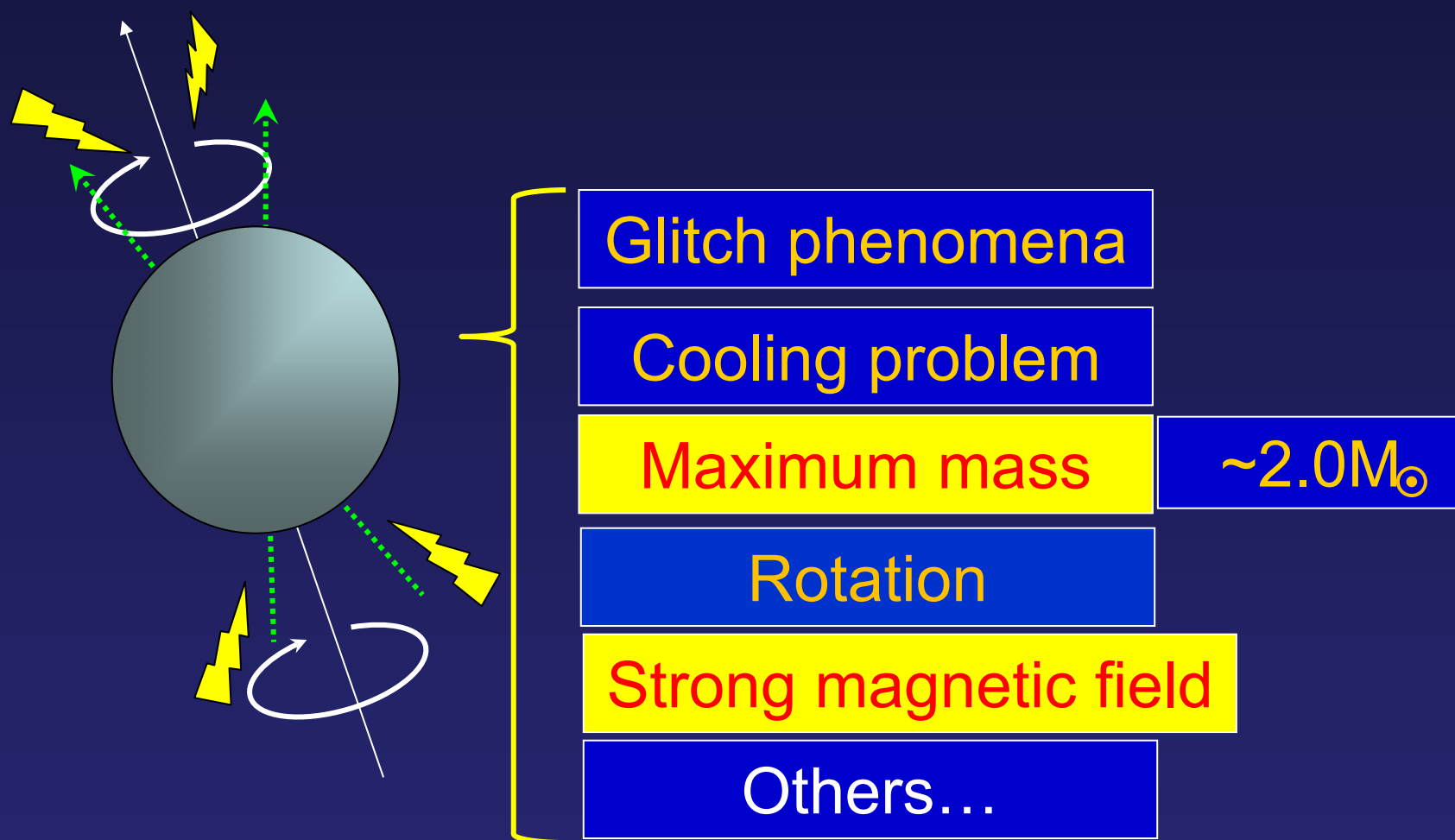


Quark matter equations of state under strong magnetic fields and hybrid stars

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Neutron stars have many physical phenomena



