Comprehensive Bayesian Exploration of Froggatt-Nielsen Mechanism

Keiichi Watanabe (ICRR, U. Tokyo)

Based on arXiv: 2412.19484 (accepted in JHEP)

with M. Ibe (ICRR) and S. Shirai (IPMU)

Underground Rare Events Young Researchers' Workshop
@ U. Toyama, 2025.3.6

Contents

Standard Model (SM)

Quark

u c t

d s b

Gauge boson

γ g

W Z

Lepton

e μ τ

ν_e ν_μ ν_τ

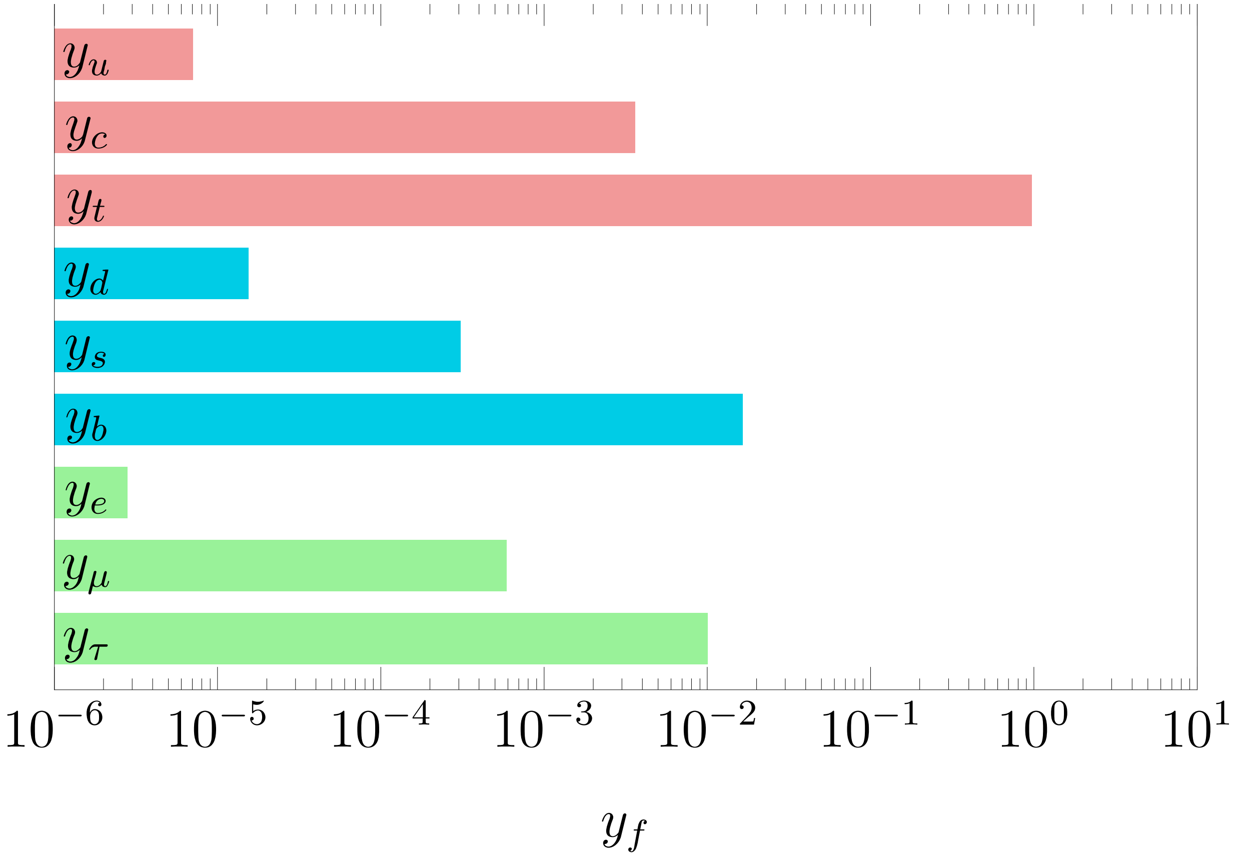
Higgs

h

- Very successful
- Repetitive structure of (quarks and leptons) $\times$ **3**

Fermion mass structure

- There is a hierarchical mass structure

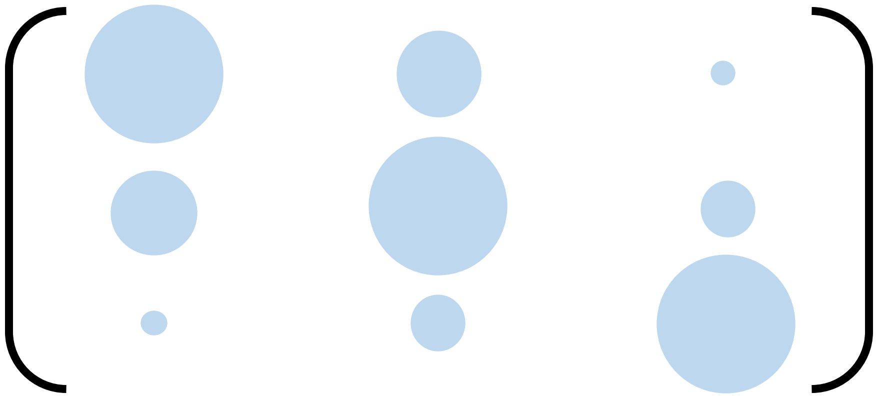


e.g. , $\dfrac{y_t}{y_u} = O(10^5)$

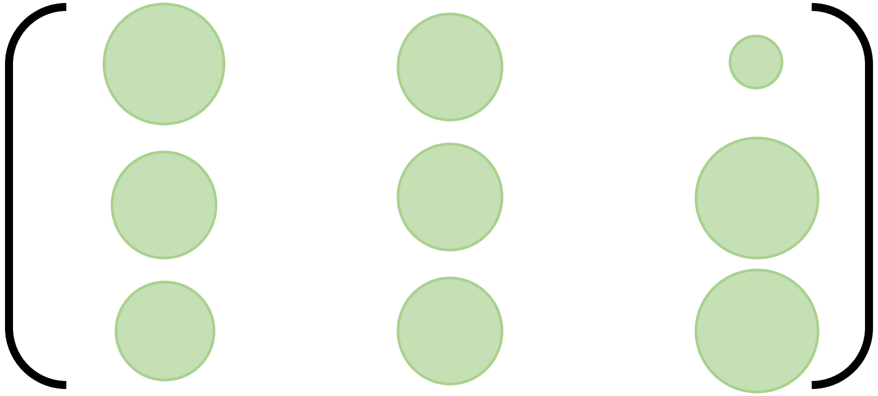
Structure of fermion mixings

- Mixing matrices have distinctive structures

$$|V_{\mathrm{CKM}}| =$$



$$|V_{\mathrm{PMNS}}| =$$



Flavor puzzle

- Hierarchical masses and angles do not arise from $O(1)$ fundamental Yukawa's,

