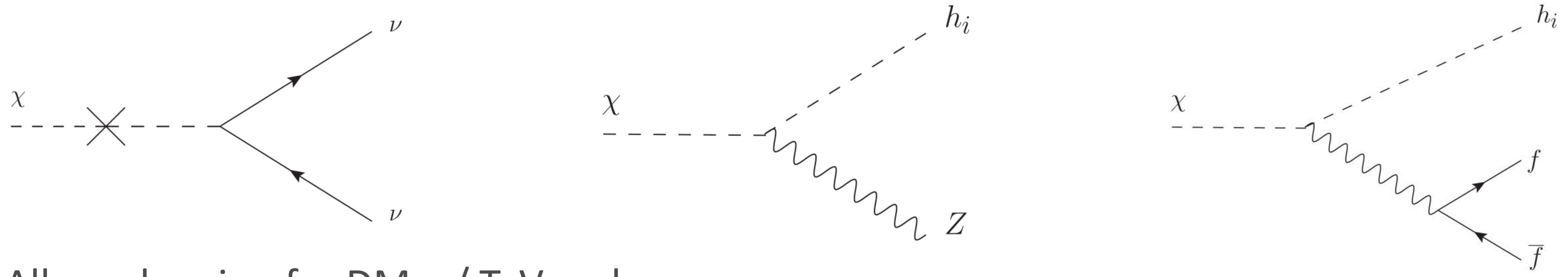


$$\boxed{i\mathcal{M} \propto -\frac{\sin\theta\cos\theta(m_{h_1}^2 - m_{h_2}^2)}{v_s m_{h_1}^2 m_{h_2}^2} q^2 + \mathcal{O}(1/v_\phi)}$$

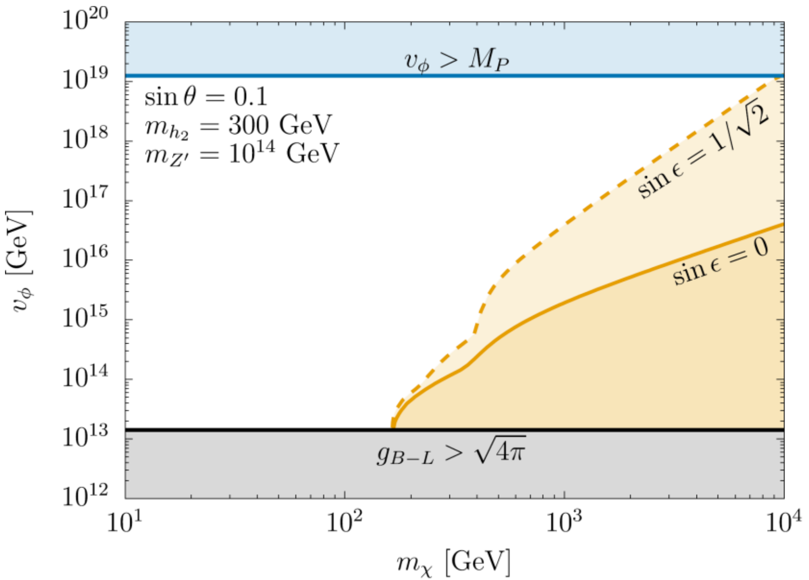
Gauged $U(1)_{B-L}$ model

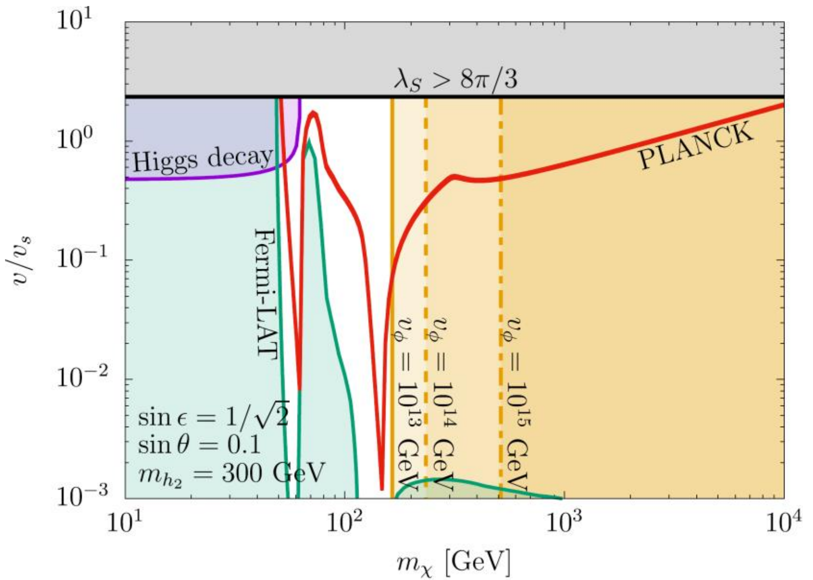
[YA-Toma-Tsumura '20]

- In return for the UV completion motivated by QG, this pNGB DM **is not stabilized**
- The new interactions and scalar mixing give the following decay processes



- Allowed region for DM w/ TeV scale mass





Constraint on the DM lifetime

$$\boxed{\tau_{\rm DM} \gtrsim 10^{27}\ {\rm s}}$$

[Baring-Ghosh-Queiroz-Sinha '16]

pNGB DM inspired by grand unification

[YA-Toma-Tsumura-Yamatsu ’21]

$SO(10)$ pNGB DM model

- What is the further UV completion of the gauged $U(1)_{B-L}$ pNGB DM model?