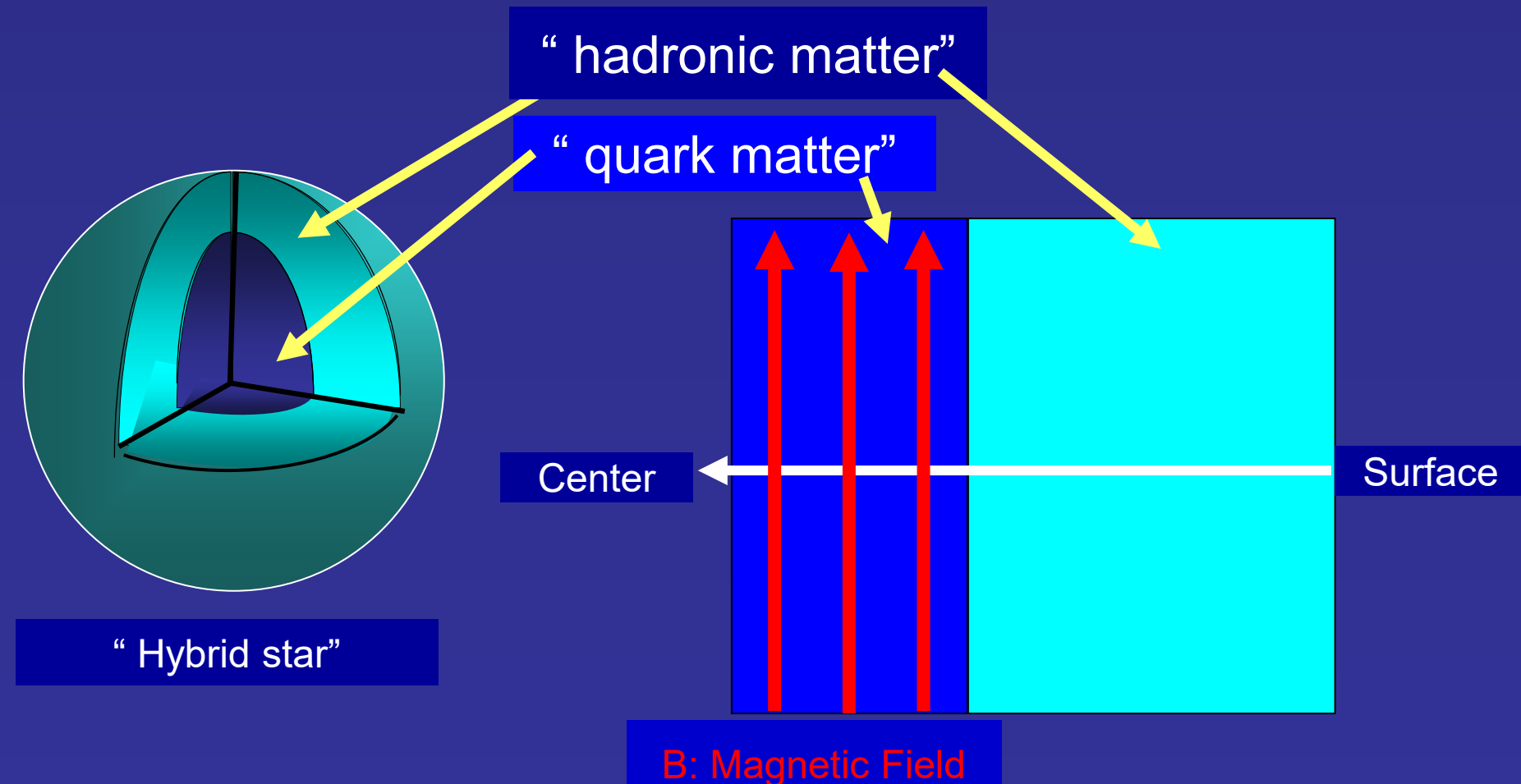
We focus on the strong magnetic field. A neutron star possesses a strong magnetic field, with the value on its surface being approximately 10^{12} G. On the other hand, there are stars with even stronger magnetic fields, reaching up to 10^{15} G. We refer to these stars as “Magnetars”.

$B(\text{surface})$: Radio Pulsars $\sim 10^{12}$ G
Magnetars $\sim 10^{15}$ G

$$|\mu| = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-9} \text{ eV/G}$$

$$1.0 \times 10^{19} \text{ G} \sim (240 \text{ MeV})^2$$

Much stronger field suggested inside the star: $\sim 10^{18}$ G
Lai and Shapiro, ApJ.383(1991)745



Hybrid stars feature a quark core within a star, surrounded by hadronic matter. In our analysis, we specifically consider the magnetic field contribution for quark matter. This focus is based on the understanding that the magnetic field's impact on quark matter is likely to be more significant than its effect on hadronic matter.

The n -th energy level of quark with flavor f in the strong magnetic field is given by

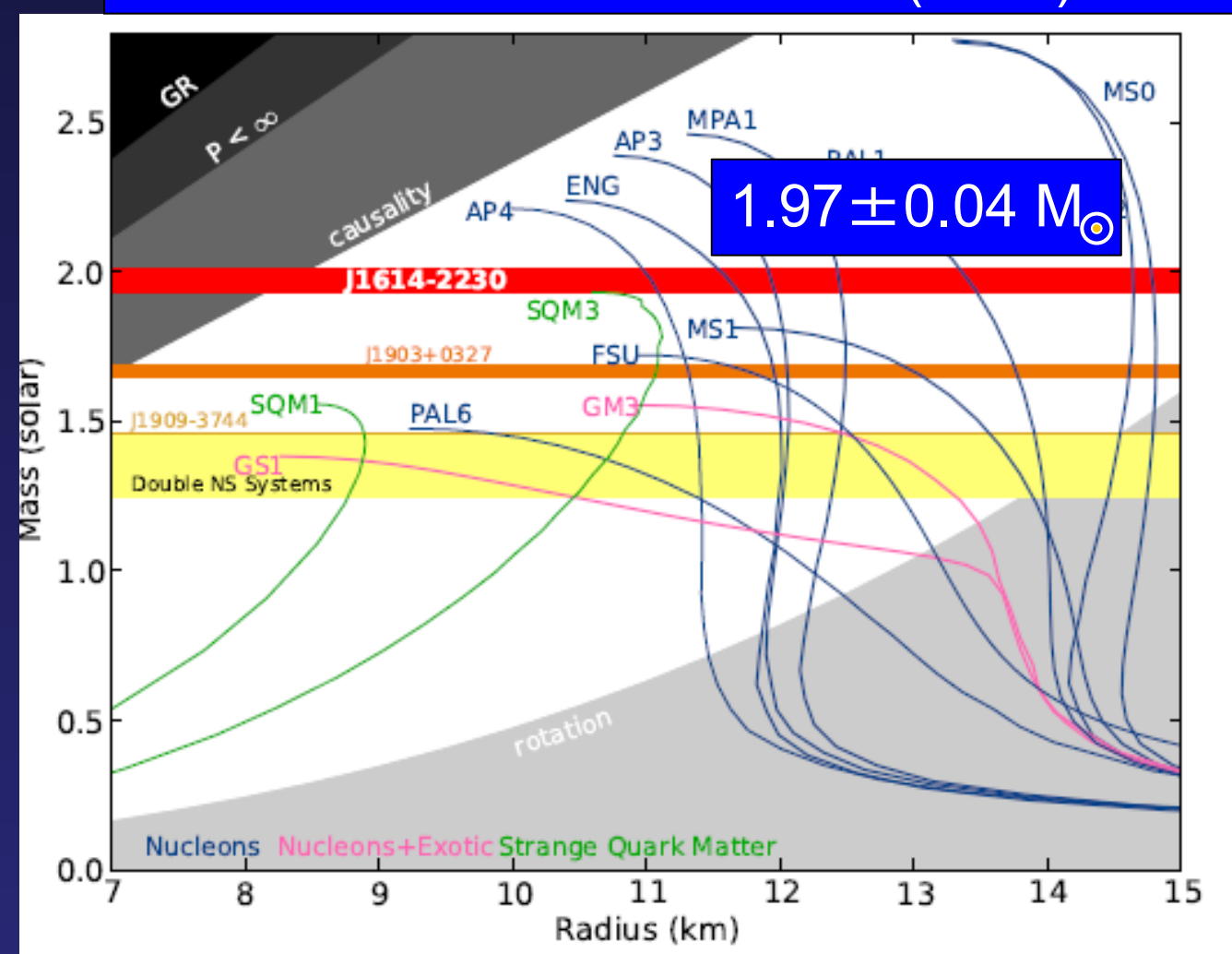
$$E_n^f = \sqrt{c^2 p_z^2 + \hbar c |e_f B| [2n + 1 + \text{sgn}(e_f B) s]}$$

