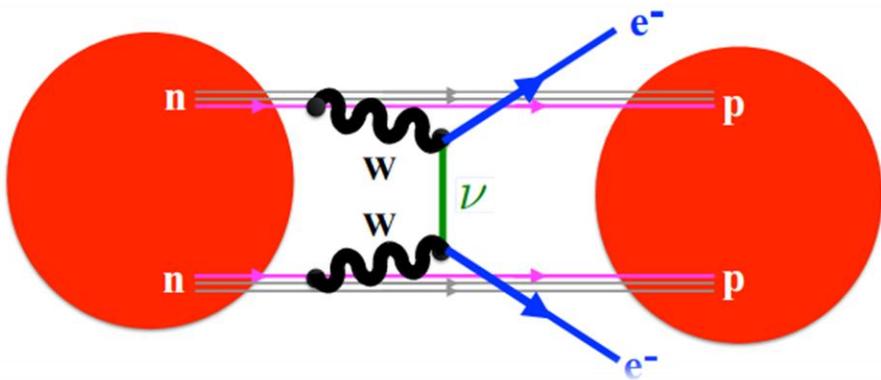


二重β崩壊の核行列要素



Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)		
	I	II	III		
mass	≈2.2 MeV/c ²	≈1.28 GeV/c ²	≈173.1 GeV/c ²	0	≈124.97 GeV/c ²
charge	2/3	2/3	2/3	0	0
spin	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H higgs
	d down	s strange	b bottom	γ photon	
	e electron	μ muon	τ tau	Z Z boson	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

QUARKS (rows 1-3)
LEPTONS (rows 4-6)
GAUGE BOSONS VECTOR BOSONS (rows 7-8)
SCALAR BOSONS (row 9)

岩田順敬

大阪経済法科大学

Outlines

Neutrinoless double beta decay ($0\nu\beta\beta$)



Nuclear matrix element (NME)



International NME research CORE



The screenshot shows the website for the NME2023 workshop. On the left is a logo featuring a city skyline at night. To the right of the logo, the text reads "NME2023" in large blue letters, followed by "RCNP Workshop for 'Theoretical and Experimental Approaches for Nuclear Matrix Elements of Double Beta Decay'" in a smaller, italicized font. In the top right corner, there is a blue button with the text "日本語". Below the main text, there is a horizontal navigation bar with five buttons: "Index" (highlighted in blue), "Bulletin", "Site", "Program", and "Registration".

Absolute Majorana Neutrino Masses

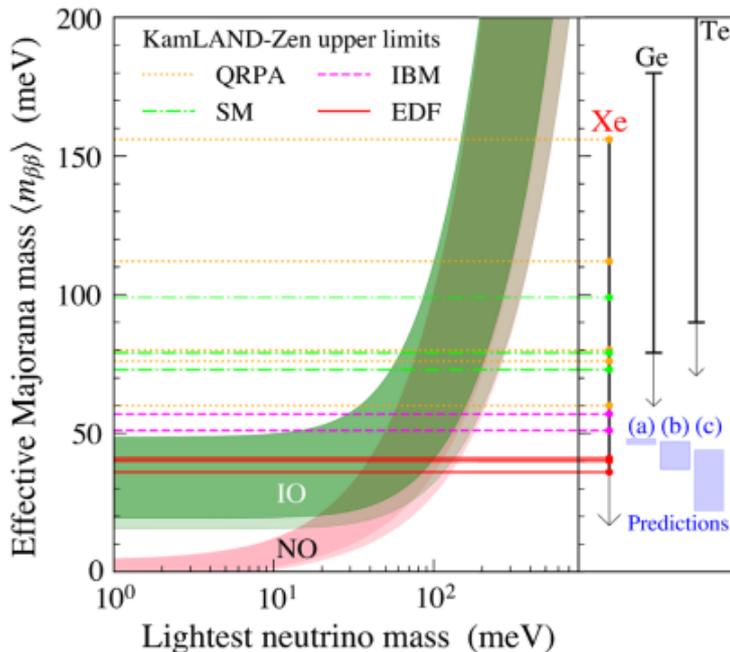
Decay rate of $0\nu\beta\beta$

$$\Gamma^{0\nu} = \frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q, Z) |m_{\beta\beta}|^2 |M^{0\nu}|^2$$

Majorana neutrino mass ($m_{\beta\beta}$):

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \\ \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e^{1/2i\alpha_1} & 0 & 0 \\ 0 & e^{1/2i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$,
 $0 \leq \theta_{ij} \leq \pi/2$, $0 \leq \delta_{13} \leq 2\pi$, α_1 and α_2 are Majorana
 CP-violating phases.