[YA-Toma-Tsumura '20, Okada *et al.* '20]

$$\langle \Phi \rangle \sim 10^{13}\ \mathrm{GeV}$$

- Charge quantization of $U(1)_Y$, anomaly cancellation in SM

[Gerogi-Glashow '74]

$\Rightarrow$ Grand unification theory (GUT) $M_U \sim 10^{15\text{–}17}\ \mathrm{GeV}$

- The symmetry $G_{\mathrm{SM}} \times U(1)_{B-L}$ can appear in the GUT symmetry breaking pattern

Motivation

Can the pNGB DM model be embedded to GUT?
Does the existence of DM (+ QG assumption?) imply the grand unification?

- In this work, we focus on the Pati-Salam gauge group G_{PS} [Pati-Salam '74]

$$G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \supset G_{\mathrm{SM}} \times U(1)_{B-L}$$

$SO(10)$ pNGB DM model

- Particle contents of our model

	Ψ_{16}						Φ_{10}	Φ_{16}	$\Phi_{\overline{126}}$
$SO(10)$	$\mathbf{16}$						$\mathbf{10}$	$\mathbf{16}$	$\overline{\mathbf{126}}$
	$\psi_{(\mathbf{4},\mathbf{2},\mathbf{1})}$		$\psi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$				$\phi_{(\mathbf{1},\mathbf{2},\mathbf{2})}$	$\phi_{(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})}$	$\phi_{(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})}$
G_{PS}	$(\mathbf{4},\mathbf{2},\mathbf{1})$		$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$				$(\mathbf{1},\mathbf{2},\mathbf{2})$	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{10}},\mathbf{1},\mathbf{3})$
	Q_L	L	u_R^c	d_R^c	e_R^c	ν_R^c	H	S	Φ
$SU(3)_C$	$\mathbf{3}$	$\mathbf{1}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$+1/6$	$-1/2$	$-2/3$	$+1/3$	$+1$	0	$+1/2$	0	0
$U(1)_{B-L}$	$+1/3$	-1	$-1/3$	$-1/3$	$+1$	$+1$	0	$+1$	$+2$

- Symmetry breaking pattern

$$SO(10) \xrightarrow{\langle\Phi_{210}\rangle\neq 0} \boxed{G_{\mathrm{PS}}(\supset G_{\mathrm{SM}}\times U(1)_{B-L}) \xrightarrow{\langle\Phi_{\overline{126}}\rangle\neq 0} G_{\mathrm{SM}} \xrightarrow{\langle\Phi_{10}\rangle\neq 0} SU(3)_C\times U(1)_{\mathrm{EM}}}$$

$$G_{\mathrm{SM}}\times U(1)_{B-L} \xrightarrow{\langle\Phi\rangle\neq 0} G_{\mathrm{SM}} \xrightarrow{\langle H\rangle\neq 0} SU(3)_C\times U(1)_{\mathrm{EM}}$$

[YA-Toma-Tsumura '20]

$SO(10)$ pNGB DM model

- Gauge kinetic mixing ~ mixing angle (cf. Weinberg angle in the SM)

$$G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \xrightarrow{\langle \Phi_{\overline{\mathbf{126}}} \rangle \neq 0} SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\cup \qquad\qquad\qquad \cup$$

$$U(1)_{B-L} \qquad \times \qquad U(1)_R$$

Gauge fields

$$\begin{array}{c} U(1)_Y \\ U(1)_{B-L} \end{array} : \begin{pmatrix} B_\mu \\ X_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\epsilon \\ 0 & 1/\cos\epsilon \end{pmatrix} \begin{pmatrix} B'_\mu \\ C'_\mu \end{pmatrix} : \begin{array}{c} U(1)_R \\ U(1)_{B-L} \end{array}$$

U(1) generator (in GUT)

$$I_Y = \sqrt{\frac{3}{5}} I_{3R} + \sqrt{\frac{2}{5}} I_{B-L}$$

- The gauge kinetic mixing angle is determined as

$$\boxed{\epsilon = -\arctan\sqrt{\frac{2}{3}}}$$

$$\mathcal{L} \supset -\frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} X_{\mu\nu}^2 - \frac{\sin\epsilon}{2} X_{\mu\nu} B^{\mu\nu}$$

$SO(10)$ pNGB DM model

- Gauge coupling unification
  - RGE $\dfrac{d}{d\log\mu}\alpha_i^{-1}(\mu) = -\dfrac{b_i}{2\pi}$

$$b_i = -\frac{11}{3}\sum_{\text{vector}} T(R_V) + \frac{2}{3}\sum_{\text{Weyl}} T(R_F) + \frac{1}{6}\sum_{\text{Real}} T(R_S)$$