$n=0$: Lowest Landau Level (LLL), H. Sotani and T. Tatsumi, MNRS 447, 3155 (2015)

$n=1$: 2nd Landau Level (2LL)

Observed maximum mass of stars

P. Demorest et al. Nature 467 (2010) 1081.



J. Antoniadis et al., Science 340 (2013) 6131

$2.01 \pm 0.04 M_\odot$: PSR J0348+0432

Equation of state (EOS) for high density matter

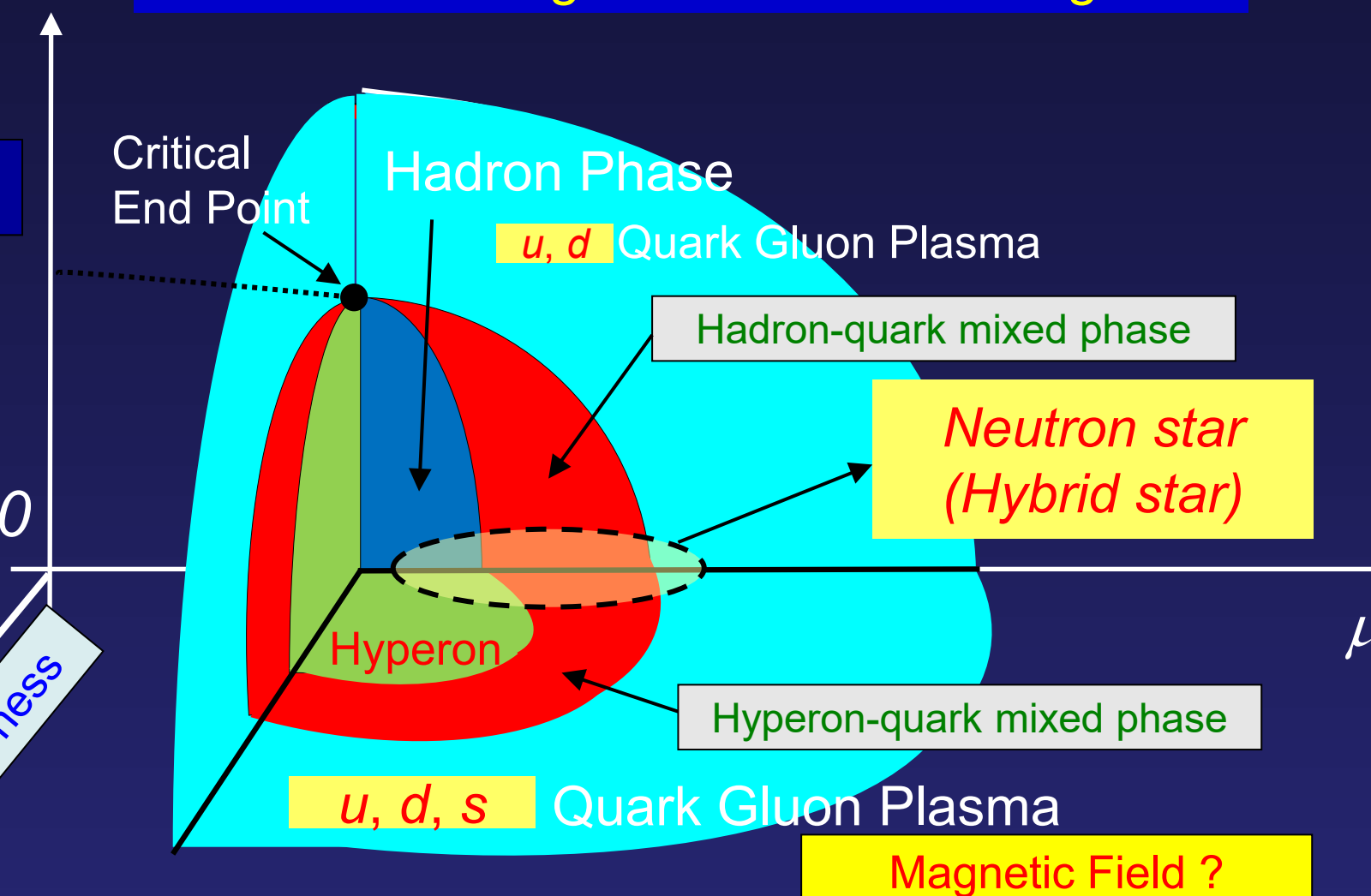
Hyperon matter
Quark matter

“Hyperon PUZZLE”