Upper limit for $m_{\beta\beta}$ of 36-156 meV has been determined from $0\nu\beta\beta$ decay experiment of ^{136}Xe at KamLAND-Zen (PHYSICAL REVIEW LETTERS 130, 051801 (2023)) with lower limit of $T_{1/2}^{0\nu}$ 2.3×10^{26} yr using different nuclear matrix elements

Neutrinoless Double Beta Decay ($0\nu\beta\beta$)

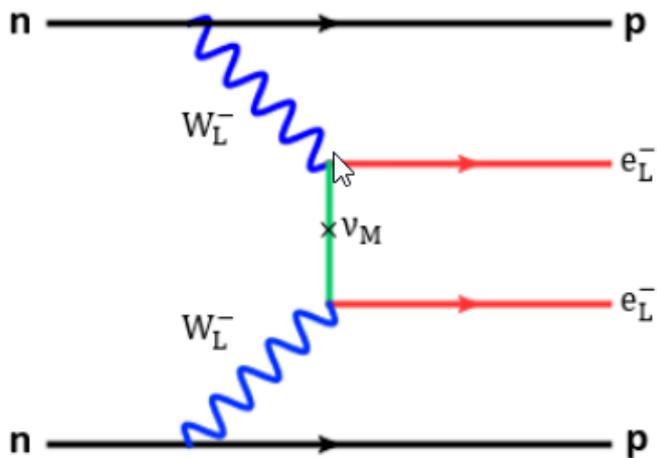


Fig1: Feynman diagram for light neutrino exchange $0\nu\beta\beta$

**Neutrinoless
Double Beta decay**

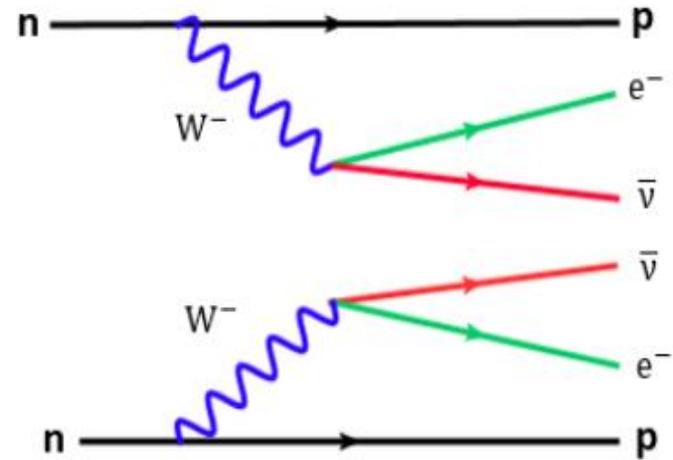
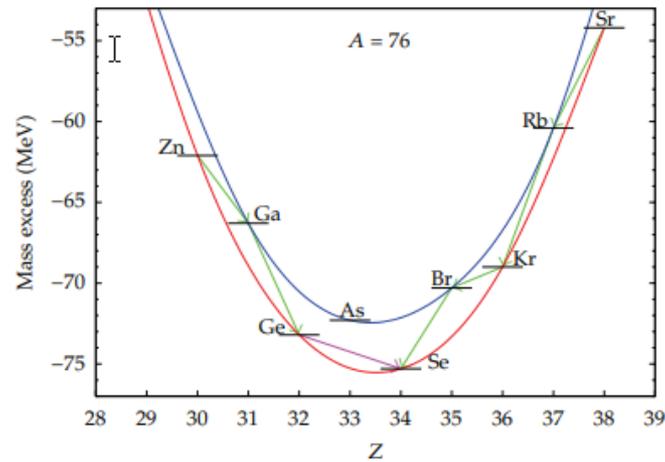