$$\mathcal{L} = y_{ij}\, F_i\, F_j\, h, \qquad y = \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

- $y_f \ll 1$ (except for y_t) is a significant issue

Contents

1. Flavor structure in SM

2. U(1) flavor symmetry

3. “Good” charge assignments

4. Summary

Froggatt-Nielsen (FN) mechanism

C.D.Froggatt and H.B.Nielsen, Nucl,Phys.B 147 (1979)

- SM fermions with new U(1) charges.

  For example,

$$Q_1 \ : \ f_{Q1}$$

$$Q_2 \ : \ f_{Q2}$$

$$\bar{u}_1 \ : \ f_{\bar{u}1}$$

- Only limited Yukawa interactions are allowed

$$\mathcal{L} = \kappa_{ij}\, F_i\, F_j\, h, \qquad \kappa_{ij} = O(1), \qquad \text{for } f_{Fi} + f_{Fj} = 0$$

Froggatt-Nielsen (FN) mechanism

- With a new scalar $\Phi : -1$, new operators can be written

$$\mathcal{L} = \kappa_{ij} \left(\frac{\Phi}{M_*}\right)^{f_{Fi}+f_{Fj}} F_i\, F_j\, h \quad , \qquad \kappa_{ij} = O(1)$$

Here, M_* is a very high scale.

