

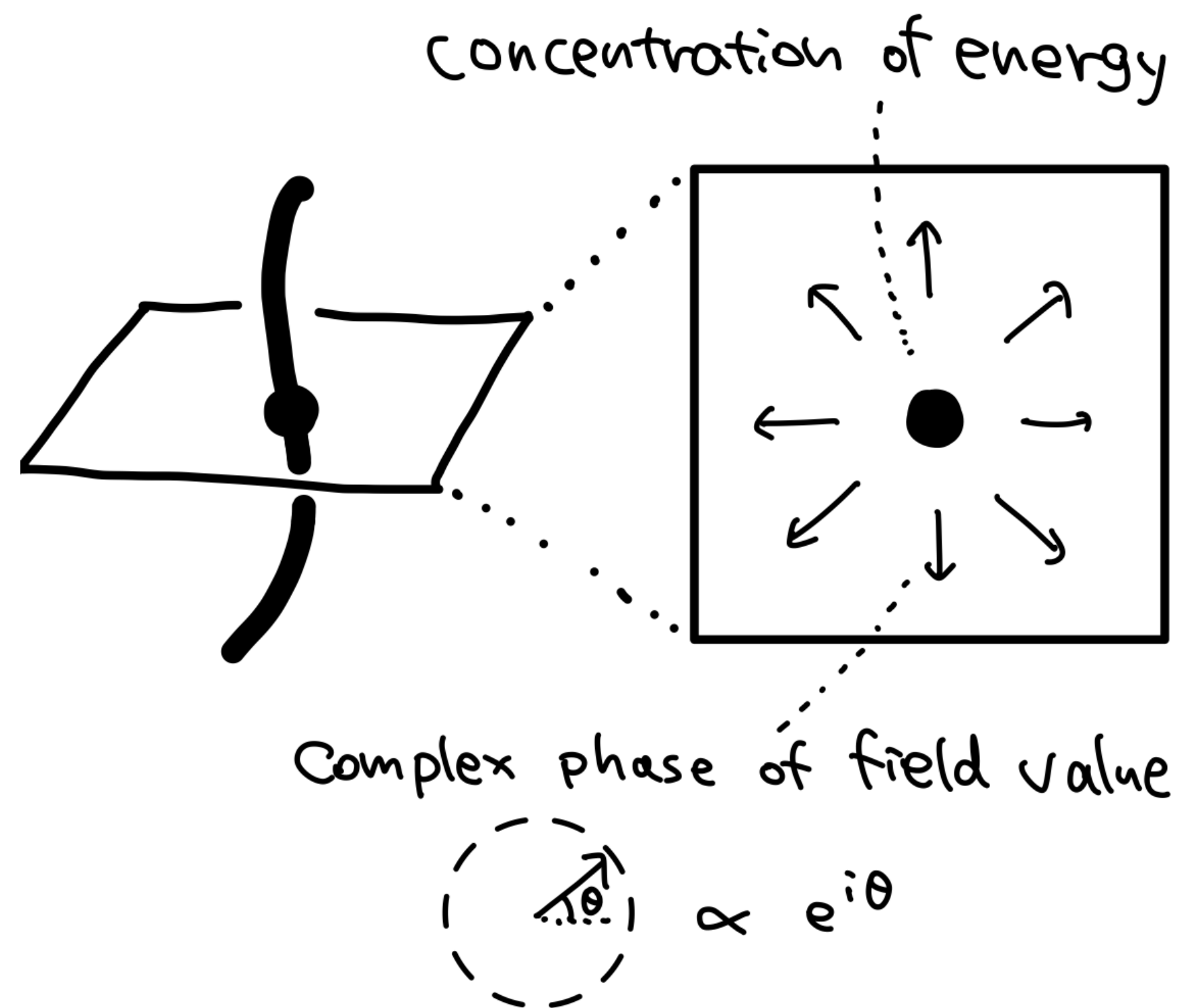
Revisiting the Role of Zero Modes around Solitons