Fig 3: Feynman diagram for two-neutrino double beta decay

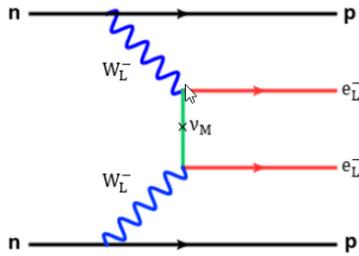
**Two-Neutrino
Double Beta decay**

Observed in
the experiment



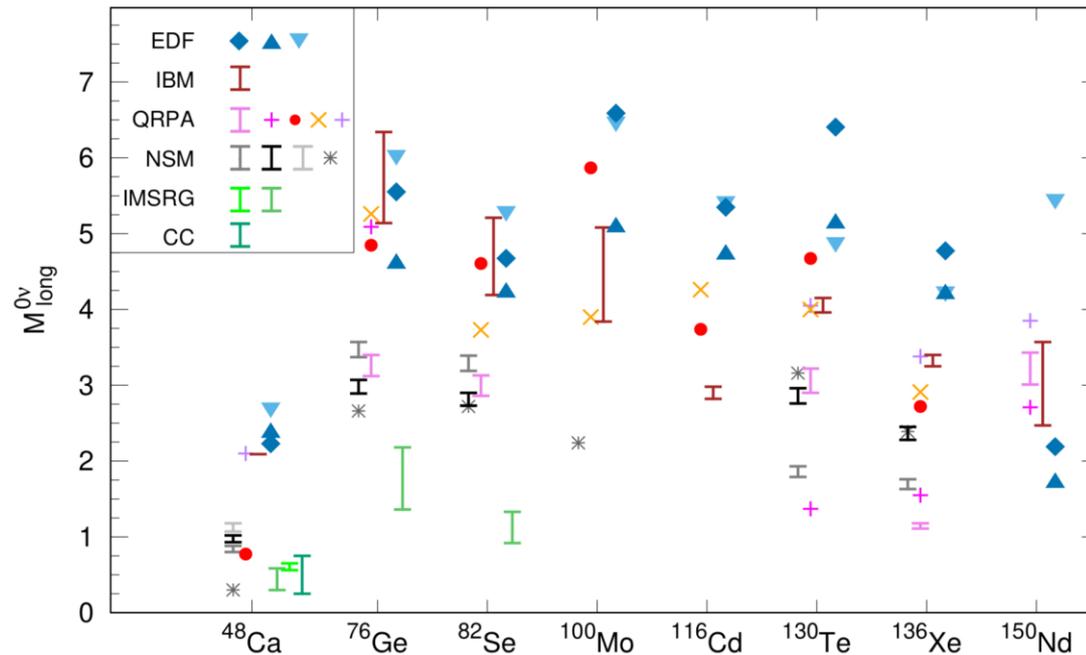
Decay Rate

Decay rate of $0\nu\beta\beta$

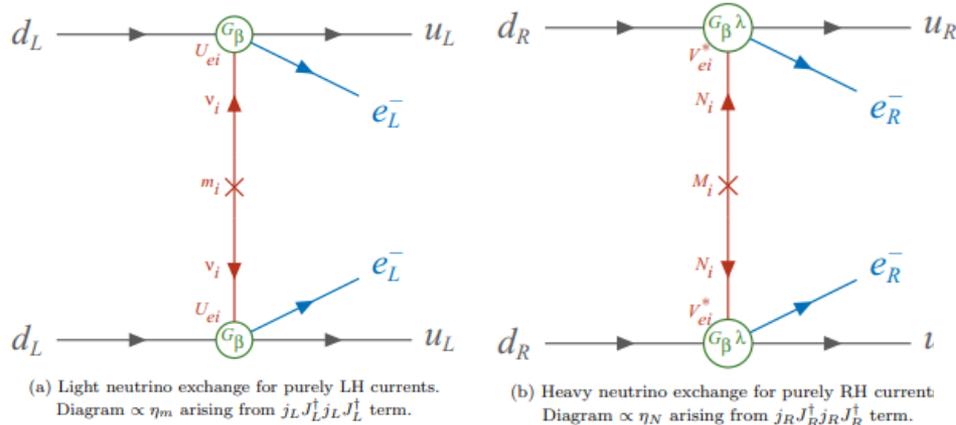


$$\Gamma^{0\nu} = \frac{1}{T_{1/2}^{0\nu}} = G^{0\nu}(Q, Z) |m_{\beta\beta}|^2 |M^{0\nu}|^2$$