# of particles and their rep. determines the RG flow of gauge couplings.

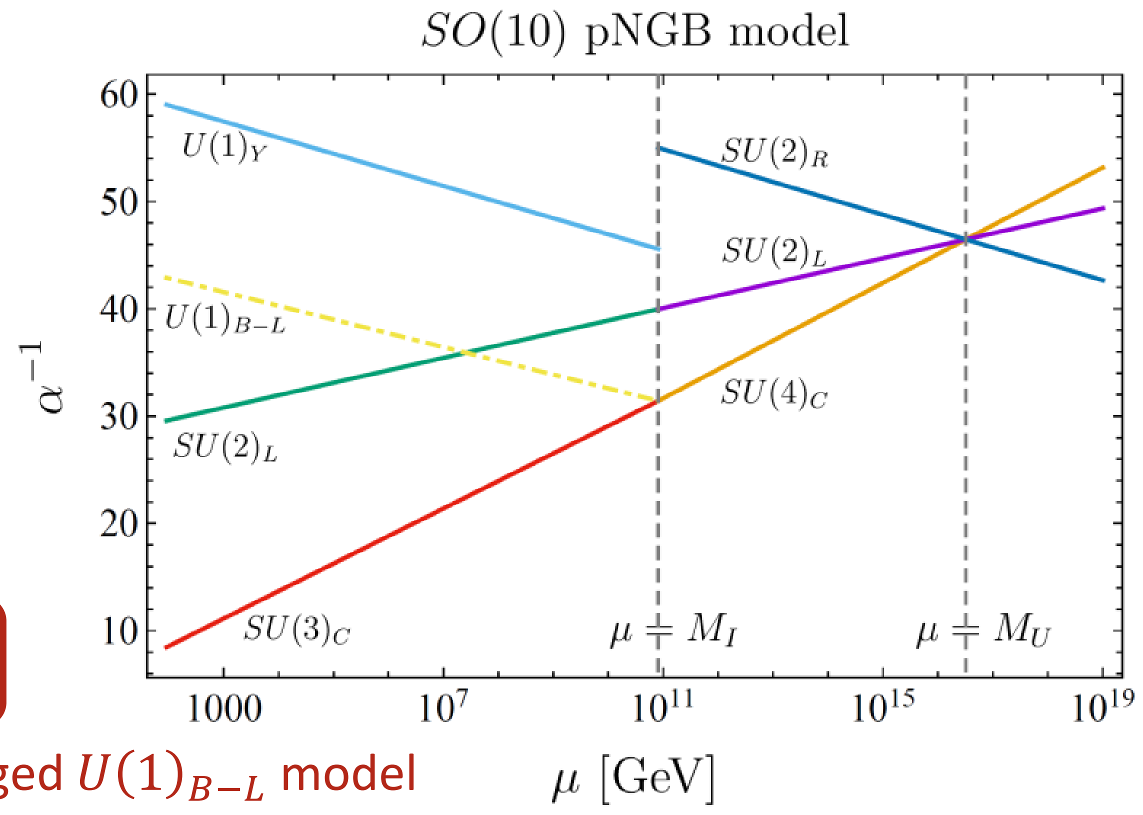
$\mu \le M_I$

$$\begin{pmatrix} b_{3C} \\ b_{2L} \\ b_{1Y} \end{pmatrix} = \begin{pmatrix} -7 \\ -19/6 \\ +41/10 \end{pmatrix}$$

$$g_{B-L}(M_I) = 0.3872$$

$$\boxed{M_I = 10^{10.91}\ \text{GeV}}$$

Smaller than that in gauged $U(1)_{B-L}$ model
→ DM tends to be short-lived.



$M_I \le \mu \le M_U$

$$\begin{pmatrix} b_{4C} \\ b'_{2L} \\ b_{2R} \end{pmatrix} = \begin{pmatrix} -22/3 \\ -19/6 \\ +25/6 \end{pmatrix}$$

$$\boxed{\begin{aligned} M_U &= 10^{16.51}\ \text{GeV}, \\ \alpha_U^{-1}(M_U) &= 46.47 \end{aligned}}$$

Cf. proton decay

[Super-Kamiokande Collaboration '10]

$SO(10)$ pNGB DM model

Gauged $U(1)_{B-L}$ model [ATT 20]

