Unitarity test of lepton mixing via energy dependence of neutrino oscillation

Ryuichiro Kitano (YITP, Kyoto Univ.), Joe Sato (Yokohama Natl. Univ.), Sho Sugama (Yokohama Natl. Univ.)

Ref. : arXiv: 2508.20389 [hep-ph]

Introduction

Motivation

- Neutrino mass remains an open question
- Studying the lepton mixing matrix holds the key to this problem
- Testing the unitarity of the PMNS matrix is one method to explore the general lepton mixing matrix

Neutrino oscillation and unitarity

◆ Lepton mixing matrix (LMM)

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

or mixing beyond 3×3 ?

$$\begin{pmatrix}V_{e1} & V_{e2} & V_{e3} & \cdots\\ V_{\mu1} & V_{\mu2} & V_{\mu3} & \cdots\\ V_{\tau1} & V_{\tau2} & V_{\tau3} & \cdots\\ \vdots & \vdots & \vdots & \ddots\end{pmatrix}$$

If the LMM is larger than 3×3, the 3×3 U_{PMNS} become non-unitary
→ **Unitarity Test**

◆ Framework of standard 3 flavor ν oscillation

$$U_{\mathrm{PMNS}} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

If U_{PMNS} is 3×3 and unitary $\Longrightarrow$ there are only 4 params. $(\theta_{12},\theta_{13},\theta_{23},\ \delta)$

But this parametrization always guarantees the unitarity
→Must remove this parametrization to test the unitarity

◆ Oscillation probability and Unitarity

Oscillation probability = (**Coefficient**) × (**Energy-dependent function**)

In vacuum, 3-generation

Up to 2nd. order in $\Delta m^2_{21}/\Delta m^2_{31}$, U_{e3}

$$P(\nu_{\mu\to e}) = C_1\cdot \sin^2(\Delta_{31}) + C_2\cdot\Delta_{21}\sin(2\Delta_{31}) + C_3\cdot\Delta_{21}^2 + C_4\cdot\Delta_{21}\sin^2(\Delta_{31})$$
$$= C_1\cdot B_1(E) + C_2\cdot B_2(E) + C_3\cdot B_3(E) + C_4\cdot B_4(E)$$

$$\Delta_{jk}\equiv\frac{\Delta m^2_{jk}}{4E}$$

If U is 3×3 unitary

$$C_1 = 4\left|U_{\mu3}U^*_{e3}\right|^2,\ C_2 = 4\mathrm{Re}\left[U_{\mu3}U^*_{e3}U^*_{\mu2}U_{e2}\right],$$
$$C_3 = -4\mathrm{Re}\left[U_{\mu2}U^*_{e2}U^*_{\mu1}U_{e1}\right],\ C_4 = -8\mathrm{Im}\left[U_{\mu3}U^*_{e3}U^*_{\mu2}U_{e2}\right]$$

New relation ξ corresponding to unitarity condition of PMNS matrix

$$U_{\mu1}U^*_{e1} + U_{\mu2}U^*_{e2} + U_{\mu3}U^*_{e3} = 0 \quad\Rightarrow\quad \boldsymbol{\xi \equiv C_1(C_3 - C_2) - C_2^2 - \frac{C_4^2}{4} = 0}$$

If unitary → ξ = 0
If non-unitary → ξ ≠0

ξ is a new criterion of unitarity test independent of the parametrization

Statistical analysis

Definition of χ^2 :
$$\chi^2 \equiv \sum_j\left[\frac{P^{\mathrm{obs}}(E_j) - \sum_{k=1}^4 C_k\cdot B_k(E_j)}{\Delta P^{\mathrm{obs}}(E_j)}\right]^2 = \left(\boldsymbol{P}^{\mathrm{obs}} - \mathcal{B}\boldsymbol{C}\right)^T W\left(\boldsymbol{P}^{\mathrm{obs}} - \mathcal{B}\boldsymbol{C}\right)$$

j runs over energy bins

$$\boldsymbol{P}^{\mathrm{obs}} \equiv \begin{pmatrix}P^{\mathrm{obs}}(E_j)\\ \vdots\\ P^{\mathrm{obs}}(E_n)\end{pmatrix},\ \mathcal{B}\equiv\begin{pmatrix}B_1(E_1) & \cdots & B_4(E_1)\\ \vdots & \ddots & \vdots\\ B_1(E_n) & \cdots & B_4(E_n)\end{pmatrix},\ \boldsymbol{C}\equiv\begin{pmatrix}C_1\\ \vdots\\ C_4\end{pmatrix}$$

$$W \equiv \mathrm{diag}\left(\frac{1}{\left(\Delta P^{\mathrm{obs}}(E_1)\right)^2},\ \cdots\ ,\frac{1}{\left(\Delta P^{\mathrm{obs}}(E_n)\right)^2}\right)$$