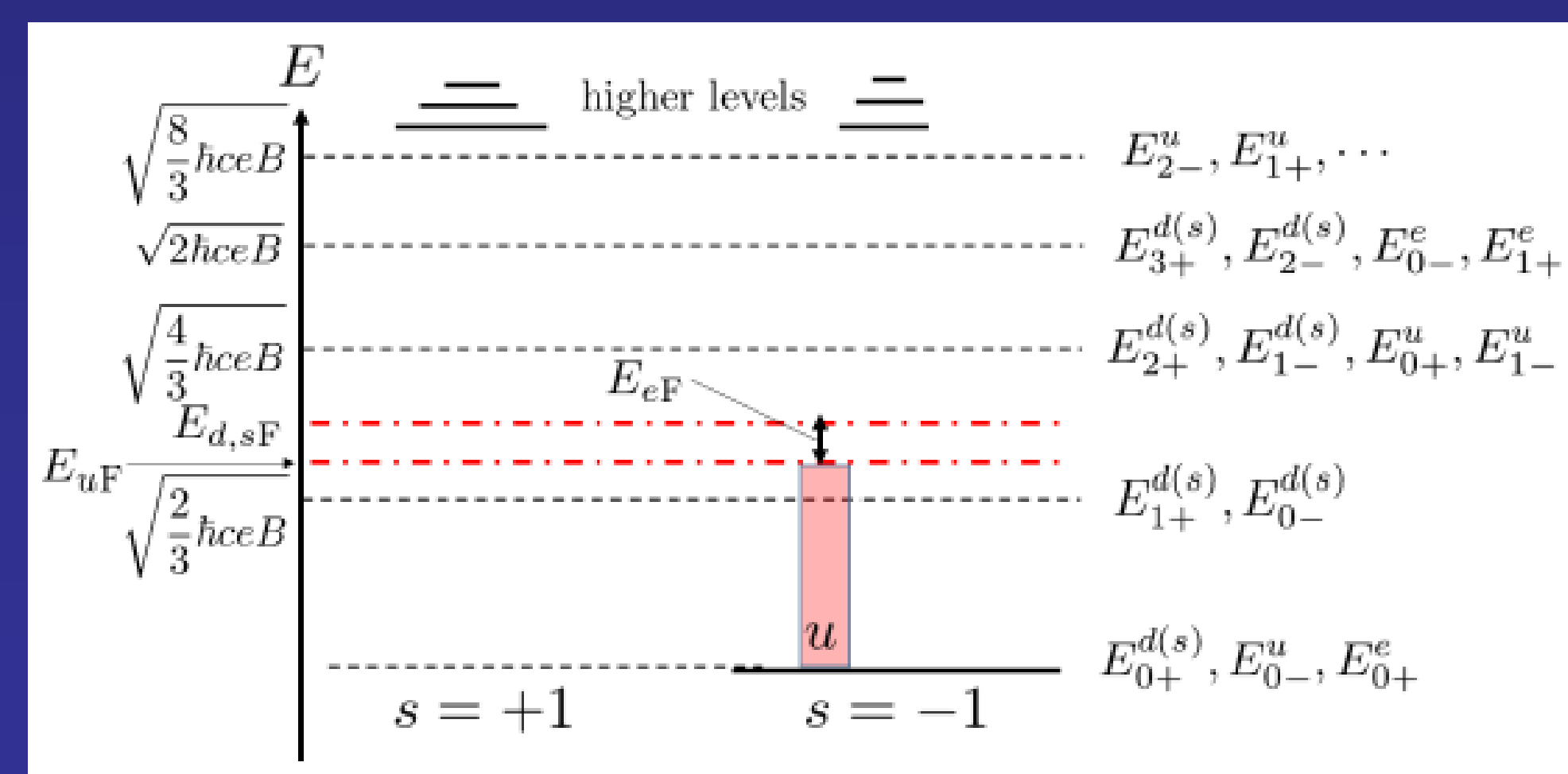
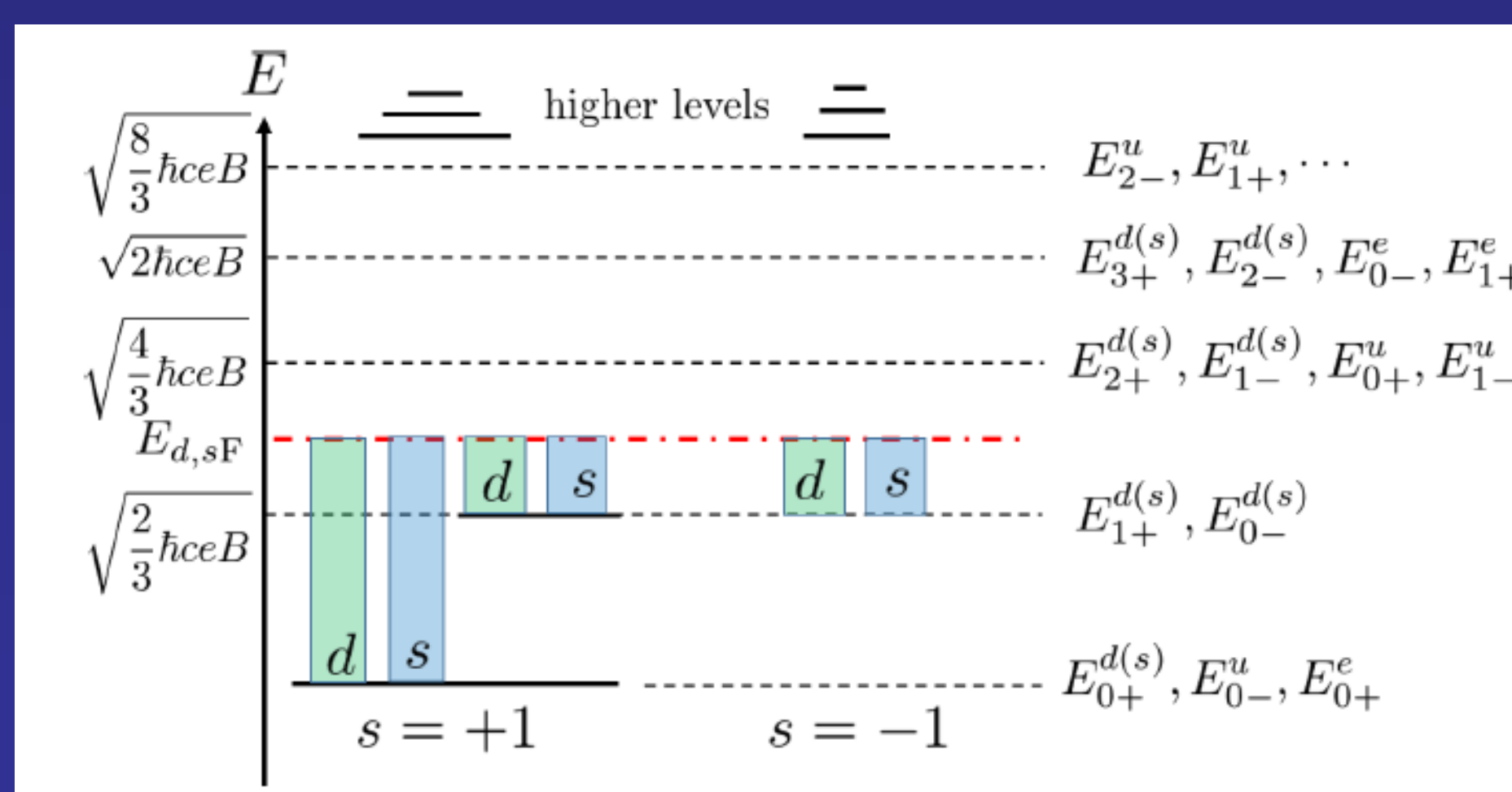
stiff EOS
Three nucleon interaction
Strong magnetic field

