超新星の一般相対論的ボルツマン輻射流体計算

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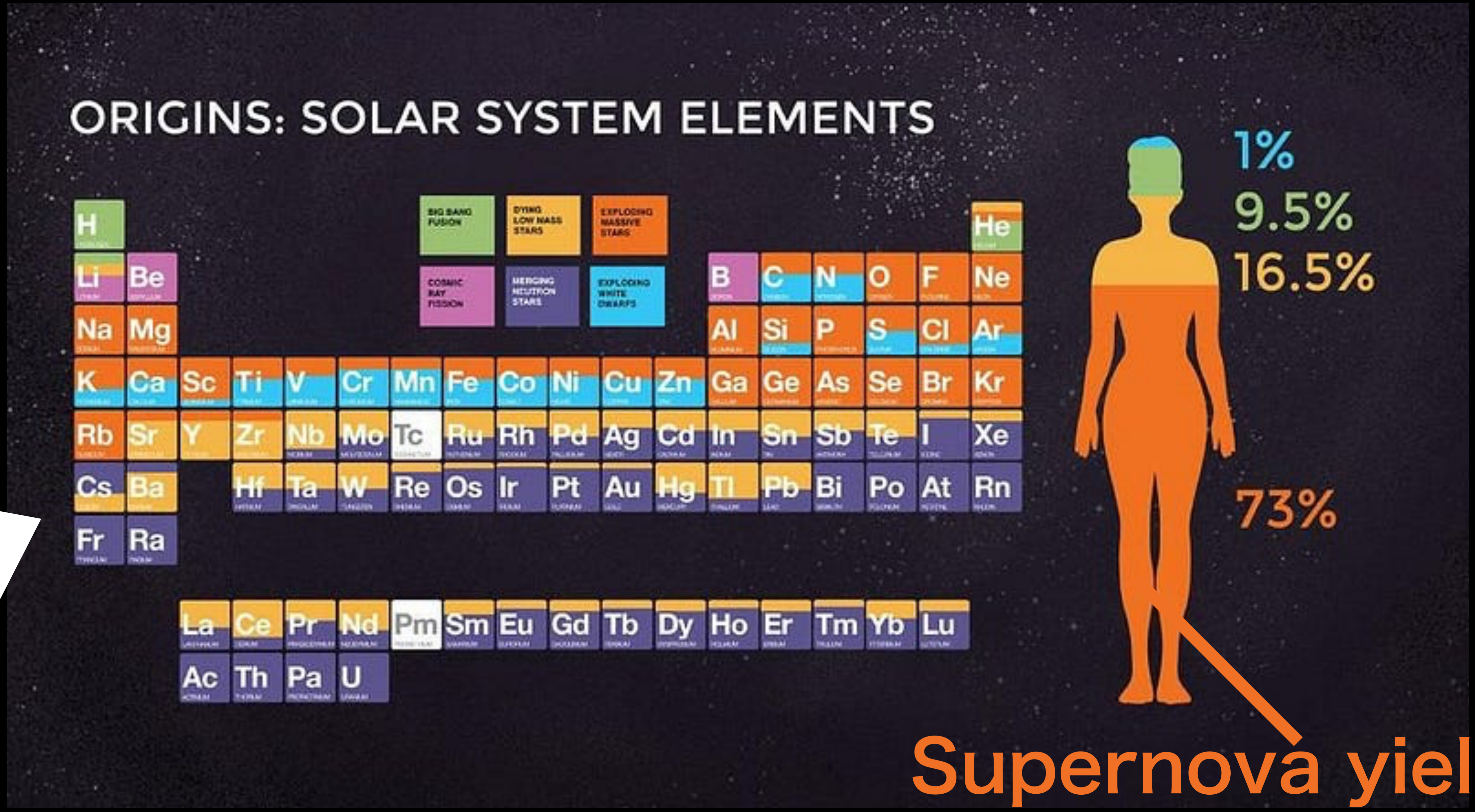
Core-collapse Supernovae (CCSNe)

- Energetic explosion at the end of stellar evolution.
- Plays central role for the evolution of the universe.