Large difference in nuclear matrix element calculations: factor ~ 3

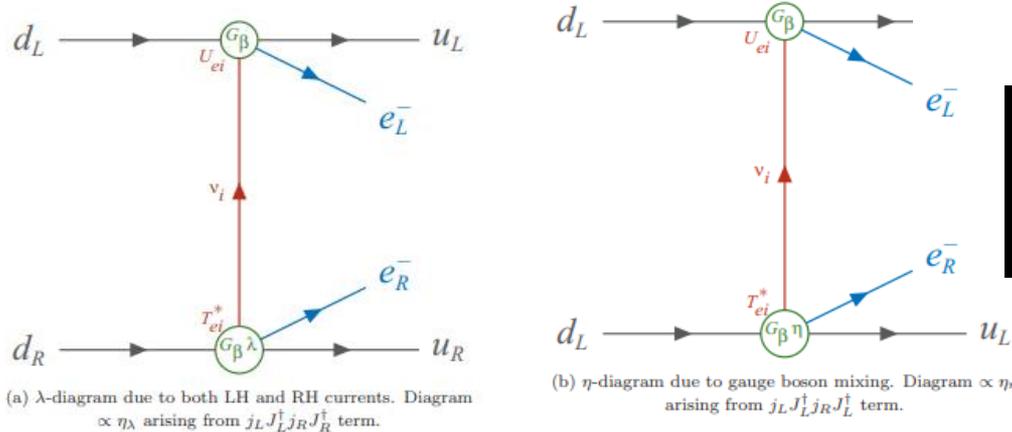


Recent Topic: Left-Right Symmetric Mechanisms of $0\nu\beta\beta$ decay



If we consider all the diagrams of $0\nu\beta\beta$ decay in left-right symmetric model, the decay rate can be written as

$$\begin{aligned}
 [T_{1/2}^{0\nu}]^{-1} &= g_A^4 \left[C_m |\eta_m|^2 + C_N |\eta_N|^2 + C_\lambda |\eta_\lambda|^2 + C_\eta |\eta_\eta|^2 \right. \\
 &\quad + C_{mN} |\eta_m| |\eta_N| \cos(\phi_m - \phi_N) + C_{m\lambda} |\eta_m| |\eta_\lambda| \cos(\phi_m - \phi_\lambda) \\
 &\quad + C_{m\eta} |\eta_m| |\eta_\eta| \cos(\phi_m - \phi_\eta) + C_{N\lambda} |\eta_N| |\eta_\lambda| \cos(\phi_N - \phi_\lambda) \\
 &\quad \left. + C_{N\eta} |\eta_N| |\eta_\eta| \cos(\phi_N - \phi_\eta) + C_{\lambda\eta} |\eta_\lambda| |\eta_\eta| \cos(\phi_\lambda - \phi_\eta) \right]
 \end{aligned}$$

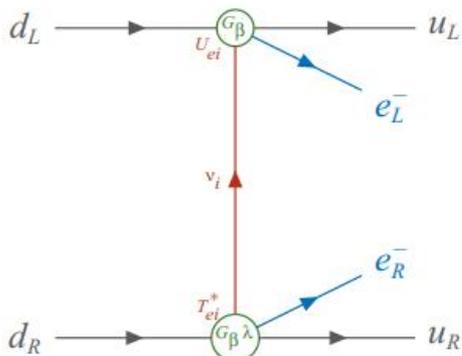


We aim to calculate all relevant nuclear matrix elements using closure and nonclosure approach

Recent Topic: Left-Right Symmetric Mechanisms of $0\nu\beta\beta$ decay λ - mechanism

TABLE 3 | Results for half-life and bounds on neutrino mass and lepton number violating parameters. The $T_{1/2}^{0\nu\text{-exp}}$ is taken from the experimental lower limit on half-life from Ref. (Arnold et al., 2005) for ^{82}Se and from Ref. (Arnold et al., 2016) for ^{48}Ca . All results are for AV18 type SRC parameterization. We have assumed CP conservation ($\psi = 0$). The results are compared with QRPA calculations for λ mechanism of Ref. (Šimkovic et al., 2017).

Quantity	^{82}Se	^{82}Se Ref. Šimkovic et al. (2017)	^{48}Ca	^{48}Ca Ref. Šimkovic et al. (2017)
$T_{1/2}^{0\nu\text{-exp}}$ [Years]	2.5×10^{23}	2.5×10^{23}	2.0×10^{22}	2.0×10^{22}
C_{mm} [Years] $^{-1}$	31.21×10^{-14}	51.3×10^{-14}	4.06×10^{-14}	2.33×10^{-14}
$C_{m\lambda}$ [Years] $^{-1}$	10.46×10^{-14}	-27.0×10^{-14}	3.37×10^{-14}	-1.04×10^{-14}
$C_{\lambda\lambda}$ [Years] $^{-1}$	36.19×10^{-14}	150.0×10^{-14}	5.39×10^{-14}	10.1×10^{-14}
$m_{\beta\beta}$ [eV]	1.83	1.43	17.92	23.7
η_λ	3.32×10^{-6}	1.63×10^{-6}	30.44×10^{-6}	22.30×10^{-6}



(a) λ -diagram due to both LH and RH currents. Diagram $\propto \eta_\lambda$ arising from $j_L J_L^\dagger j_R J_R^\dagger$ term.