Two solar mass neutron stars have been observed. The study of the Equation of State (EOS) for high-density matter faces challenging issues, such as the ‘Hyperon Puzzle.’ The transition from nuclear matter to hyperonic matter and/or quark matter at high-density regions poses difficulties, as these hyperonic or quark matters tend to soften the EOS, a problem commonly referred to as the ‘Hyperon Puzzle.’ To address this issue and enhance the stiffness of the EOS, various studies have explored the effects of three-nucleon interactions or the influence of a strong magnetic field. In this study, we aim to investigate the impact of the strong magnetic field.

Schematic image of QCD Phase diagram



2nd Landau Levels(2LL)+ β equilibrium



Energy density(2LL)

These figures are the image of energy levels for the situation when u quarks and electrons are in the LLL state, while d and s quarks are in up to the 2LL state, where $E_{\beta F}(=E_F)$, E_{dF} , and E_{sF} denote the Fermi energy of d , s , u quarks, and electrons, respectively.

$$\varepsilon_f = \frac{3|e_f B|}{4\pi^2 \hbar^2} \left[p_{dF}^{(1)2} + 2p_{dF}^{(1)} p_{dF}^{(2)} + \frac{4\hbar|e_f B|}{c} \ln |p_{dF}^{(1)} + p_{dF}^{(2)}| - \frac{2\hbar|e_f B|}{c} \ln \left(\frac{2\hbar|e_f B|}{c} \right) \right]$$

u, e : LLL, d, s : 2LL

$$\varepsilon_{2LL\beta} = a \left\{ \frac{3n_b}{2a} - \left(p_{dF}^{(1)} + 2\sqrt{p_{dF}^{(1)2} - \frac{2}{3}eB} \right)^2 \right. \\ \left. + a \left[p_{dF}^{(1)2} + 2p_{dF}^{(1)} p_{dF}^{(2)} + \frac{4eB}{3} \ln |p_{dF}^{(1)} + p_{dF}^{(2)}| - \frac{2eB}{3} \ln \left(\frac{2eB}{3} \right) \right] \right. \\ \left. + \frac{a}{2} \left\{ \frac{2}{7} \left(p_{dF}^{(1)} - 2\sqrt{p_{dF}^{(1)2} - \frac{2}{3}eB} \right)^2 + B \right\} \right.$$

$$\text{Baryon number} \quad \frac{1}{3} (n_u + n_d + n_s) = n_b$$

$$\text{Charge neutrality} \quad \frac{2}{3} n_u - \frac{1}{3} n_d - \frac{1}{3} n_s - n_e = 0$$