FN mechanism

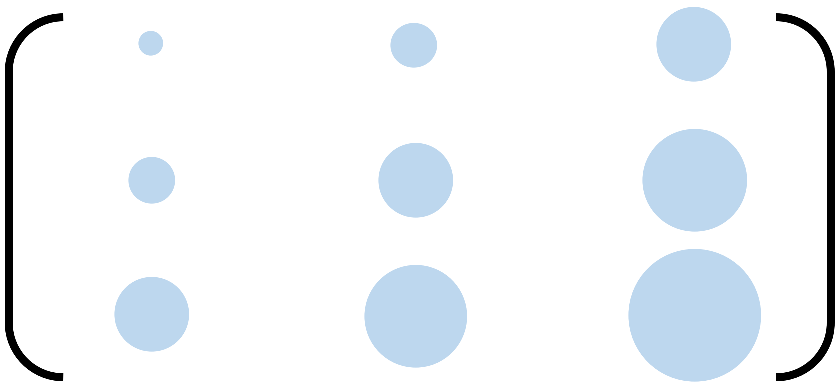
- If Φ gets VEV and U(1) is broken, Yukawa interactions arise

$$\kappa_{ij} \left(\frac{\Phi}{M_*}\right)^{f_{Fi}+f_{Fj}} F_i\, F_j\, h \quad \xrightarrow[\Phi \;\to\; \langle\Phi\rangle]{} \quad \underline{\kappa_{ij} \left(\frac{\langle\Phi\rangle}{M_*}\right)^{f_{Fi}+f_{Fj}}}\, F_i\, F_j\, h,$$

$$y_{ij} = \kappa_{ij} \times \epsilon^{f_{Fi}+f_{Fj}}, \qquad \epsilon = \langle\Phi\rangle / M_* = O(0.1)$$

Yukawa structure

$$\kappa = \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix} \longrightarrow$$

$y =$ 

- κ distribution $\longrightarrow$ $y_f,\, s_\theta$ prediction

L. Hall, H. Murayama and N. Weiner,
PRL 84 (2000) 2572-2575

N. Haba and H. Murayama,
PRD 63 (2001) 053010

Motivation

- What charges are good ?

  Are there any charge assignments that are entirely unknown?

- How can we distinguish many good FN charges ?

  We aim to get closer to understanding the origin of flavor structure.

Contents

How to find good charges

- Compare the plausibility of
  multiple different FN charge assignments

- We adopt **Bayesian inference**

(J. Bergstrom, D. Meloni and L. Merlo, PRD 89 (2014) 9, 093021)

Bayes theorem

$$\mathrm{P}(M_i|Data)$$
$$\propto \mathrm{P}(Data|M_i) \cancel{\times \mathrm{P}(M_i)}$$
$$\boldsymbol{Z_i}$$

$\mathrm{P}(M_i|Data)$: posterior probability of models

$\mathrm{P}(M_i)$: prior probability of models

$\mathrm{P}(Data|M_i)$: marginal likelihood

Bayes factor

$$\frac{\mathrm{P}(M_i|Data)}{\mathrm{P}(M_j|Data)} = \frac{Z_i}{Z_j} \times \frac{\mathrm{P}(M_i)}{\mathrm{P}(M_j)}$$