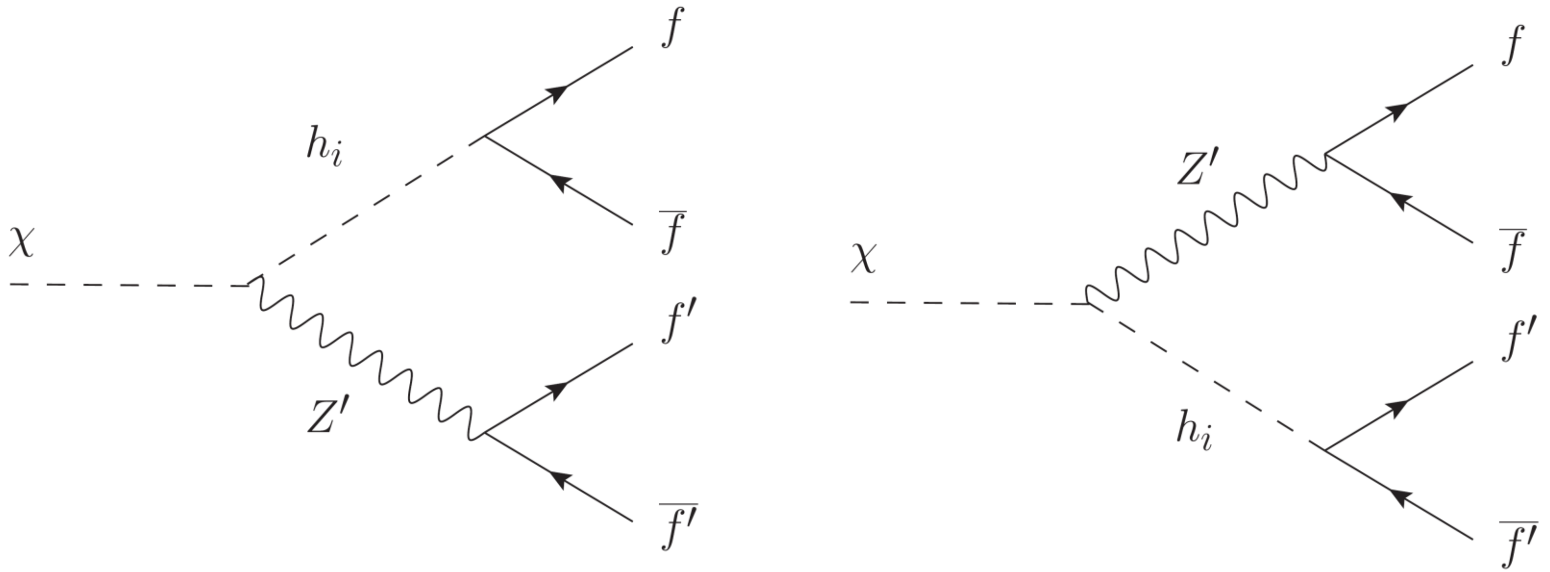
$m_\chi \sim 1 \text{ TeV}$ Three-body decay dominant

$v_\phi \sim M_I \sim 10^{13} \text{ GeV}$

- Long-lived DM in $SO(10)$ pNGB DM model

DM should be lighter than that in the previous work.