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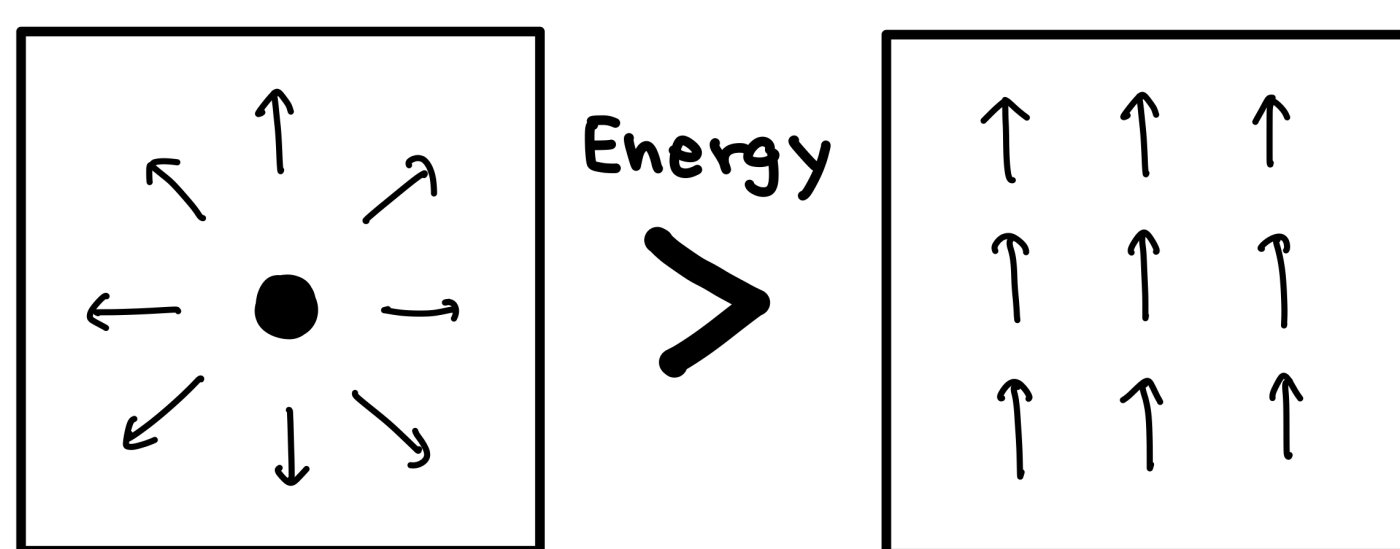
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What is “Topological Soliton”?

- ▶ Example: **(cosmic) string**



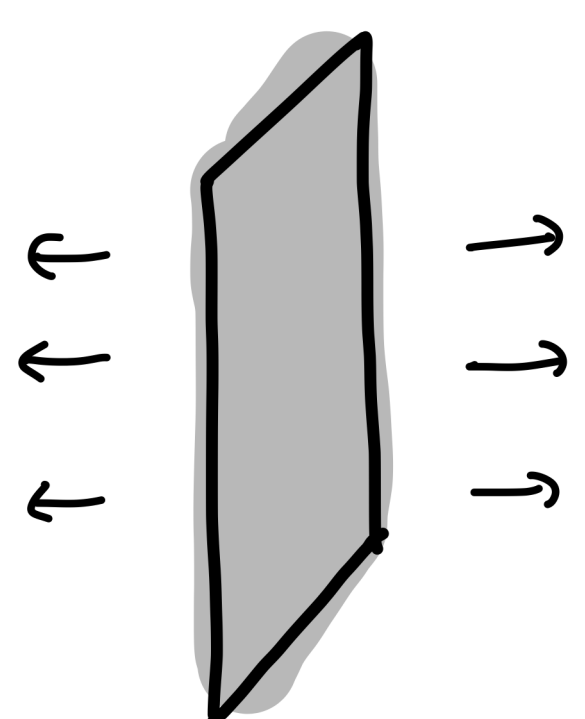
String has a larger energy than the ground state (so-called trivial vacuum).