$$\boldsymbol{B_{ij}}$$

Bayes factor: Comparison of the plausibility of two models

Jeffreys scale

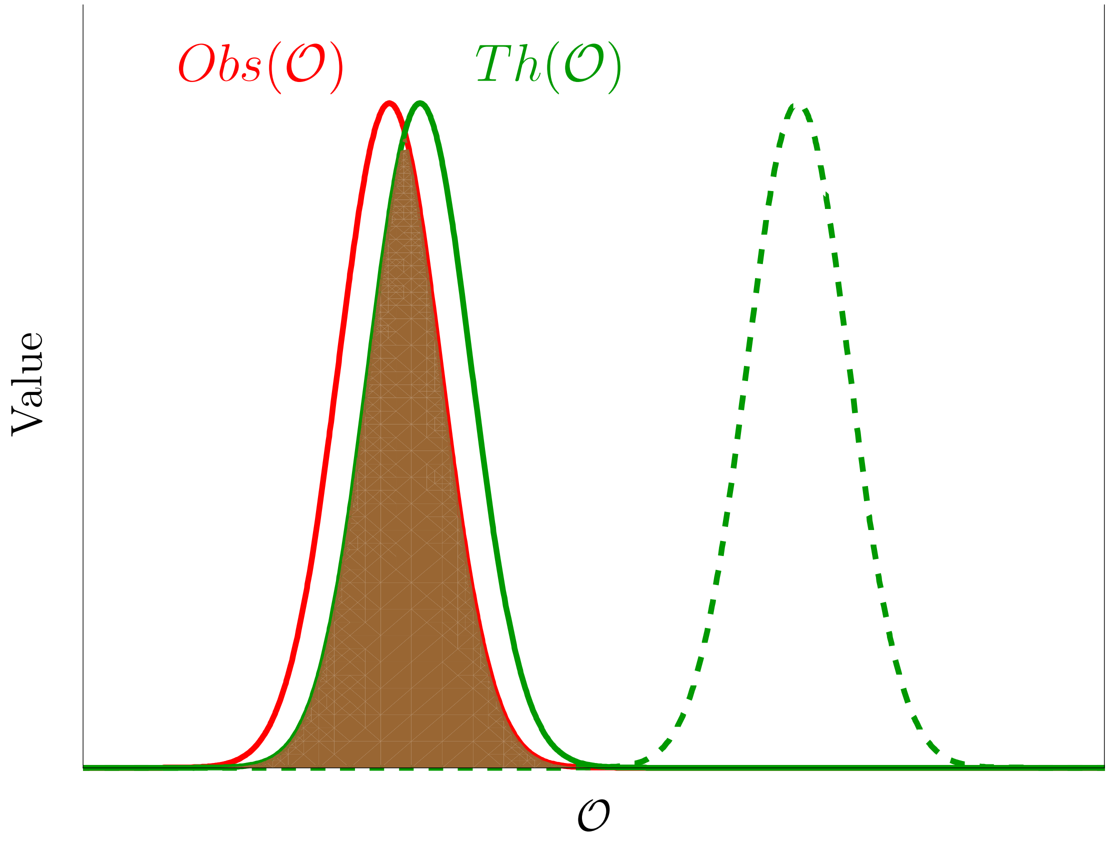
$$\frac{P(M_i|Data)}{P(M_j|Data)} = \frac{Z_i}{Z_j} \times \frac{P(M_i)}{P(M_j)}$$

$\boldsymbol{B_{ij}}$

B_{ij}	Strength of evidence
1 to 3.2	Not worth more than a bare mention
3.2 to 10	Substantial
10 to 100	Strong
> 100	Decisive

Z (picture)

$$Z = \mathrm{P}(Data|\kappa, \epsilon, f_F) = \int d\mathcal{O}\, Obs(\mathcal{O}) \times Th(\mathcal{O})$$