- The following four-body decay process is dominant to the DM decay



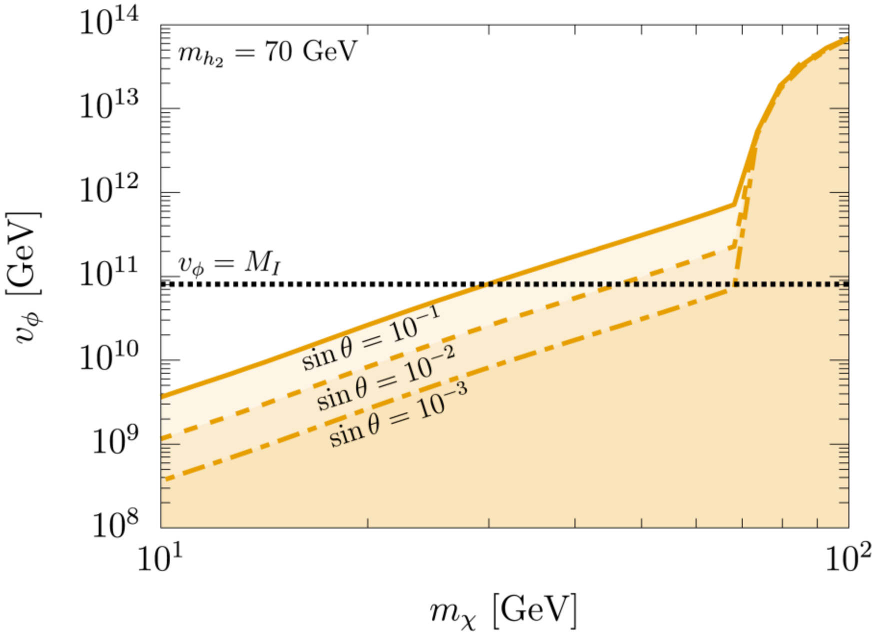
- DM lifetime constraint

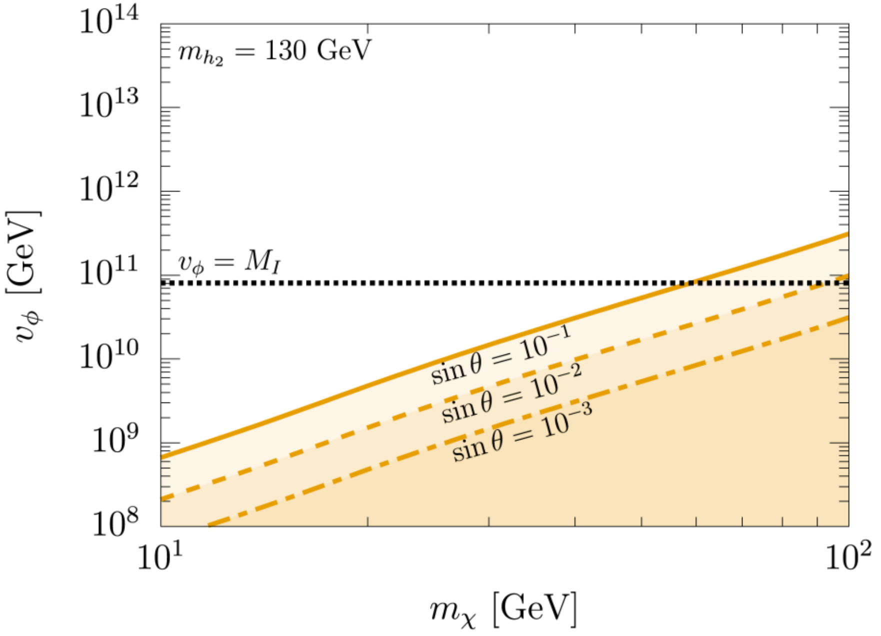
$$\boxed{\tau_{\rm DM} \gtrsim 10^{27} \text{ s}}$$

[Baring-Ghosh-Queiroz-Sinha '16]