λ - mechanism



Interacting Shell Model Calculations for Neutrinoless Double Beta Decay of ^{82}Se With Left-Right Weak Boson Exchange

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Nuclear Matrix Element(NME) $M^{0\nu}$

$$M^{0\nu} = M_{GT}^{0\nu} - \frac{g_V^2}{g_A^2} M_F^{0\nu} + M_T^{0\nu}$$

- F- Fermi
- GT- Gamow-Teller
- T- Tensor
- g_V and g_A are vector and axial vector constant

$$M_\alpha^{0\nu} = \langle f | \tau_{-1} \tau_{-2} O_{12}^\alpha | i \rangle \quad \alpha = (F, GT, T)$$

$0\nu\beta\beta$ transition operators

- Fermi Type: $O_{12}^F = S_F H_F(r) = H_F(r)$
- Gamow Teller type:
 $O_{12}^{GT} = S_{GT} H_{GT}(r) = \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r)$
- Tensor Type:
 $O_{12}^T = S_T H_T(r) = [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_T(r)$

Models of Nuclear Matrix Element calculations

- **Nuclear Shell Model (NSM) (We use this)**
- Quasiparticle Random Phase Approximation (QRPA)
- Projected Hartree Fock Bogliovob Method (PHFB)
- Interacting Boson Model 2 (IBM2)
- Energy Density Functional Theory (EDF)

Shell structure of nucleus

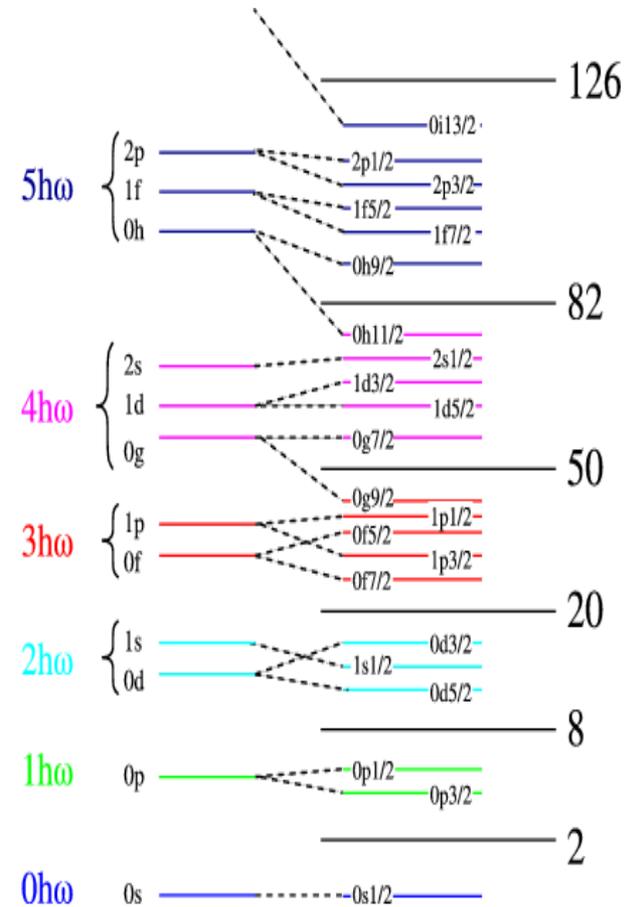


Fig 3: Nuclear shell structure

Method: Running nonclosure and closure

Here....Neutrino potential are calculated explicitly in terms of excitation energy of ^{48}Sc

$$H_\alpha(r, E_k^*) = \frac{2R}{\pi} * \int_0^\infty \frac{1}{q + E_0 + E_k^*} j_p(qr) g_\alpha(q) q dq$$

$$\langle n'l' | H_\alpha(r, E_k^*) | nl \rangle = \int_0^\infty R_{n'l'} R_{nl} r^2 dr * H_\alpha(r, E_k^*)$$