Can be solved analytically

Best fit points of coefficient $C_1 \sim C_4$

$$\frac{d}{d\boldsymbol{C}}\chi^2 = 0 \Longrightarrow \boldsymbol{C} = \left(\mathcal{B}^TW\mathcal{B}\right)^{-1}\mathcal{B}^TW\boldsymbol{P}^{\mathrm{obs}}$$

Unitarity test

◆ Set up

Generate 10^6 virtual-experiments and χ^2 fit with 3-gen model

Generate events by 4-gen ν oscillation model

Generate events by 3-gen ν oscillation model

↓ ↓

Fitting 3-gen model to each events

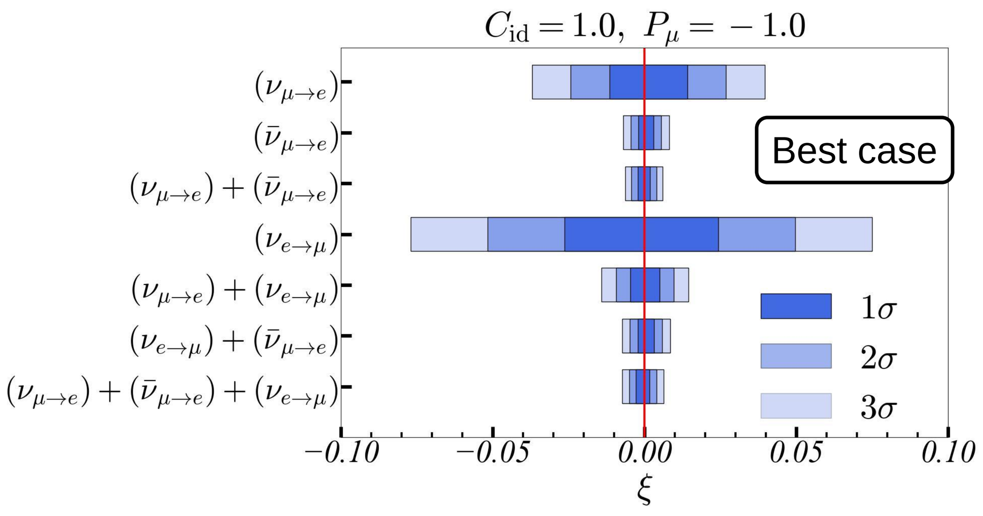
※Probability distribution is assumed to be binomial distribution

- We assume T2HK and neutrino factory with ν_e at J-PARC
- Consider the charge identification efficiency at Hyper-K detector (C_{id})
- The muon beam polarization must be considered at the neutrino factory (P_μ)