Explosive nucleosynthesis

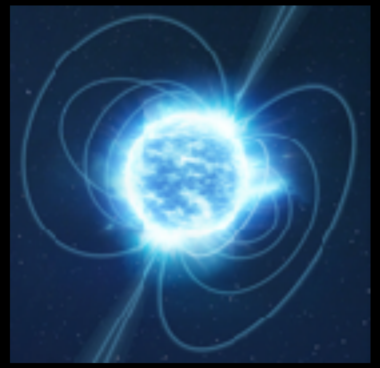


Supernova yield

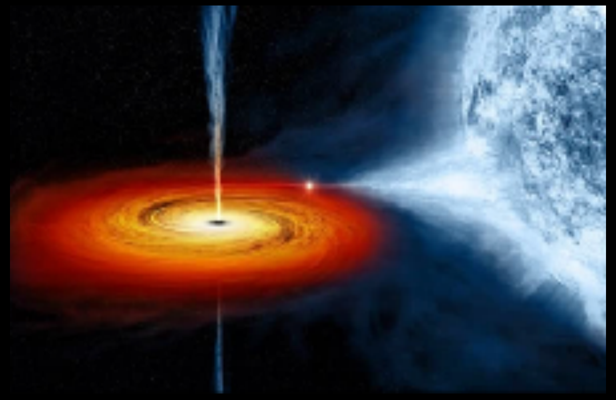
Supernova explosion

Formation of compact stars

Neutron stars



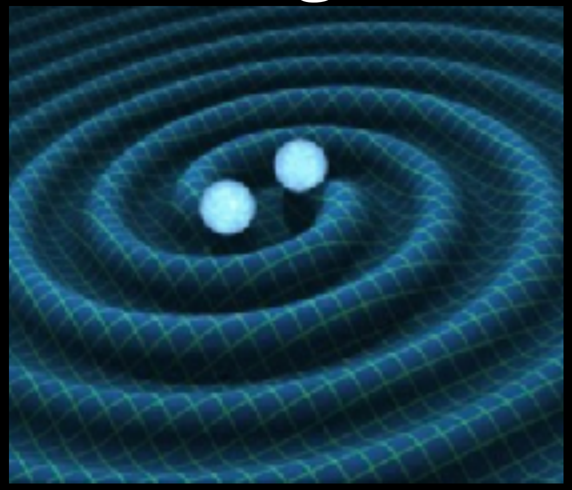
Black holes



High energy astrophysical phenomena

GRB

Compact binary mergers



Scenario of CCSN

Probe Physics with Multi-messenger Observation

- Multi-messenger observation with neutrinos/GWs provide important information

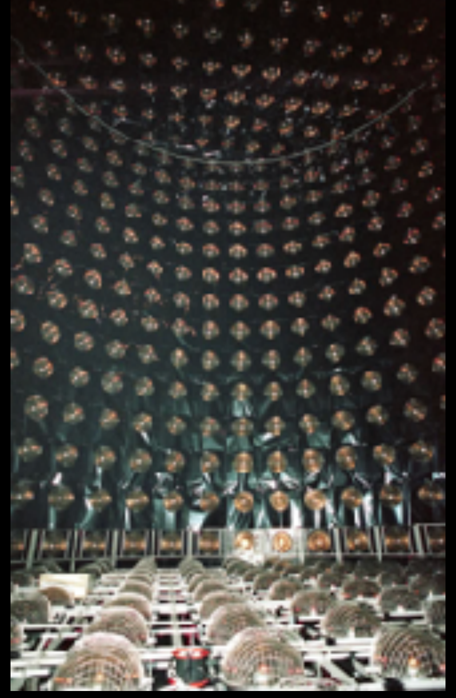
1987

Neutrinos from SN1987A

SN1987A

(Credit: Anglo-Australian Observatory)

Kamiokande-II

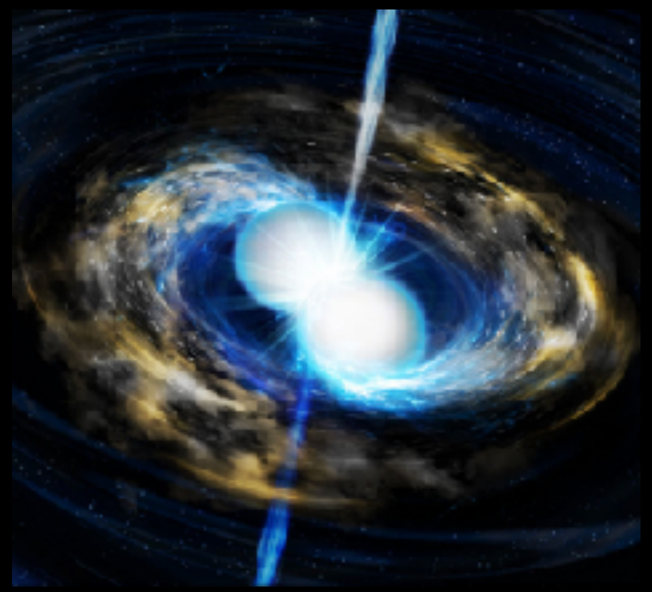


(Credit: ICRR)

- Neutrino deposited in CCSNe
- Upper limit on neutrino mass, charge, # of flavors

2015, 2017

GW from BBHM GW150914、BNSM GW170817



(Credit: NAOJ)