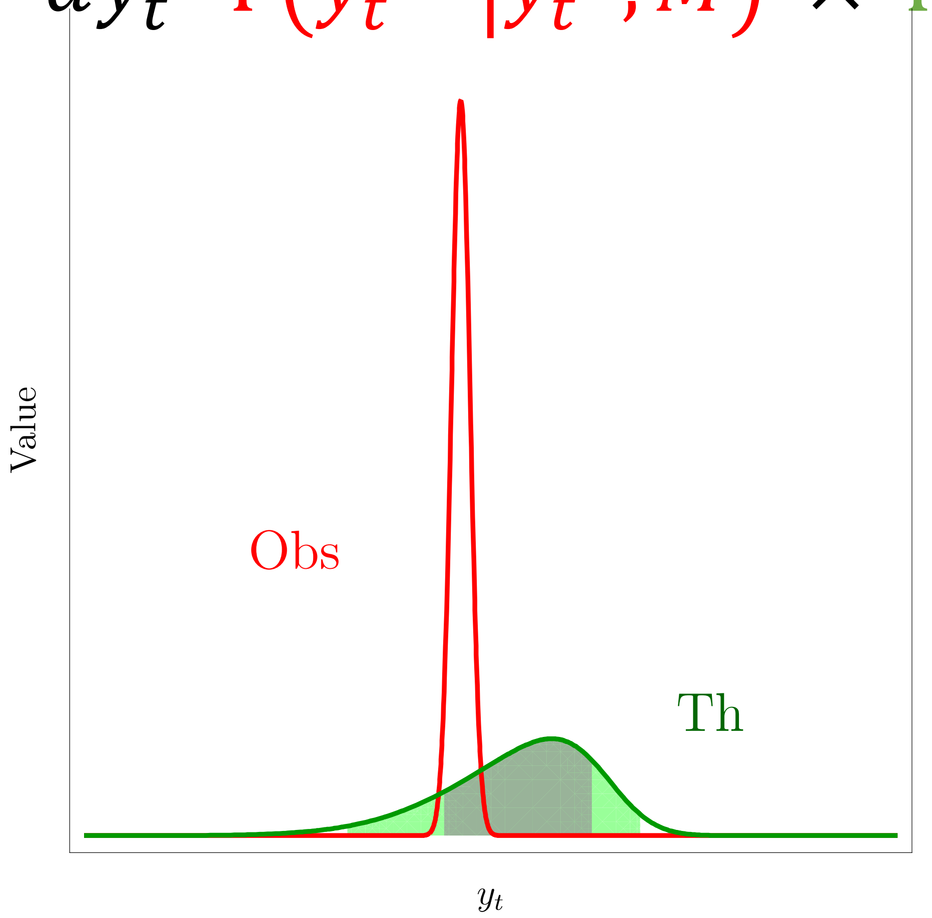


Large overlap $\longrightarrow$ Large Z

Small overlap $\longrightarrow$ Small Z

Concrete function forms

$$Z = \int dy_t^{\mathrm{th}}\, \mathrm{P}\left(y_t^{\mathrm{obs}} \middle| y_t^{\mathrm{th}}, M\right) \times \mathrm{P}\left(y_t^{\mathrm{th}} \middle| M\right)$$



Parameters

Fermion Yukawa $y_{u,c,t}$, $y_{d,s,b}$, $y_{e,\mu,\tau}$

CKM and PMNS parameters

$s_{12}^{\mathrm{CKM}}, s_{23}^{\mathrm{CKM}}, s_{13}^{\mathrm{CKM}}, \delta^{\mathrm{CKM}},$

$s_{12}^{\mathrm{PMNS}}, s_{23}^{\mathrm{PMNS}}, s_{13}^{\mathrm{PMNS}}, \delta^{\mathrm{PMNS}}$

Neutrino mass ratio (normal ordering, Majorana)

$\Delta m_{12}^2/\Delta m_{13}^2 = (m_1^2 - m_2^2)/(m_1^2 - m_3^2)$

Results (quark + lepton)

- We found $O(10^2)$ charges which can explain the SM flavor structure

95% range

f_Q	$f_{\bar{u}}$	$f_{\bar{d}}$	f_L	$f_{\bar{e}}$	$\log_{10}(\mathcal{Z}_C/\mathcal{Z}_{0,C})$	ϵ
$5,3,0$	$7,3,1$	$6,5,5$	$10,-9,-9$	$1,3,5$	150.55 ± 0.03	$0.301 \to 0.322$
$5,3,0$	$8,4,1$	$6,6,6$	$10,-9,-9$	$2,2,4$	150.49 ± 0.02	$0.336 \to 0.349$
$5,3,0$	$7,3,1$	$6,5,5$	$10,9,-9$	$3,1,-5$	150.48 ± 0.02	$0.301 \to 0.321$
$5,3,0$	$7,3,1$	$6,5,5$	$3,-2,-2$	$9,9,6$	150.42 ± 0.05	$0.307 \to 0.328$
$5,3,0$	$8,4,1$	$6,6,6$	$3,-2,-2$	$9,9,7$	150.41 ± 0.02	$0.335 \to 0.349$

- Posterior ratio = $\mathbf{10^{150}}$ × Prior ratio
- This is a triumph of FN mechanism！

Contents

Summary

- We first quantitatively discussed good FN charges explaining the flavor structure of the SM
- We found that there are $O(10^2)$ such charges
- Prediction of flavor changing processes can depend on the charge assignments (next Chitose-kun's talk)