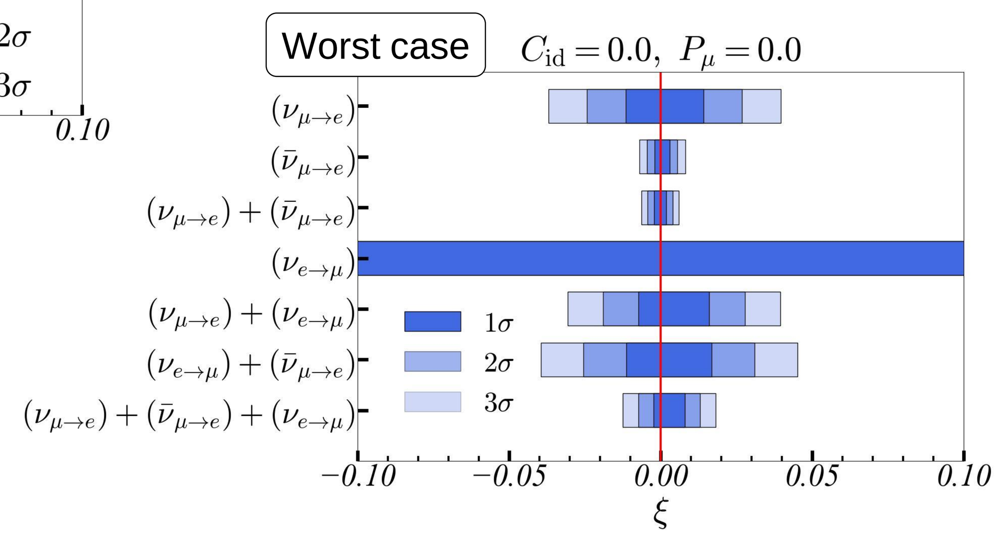
Result

◆ Fit **3-gen** model to **3-gen** events

Test of consistency between 3-gen model and 3-gen events in terms of ξ



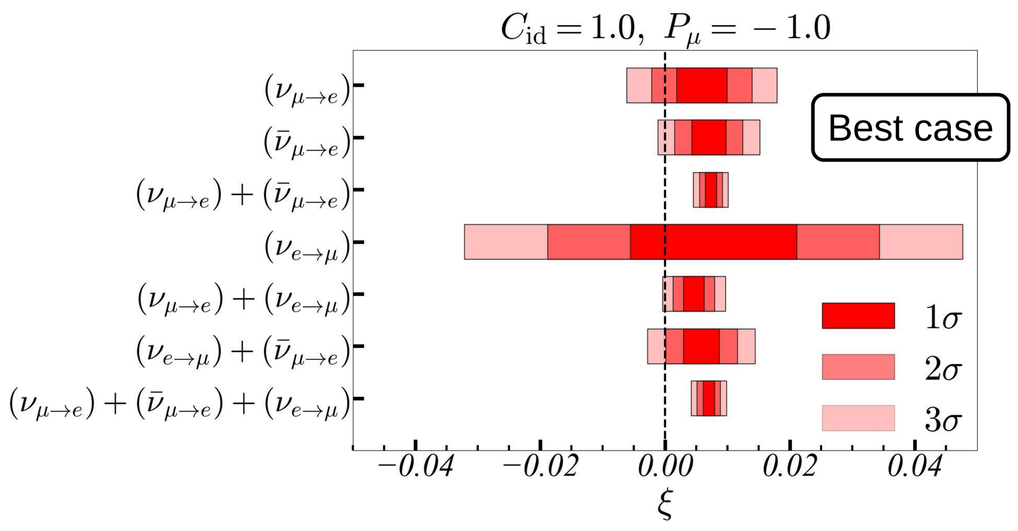
- All analyses are consistent with $\xi = 0$
- Combining multiple channels yields a much smaller statistical error



- Charge identification and muon polarization are important factor, particularly in the analysis using $(\nu_{e\to\mu})$

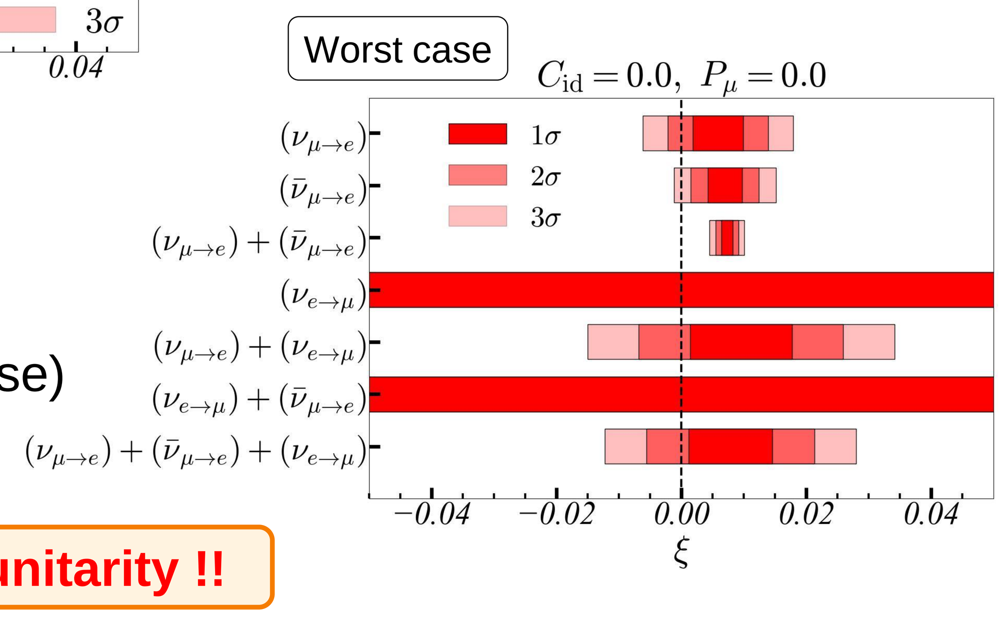
◆ Fit **3-gen** model to **4-gen** events

Test of inconsistency between 3-gen model and 4-gen events in terms of ξ



- Combining multiple channels yields a much smaller statistical error

- Inconsistent with $\xi = 0$ at more than 3σ,
  - $(\nu_{\mu\to e})+(\bar\nu_{\mu\to e})$
  - $(\nu_{\mu\to e})+(\bar\nu_{\mu\to e})+(\nu_{e\to\mu})$ (Best case)



Sufficient to test the violation of unitarity !!

Summary

- Studying a general lepton mixing is important to understand neutrino mass.
- One method to explore a general lepton mixing matrix is testing unitarity of it.
- We propose a method for testing unitarity that does not depend on the parameterization of the PMNS matrix, and demonstrate its application using the new criterion ξ.
- Fitting 3-gen model to 4-gen results in a 3σ inconsistency with ξ = 0, in the $(\nu_{\mu\to e})+(\bar\nu_{\mu\to e})$ from T2HK and in the $(\nu_{\mu\to e})+(\bar\nu_{\mu\to e})+(\nu_{e\to\mu})$ from T2HK+neutrino factory
- In this study, we consider neutrino oscillations in vacuum, but for a more realistic analysis, matter effects must be taken into account.