$$\beta \text{ equilibrium} \quad \mu_d = \mu_u + \mu_e$$

$$n_d = n_s$$

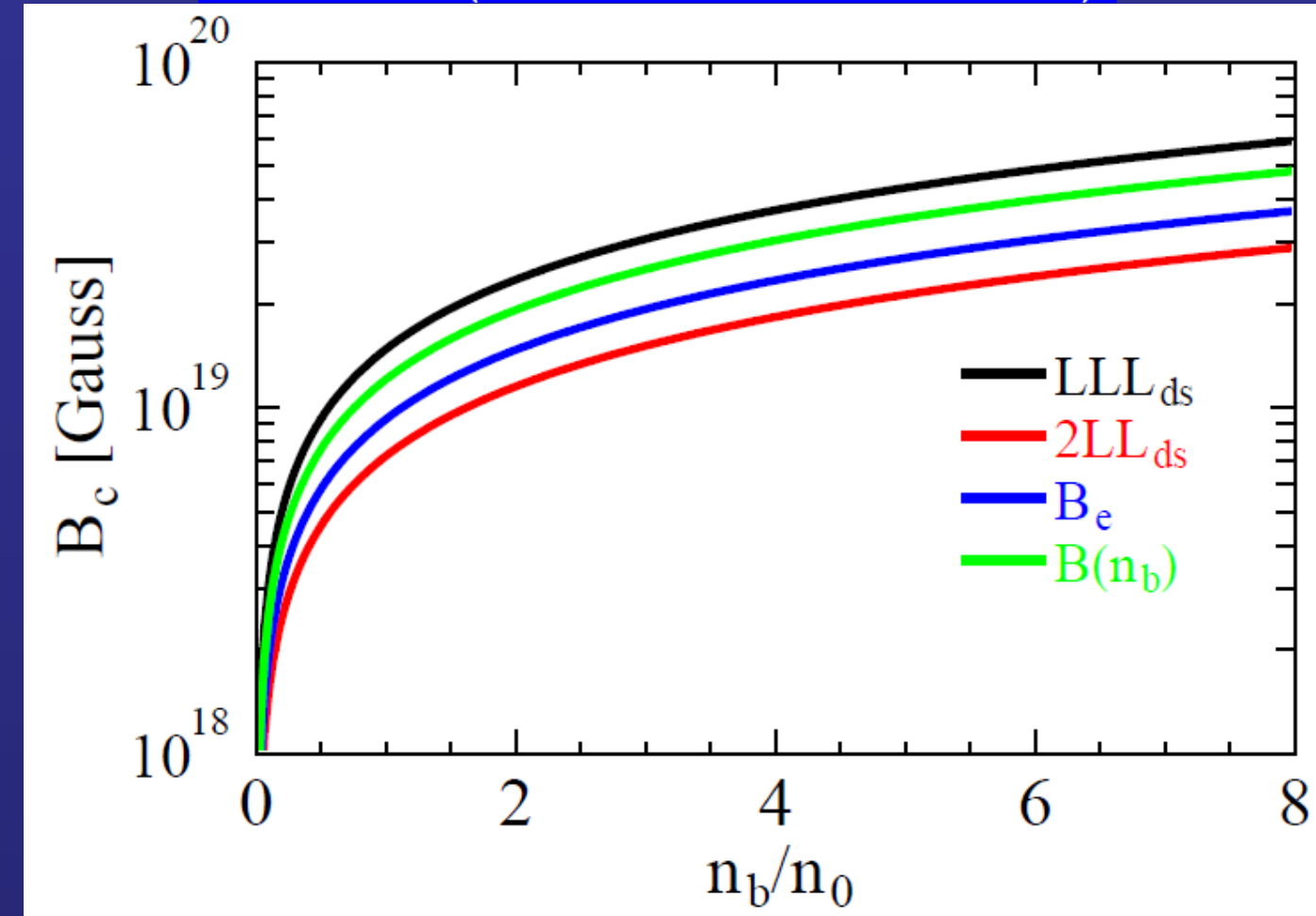
Density dependent magnetic field

$$B(n_b) = C_B \times 10^{18} \times \left(\frac{n_b}{n_0} \right)^{\frac{2}{3}} \text{ G}$$

B_c of d, s quark at LLL $\Leftrightarrow$ 2LL: $C_B=14.8$

The coefficient C_B : We can change the field strength by using different C_B .

$$p_{eF} = \frac{2}{7} \left(p_{dF}^{(1)} - 2\sqrt{p_{dF}^{(1)2} - \frac{2}{3}eB} \right) \Rightarrow B_c^{(e)} = 9.28 \times 10^{18} \times \left(\frac{n_b}{n_0} \right)^{\frac{2}{3}} \text{ G}$$



$$14.8 \times 10^{18} \times \left(\frac{n_b}{n_0} \right)^{\frac{2}{3}} \text{ G}$$

Range of magnetic field strengths

$$9.28 \times 10^{18} \times \left(\frac{n_b}{n_0} \right)^{\frac{2}{3}} \text{ G}$$

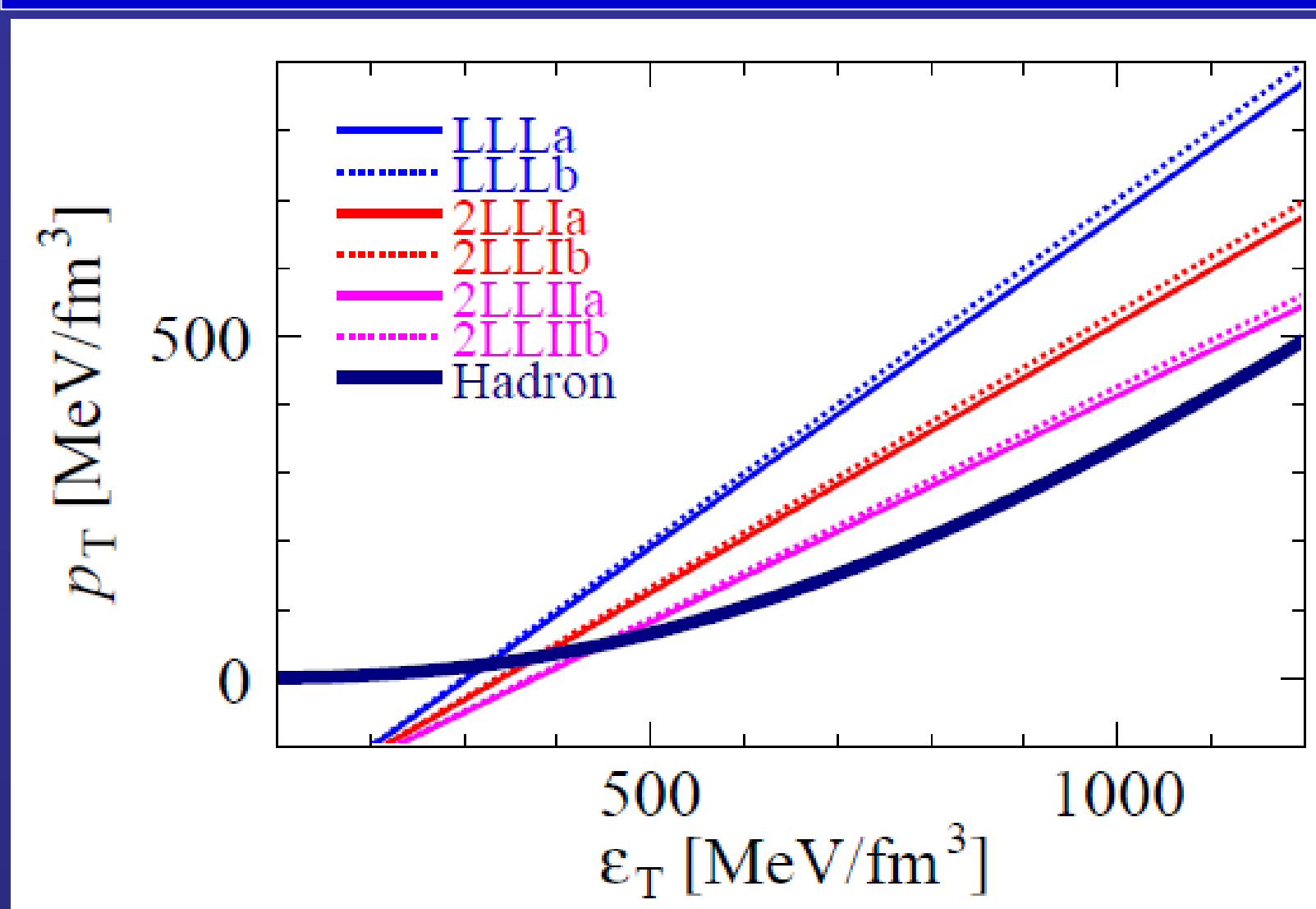
2LL(d, s)

$$B_e = 7.27 \times 10^{18} \times \left(\frac{n_b}{n_0} \right)^{\frac{2}{3}} \text{ G}$$

$$u, e \text{ : LLL, } d, s \text{ : 2LL} \Rightarrow 9.28 \leq C_B \leq 14.83$$

To incorporate the 2LL effect, we carefully introduce the magnetic field. We introduce a density dependent magnetic fields. If we consider such magnetic fields, we can incorporate the situation of u, e in LLL and d, s in “2LL”.