Long-lived DM in $SO(10)$ pNGB DM model

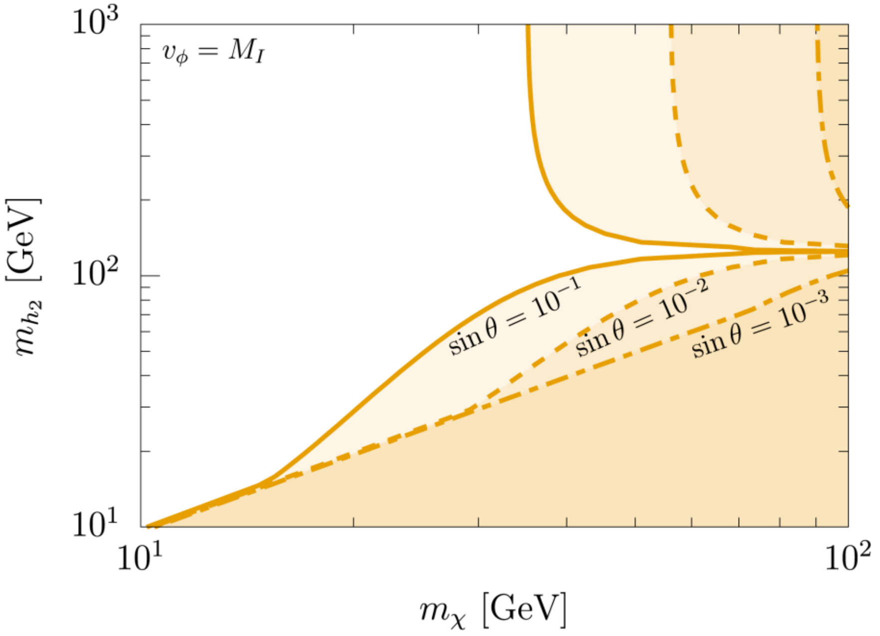
Parameter space in (m_χ, v_ϕ)-plane





θ: mixing angle of SM-like Higgs and second Higgs

Parameter space in (m_χ, m_{h_2})-plane

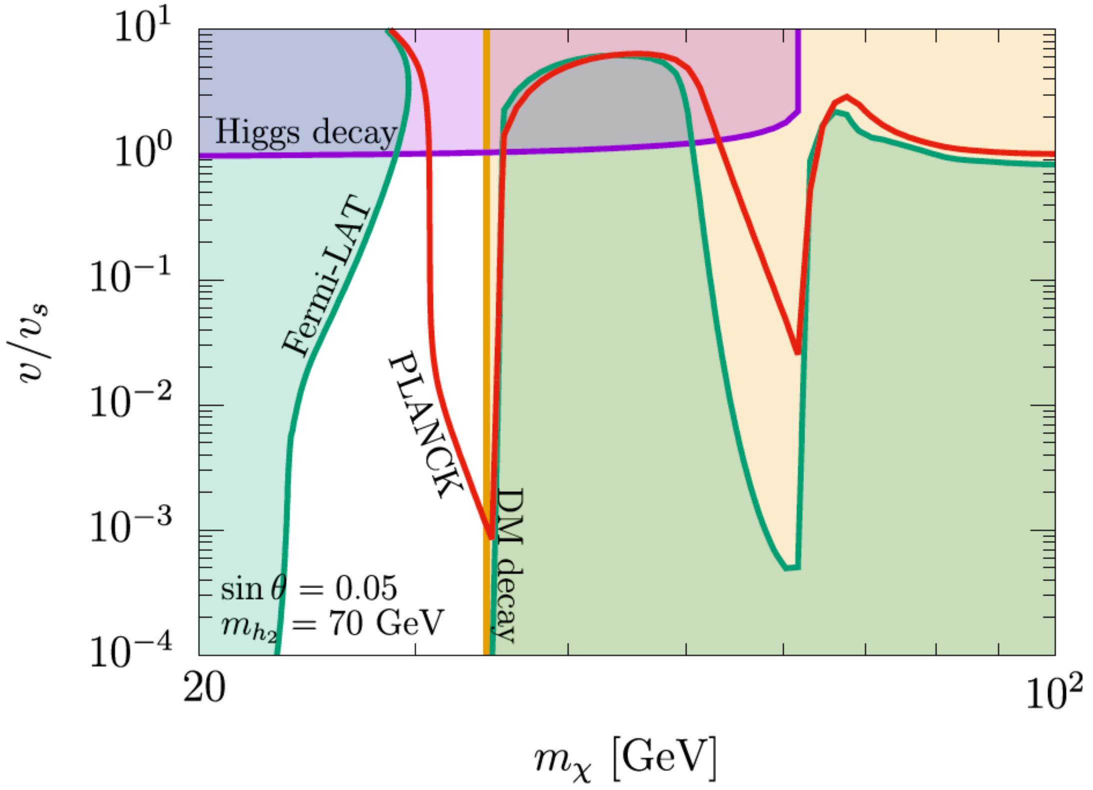


pNGB DM model inspired by grand unification

- Allowed region in $(m_\chi, v/v_s)$ plane realizing the DM relic abundance

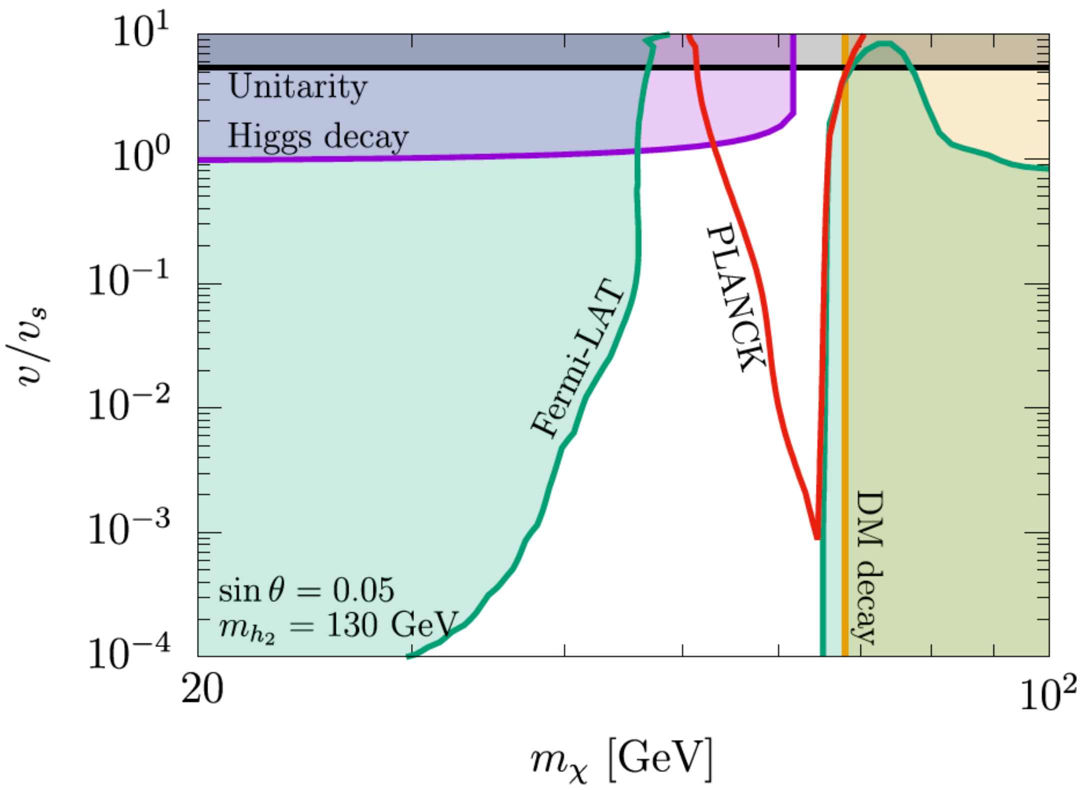
$\sin\theta = 0.05$

$m_{h_2} = 70$ GeV



$\sin\theta = 0.05$

$m_{h_2} = 130$ GeV



Summary

- We proposed an $SO(10)$ pNGB DM model in the framework of GUTs
- The gauged $U(1)_{B-L}$ pNGB DM model can be embedded to $SO(10)$ GUT

$$H \subset \Phi_{\mathbf{10}}, \quad S \subset \Phi_{\mathbf{16}}, \quad \Phi \subset \Phi_{\overline{\mathbf{126}}}$$

- The grand unification condition requires the following relations

$$\sin\epsilon = -\sqrt{2/5}, \quad v_\phi \approx 10^{11}\ \mathrm{GeV}, \quad m_\chi \lesssim \mathcal{O}(100)\ \mathrm{GeV}$$

- We find that the thermal relic abundance can be consistent with all the constraints when the DM mass is rather close to the resonances

Backup

Gauged $U(1)_{B-L}$ model

[YA-Toma-Tsumura '20]