However, the field winding around the string cannot be **deformed continuously** to trivial (non-winding) configuration: **“topological stability.”**

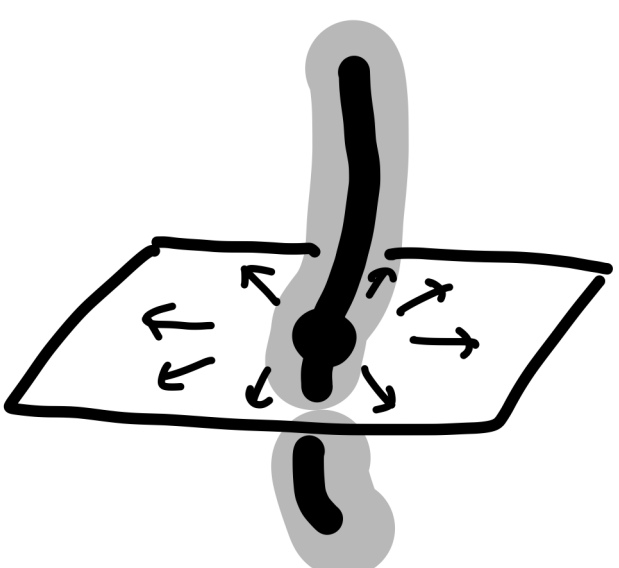
Variety of Topological Solitons

- ▶ **Domain wall**



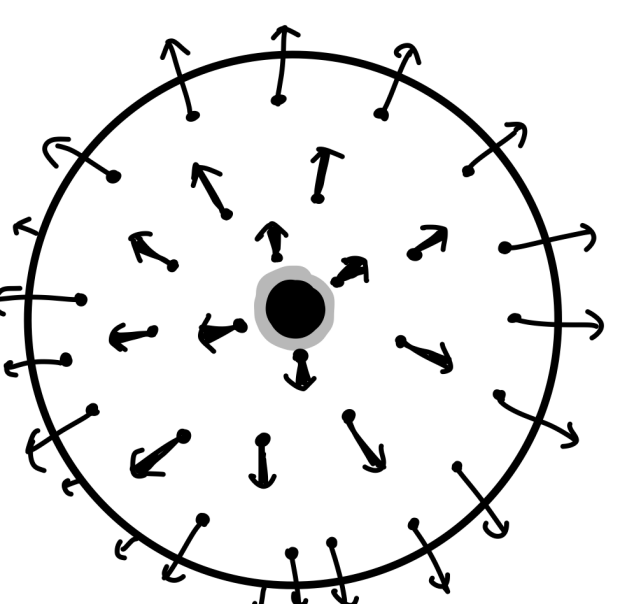
space **2-dim** + time **1-dim**

- ▶ **String**



space **1-dim** + time **1-dim**

- ▶ **Monopole**



space **0-dim** + time **1-dim**

- ▶ **Instanton**

space **0-dim** + time **0-dim**

Zero Modes around Solitons

Zero modes: Field oscillation $\phi^{(0)}$ which cost no energy.

$$E[\phi + (\text{const}) \times \phi^{(0)}] = E[\phi]$$

(ϕ : soliton configuration, $\phi^{(0)}$: zero mode)

Localized (decreasing at far away from the soliton)
zero modes are important.

→ Such mode does not exist around trivial vacuum
(because modes with spatial dependence cost energy).

- ▶ **Boson localized zero modes:**

Translation, rotation, ..., of the soliton.

- ▶ **Fermion localized zero modes:**

There often exist zero modes of fermions interacting with the soliton.

$$E[\phi, \psi \propto \psi^{(0)}] = E[\phi, \psi = 0]$$

Impact of Fermion Zero Modes

- ▶ **Fermion zero modes prevent instanton effect:**

Path integral around the soliton vanishes due to fermion zero modes.

$$Z = \int \mathcal{D}\psi e^{-E[\psi]} \propto \int d(\text{zero mode}) \times (\text{const}) = 0.$$

Effect of instanton on physical quantity vanishes when fermion zero modes are present.

- ▶ **Superconduction along string:**

Zero modes in (x, y) + String direction z + Time t
→ Massless fermion along the string.

Superconducting string

Our Previous and Ongoing Work

- ▶ **Our work:** Recently, it was pointed out that **predicted mass of “axion”** (hypothetical particle searched experimentally with various efforts!) might be enhanced by **instantons beyond Standard Model**. We discussed it closely with careful treatment of fermion zero modes. (2404.19342 [hep-ph])
- ▶ **Another ongoing work:** Revisiting instanton and fermion zero modes with **spontaneously broken gauge symmetry**, relying on both analytic and numerical approaches.