EOS for various magnetic field strength



label	C_B	pressure term
LLLa	14.83	$B^2/(24\pi)$
2LLLa	13.44	$B^2/(24\pi)$
2LLIIa	12.06	$B^2/(24\pi)$
LLLb	14.83	$B^2/(8\pi)$
2LLIb	13.44	$B^2/(8\pi)$
2LLIIb	12.06	$B^2/(8\pi)$

Solid line(a): anisotropic
Dotted line(b): isotropic

This figure shows the quark EOS for various magnetic field strength. The quark EOS is shown with solid lines for the field contribution given by “a” case and with the dotted lines for that given by “b” case, where LLL, 2LLI and 2LLII correspond to the different field strength shown in this Table. For reference, the EOS for hadronic matter is also shown with the thick-solid line. From this figure, we can observe that the quark EOS becomes softer according to the decreasing magnetic field strengths.

Energy contribution of magnetic field

L.Lope and Menezes, Journal of Cosmology and Astroparticle Physics, 1508 (2015) No.08, 002

anisotropic

$$\varepsilon_T = \varepsilon_M + \frac{B^2}{8\pi} \quad P_T = P_M + \frac{B^2}{24\pi}$$

$\Rightarrow$ Subscript “a”

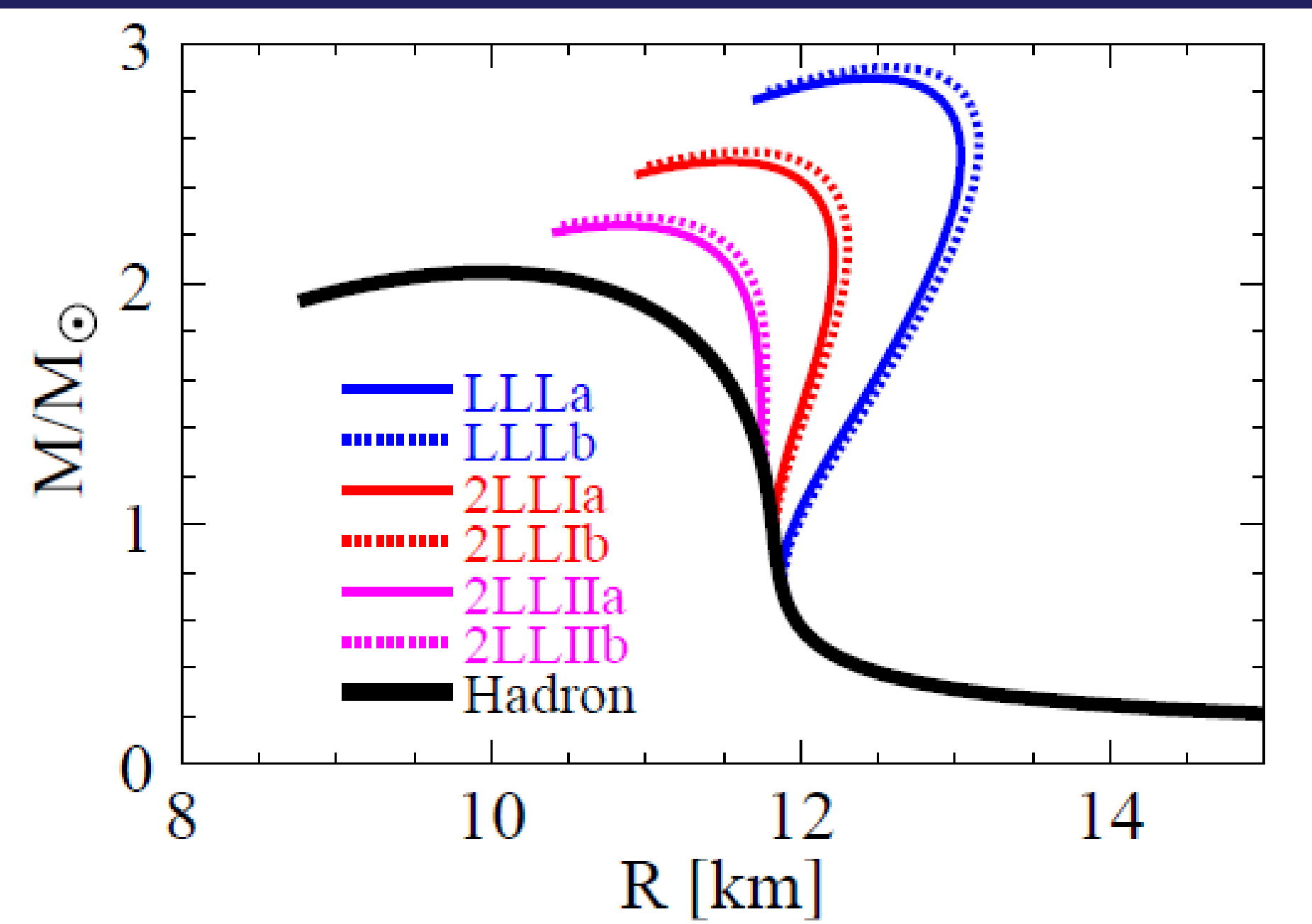
isotropic

$$\varepsilon_T = \varepsilon_M + \frac{B^2}{8\pi} \quad P_T = P_M + \frac{B^2}{8\pi}$$