- Scalar potential of gauged $U(1)_{B-L}$ model

$$V(H,S,\Phi) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 - \frac{\mu_\Phi^2}{2}|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \frac{\lambda_\Phi}{2}|\Phi|^4 + \lambda_{HS}|H|^2|S|^2 + \lambda_{H\Phi}|H|^2|\Phi|^2 + \lambda_{S\Phi}|S|^2|\Phi|^2 - \left(\frac{\mu_c}{\sqrt{2}}\Phi^* S^2 + \text{c.c.}\right)$$

- Parametrization of the scalars

$$H = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}, \quad S = \frac{v_s + s + i\eta_s}{\sqrt{2}}, \quad \Phi = \frac{v_\phi + \phi + i\eta_\phi}{\sqrt{2}}$$

- Type I see-saw mechanism identifies the scale of VEV of Φ

$$\boxed{v_\phi \sim 4.3 \times 10^{14}\ \text{GeV}\left(\frac{y^{\nu 2}}{y^\Phi}\right)} \gg v,\ v_s$$

Mass spectrum of scalar sectors

- Mass eigenstates
  - CP-even scalars

$$\tan 2\theta \approx \frac{2vv_s(\lambda_{HS}\lambda_\Phi - \lambda_{H\Phi}\lambda_{S\Phi})}{v^2(\lambda_{H\Phi}^2 - \lambda_H\lambda_\Phi) - v_s^2(\lambda_{S\Phi}^2 - \lambda_S\lambda_\Phi)}$$

SM-like Higgs boson

$$\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & \frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} \\ 0 & 1 & \frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} \\ -\frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} & -\frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

$$m_{h_1}^2 \approx \lambda_H v^2 - \frac{\lambda_{H\Phi}^2\lambda_S - 2\lambda_{HS}\lambda_{H\Phi}\lambda_{S\Phi} + \lambda_\Phi\lambda_{HS}^2}{\lambda_{S\Phi} - \lambda_{S\Phi}^2}v^2, \quad \leftarrow \text{125 GeV}$$

$$m_{h_2}^2 \approx \frac{\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2}{\lambda_\Phi}v_s^2 + \frac{(\lambda_\Phi\lambda_{HS} - \lambda_{H\Phi}\lambda_{S\Phi})^2}{\lambda_\Phi(\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2)}v^2, \quad m_{h_3}^2 \approx \lambda_\Phi v_\phi^2$$

  - CP-odd scalars

pNGB (dark matter)

$$\begin{pmatrix} \eta_s \\ \eta_\phi \end{pmatrix} = \frac{1}{(v_s^2 + 4v_\phi^2)^{1/2}} \begin{pmatrix} 2v_\phi & v_s \\ -v_s & 2v_\phi \end{pmatrix} \begin{pmatrix} \chi \\ \boxed{\tilde{\chi}} \end{pmatrix}$$

Eaten by X_μ

$$\boxed{m_\chi^2 = \frac{\mu_c(v_s^2 + 4v_\phi^2)}{4v_\phi}}$$

Gauge kinetic mixing

- E_μ: gauge field of $U(1) \subset SU(4)_C$, $W_\mu'^3$: gauge filed of $U(1) \subset SU(2)_R$
- Lagrangian

$$\mathcal{L} \supset -\frac{1}{4}(E_{\mu\nu})^2 - \frac{1}{4}(W_{\mu\nu}'^3)^2 + \frac{1}{2}\left(\frac{v_s^2}{4} + v_\phi^2\right)\left(2g_{B-L}E_\mu - g_R W_\mu'^3\right)^2$$

$$= -\frac{1}{4}(B_{\mu\nu}')^2 - \frac{1}{4}(C_{\mu\nu}')^2 + \frac{1}{2}M_{C'}^2 C_\mu'^2$$

$$= -\frac{1}{4}(B_{\mu\nu})^2 - \frac{1}{4}(C_{\mu\nu})^2 - \frac{\sin\epsilon}{2}B_{\mu\nu}C^{\mu\nu} + \frac{1}{2}M_C^2 C_\mu^2$$

- Relations of gauge fields

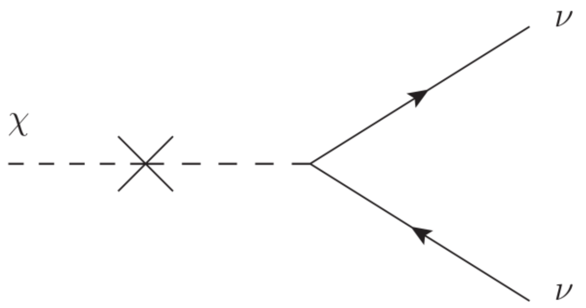
$$\begin{pmatrix} B_\mu' \\ C_\mu' \end{pmatrix} = \begin{pmatrix} \cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} W_\mu'^3 \\ E_\mu \end{pmatrix}, \quad \begin{pmatrix} B_\mu \\ C_\mu \end{pmatrix} = \begin{pmatrix} 1 & -\tan\epsilon \\ 0 & 1/\cos\epsilon \end{pmatrix} \begin{pmatrix} B_\mu' \\ C_\mu' \end{pmatrix}, \quad \begin{pmatrix} W_\mu'^3 \\ E_\mu \end{pmatrix} = \begin{pmatrix} \cos\epsilon & 0 \\ \sin\epsilon & 1 \end{pmatrix} \begin{pmatrix} B_\mu \\ C_\mu \end{pmatrix}$$