Closure approximation

$$E_0 + E_k^* = \langle E \rangle$$

Nonclosure Closure approximation

$$E_0 + E_k^* \rightarrow 1.9 \text{ MeV} + E_k^*$$

Explicit Form of NME in running nonclosure method

$$M_{\alpha\text{-running nonclosure}}^{0\nu}(E) = \sum_{k_1' k_2' k_1 k_2 J_k} \sum_{E_k^* \leq E_C} \sqrt{(2J_{k_1} + 1)(2J_{k_2} + 1)(2J_k + 1)} \\ \times (-1)^{j_{k_1} + j_{k_2} + J} \begin{Bmatrix} j_1 & j_2 & J_k \\ j_4 & j_3 & J \end{Bmatrix} \times \text{OBTD}(k, f, k_1', k_2', J_k) \\ \times \text{OBTD}(k, i, k_1, k_2, J_k) \langle k_1' k_2' : J | | \tau_{-1} \tau_{-2} \mathcal{O}_{12}^{0\nu} | | k_1 k_2 \rangle$$

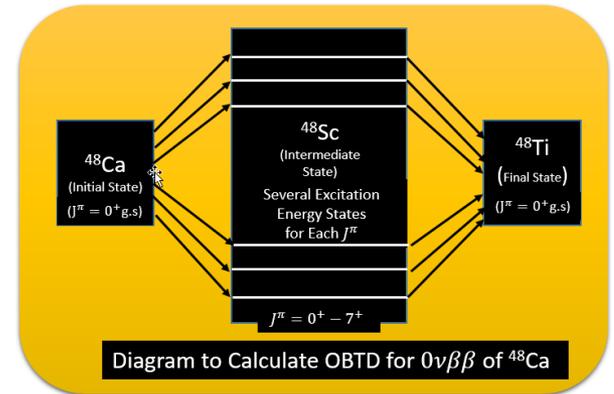


Fig 20: Schematic diagram to calculate OBTD

$$\text{OBTD}(k, f, k_1', k_2', J_k) = \frac{\langle k | \left[a_{k_1}^+ \otimes \tilde{a}_{k_1'} \right]_{J_k} | f \rangle}{\sqrt{2J_k + 1}}$$

Recent Trends in Theoretical NME research



The banner features a night cityscape on the left. The text 'NME2023' is in large blue letters, with 'RCNP Workshop for "Theoretical and Experimental Approaches for Nuclear Matrix Elements of Double Beta Decay"' below it. The RCNP logo and '大阪大学 核物理研究センター' are on the right, along with a '日本語' button. A navigation bar at the bottom has buttons for 'Index', 'Bulletin', 'Site', 'Program', and 'Registration'.

- ❑ Two-neutrino double beta decay within the mapped IBM
- ❑ Modeling neutrinoless double-beta decay with operators from chiral effective field theory
- ❑ Impact of isovector pairing fluctuations on neutrinoless double- β decay in MR-CDFT
- ❑ Improvement of reliability of nuclear matrix element of double-beta decay
- ❑ Neutrinoless DBD from valence-space in-medium similarity renormalization group
- ❑ Beta spectral shapes - A versatile tool for probing weak interactions
- ❑ Neutrinoless double-beta decay nuclear matrix elements: overview and future directions
- ❑ implication of ab initio no-core Monte Carlo Shell Model to double beta decay
- ❑ Improved neutrinoless double-beta-decay matrix elements in the pnQRPA
- ❑ Anti-Neutrino Nuclear Responses by Ordinary Muon Capture: Status and Review
- ❑ Recent progress in the nuclear matrix element calculation using the FAM for QRPA
- ❑ High precision description of two neutrino double beta decay spectra
- ❑ Study of Neutrinoless Double Beta Decay in Nuclear Shell Model
- ❑ Impact from nuclear structure aspects on the neutrinoless double-beta decay
- ❑ Uncertainties of NME of neutrinoless DBD based on Skyrme QRPA model and beyond

Theoretical NME research network with EXPERIMENTALISTS

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Sei Yoshida (Department of Physics, Osaka University, Assoc.Prof.)

Atsushi Tamii (Department of Physics, Osaka University, Prof.)

Kosuke Nomura (Hokkaido University, Assoc.Prof.)

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Theoretical NME research network with EXPERIMENTALISTS



Summary

Neutrinoless double beta decay ($0\nu\beta\beta$)



Nuclear matrix element (NME)



International NME research CORE



NME2023

RCNP Workshop for

"Theoretical and Experimental Approaches for Nuclear Matrix Elements of Double Beta Decay"

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