$\Rightarrow$ Subscript “b”

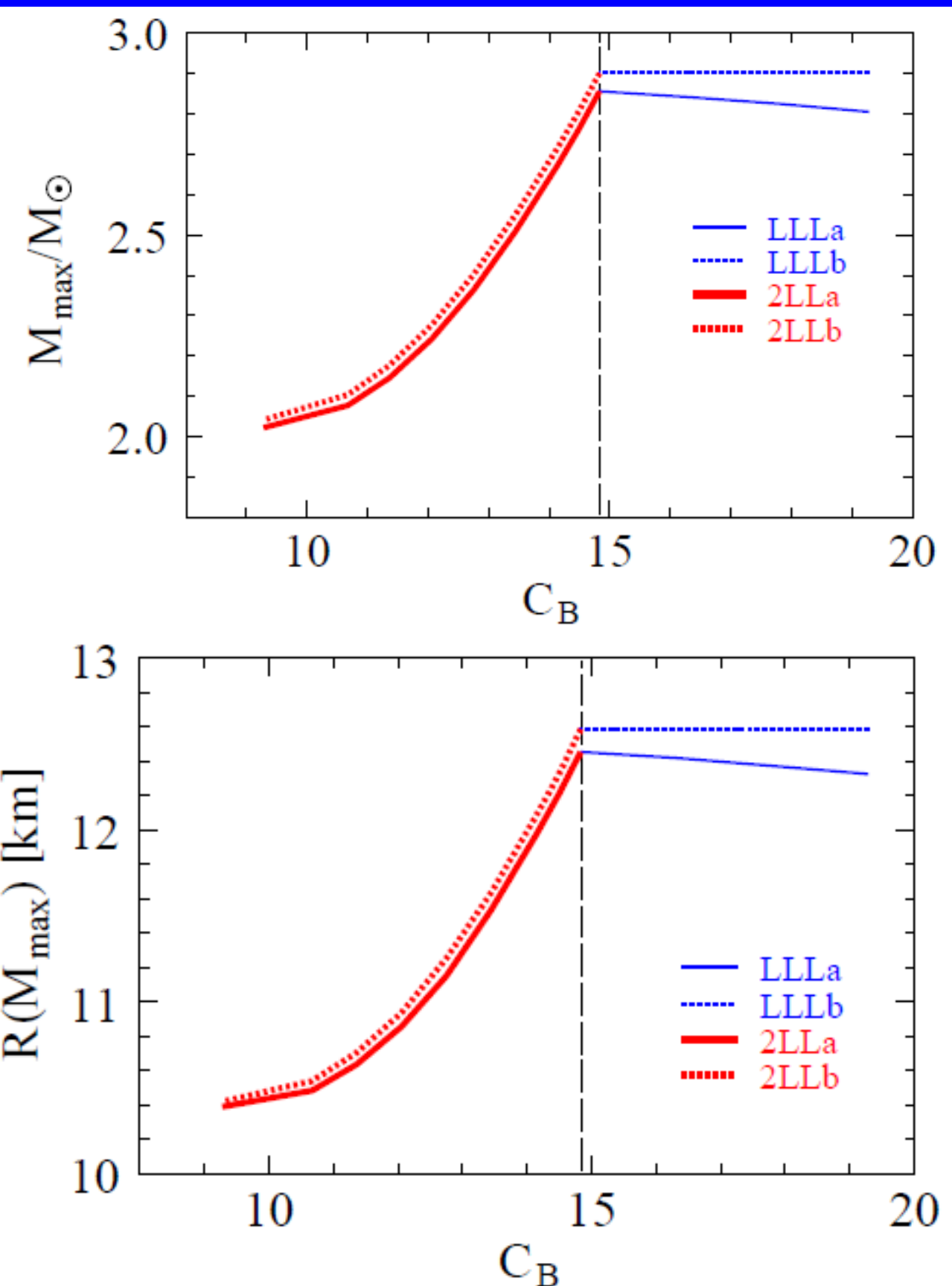
The contribution from the strong magnetic field typically relies on the magnetic field configuration within the star, often resulting in anisotropic pressure. However, due to the ongoing uncertainty surrounding the specifics of the magnetic field configuration, our study simplifies the consideration to two different field contributions, commonly assumed in many studies.

Mass-radius relation of hybrid stars with different magnetic field strength



We show the mass and radius relation for hybrid stars constructed with the EOS shown in this figure. From this figure, we can see that the mass and radius are very sensitive to the field strength, but less sensitive to the field contribution.

Maximum mass and radius of hybrid stars a function of the field strength



LLL: less sensitive to C_B
2LL: sensitive to C_B

To examine the dependence on C_B , we illustrate the maximum mass of hybrid stars as a function of C_B in the upper figure, while the radius of the hybrid star with the maximum mass is displayed in the lower figure. In both figures, the solid and dotted lines represent the results obtained with different field contributions given by Eqs. ‘a’ and ‘b’, respectively. From these figures, we observe that the maximum mass and the corresponding radius for the LLL case are less sensitive to C_B . However, for the 2LL case, they strongly depend on C_B , as evidenced by their decrease as C_B decreases.

Summary

- β equilibrium, 2LL(d, s) effect
- Two types of field contribution (“a” and “b”)
- EOS with 2LL(d, s) becomes soft
- EOS are sufficiently stiff to sustain $2.0 M_\odot$

Future plans ;

- u quark in 2LL
- 3LL(3rd Landau Level)
- Anisotropy of pressure
- Improvement of bag pressure
- Improvement of magnetic field

We need to further improve our models. In this study, we used a simple model of quark matter, namely, a degenerate free Fermi gas. To obtain more realistic results, we need to employ more sophisticated models, such as the Nambu–Jona-Lasinio (NJL) model and models of color superconductivity. Furthermore, we could not properly account for the deformation of stars caused by the anisotropic pressure induced by the strong magnetic field, and instead used approximate treatments. Properly incorporating the effects of anisotropic pressure is a challenging task because it requires us to determine the configuration of the magnetic field inside the stars.