- Covariant derivative

$$D_\mu \supset \mathrm{i}g_{B-L}E_\mu Q_{B-L} + \mathrm{i}g_R W_\mu'^3 I_3^{SU(2)_R} = \mathrm{i}g_{B-L}C_\mu Q_{B-L} + \mathrm{i}g_1 B_\mu Q_Y$$

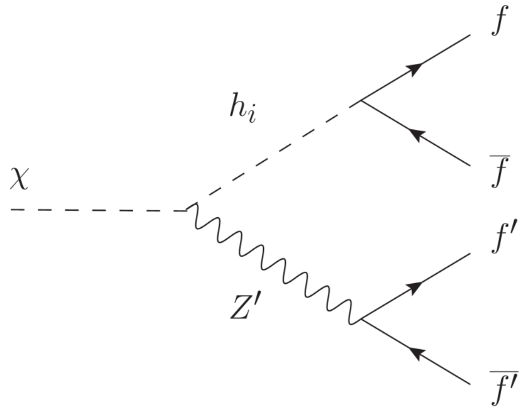
Decay width

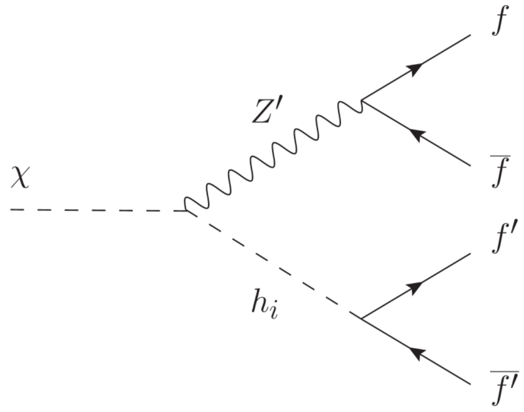
- Tow-body decay



$$\Gamma_{\nu\nu} = \frac{m_\chi}{64\pi}\frac{v_s^2}{v_\phi^4}\sum_i m_{\nu_i}^2$$
$$= 5\times 10^{-59}\ \mathrm{GeV}\left(\frac{m_\chi}{100\ \mathrm{GeV}}\right)\left(\frac{v_s}{1\ \mathrm{TeV}}\right)^2\left(\frac{10^{11}\ \mathrm{GeV}}{v_\phi}\right)^4\sum_i\left(\frac{m_{\nu_i}}{0.1\ \mathrm{eV}}\right)^2$$

- Four-body decay





$$\Gamma_{\text{4-body}} \sim \sum_{f,f'}\frac{1}{2(4\pi)^5\times 1440}g_{B-L}^2(Q_{B-L}^S)^2(U_{CZ'}^G)^2\frac{m_\chi^9}{m_{Z'}^4}\left(\frac{1}{m_{h_1}^2}-\frac{1}{m_{h_2}^2}\right)^2\sin^2 2\theta\left[y_f^2\left((b_V^{f'})^2+(b_A^{f'})^2\right)+y_{f'}^2\left((b_V^{f})^2+(b_A^{f'})^2\right)\right]$$

Path of $SO(10) \to SU(3)_C \times U(1)_{\mathrm{em}}$

$$
\begin{tikzcd}
SO(10) \arrow[d, "\mathbf{54},\, \mathbf{210}"] \arrow[drr, "\mathbf{45},\, \mathbf{210}"] & & \\
SU(4)_C \times SU(2)_L \times SU(2)_R \arrow[rr, "\mathbf{45},\, \mathbf{210}"] \arrow[d, "\mathbf{45},\, \mathbf{210}"] & & SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \arrow[dll, "\mathbf{45},\, \mathbf{210}"] \\
SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \arrow[d, "\mathbf{16},\, \mathbf{126} \Leftrightarrow \Phi_{\mathbf{16}}(\ni S),\, \Phi_{\overline{\mathbf{126}}}(\ni \Phi)"] & & \\
SU(3)_C \times SU(2)_L \times U(1)_Y \arrow[d, "\mathbf{10} \Leftrightarrow \Phi_{\mathbf{10}}(\ni H)"] & & \\
SU(3)_C \times U(1)_{\mathrm{em}} & &
\end{tikzcd}
$$

Our model

[YA-Toma-Tsumura-Yamatsu ’21]

$$
\begin{tikzcd}
SO(10) \arrow[d, "\mathbf{210}"] \\
G_I = G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \arrow[d, "\overline{\mathbf{126}},\, \Phi_{\overline{\mathbf{126}}} \ni \Phi"] \\
G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y \arrow[d, "\mathbf{10},\, \Phi_{\mathbf{10}} \ni H"] \\
SU(3)_C \times U(1)_{\mathrm{em}}
\end{tikzcd}
$$