LIGO検出器

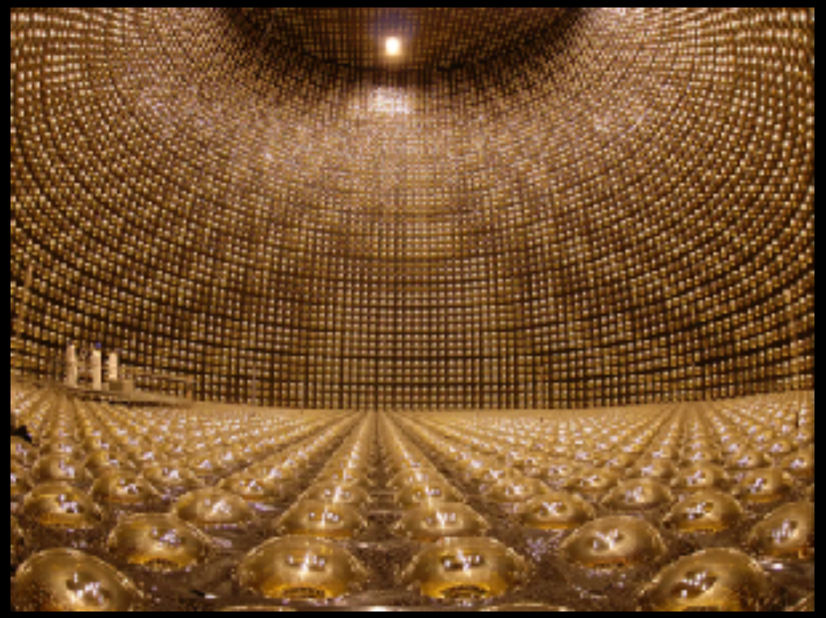
(Credit: LIGO)

- Constraint on nuclear EOS
- Constraint on modified gravity
- SGRB

20XX

Neutrinos/GW from CCSN?

Super-Kamiokande



(Credit: ICRR)

- Further constraint on EOS?
- Constraint on neutrino properties (mass hierarchy...)
- Constraint on beyond SM(e.g. axion-like particle)

Aim of CCSN Simulation

Reproducing Existing Observations

- Reproduce explosion energy, synthesized $^{56}\mathrm{Ni}$ mass inferred from electromagnetic observation
- Current stete-of-the-art simulation still cannot reproduce observed values

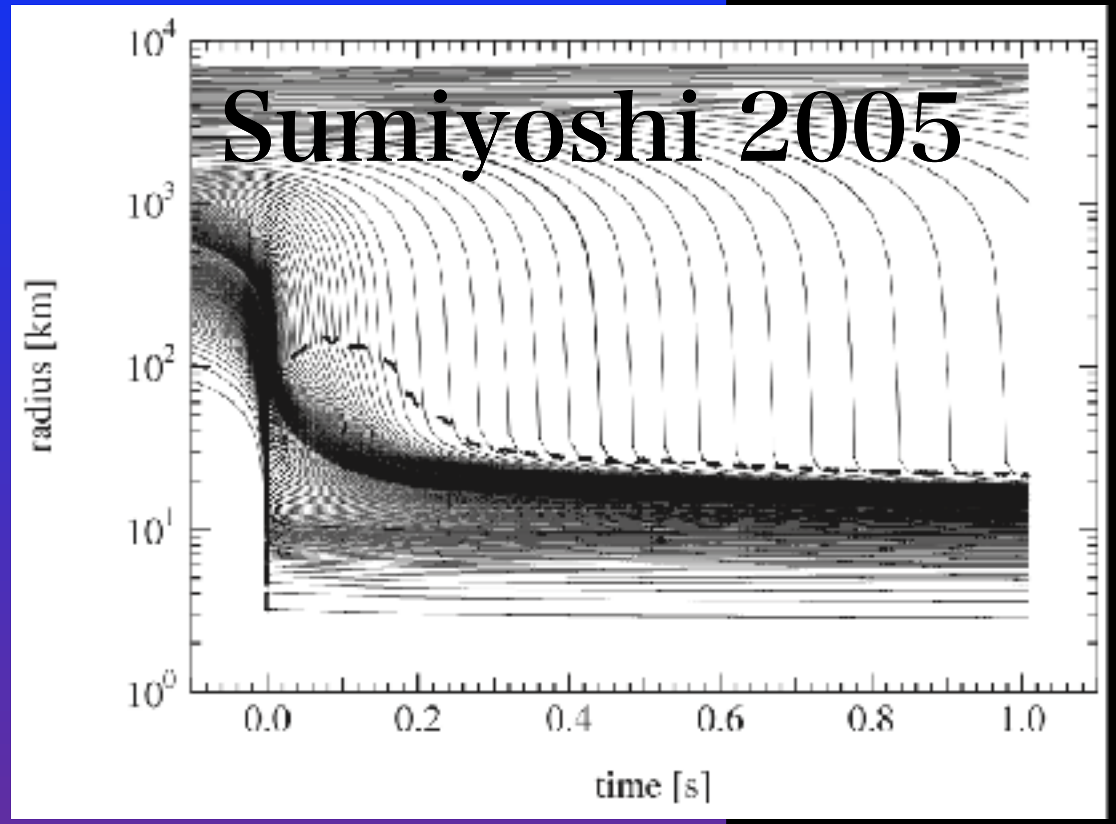
Construct Theoretical Model for Future Observations

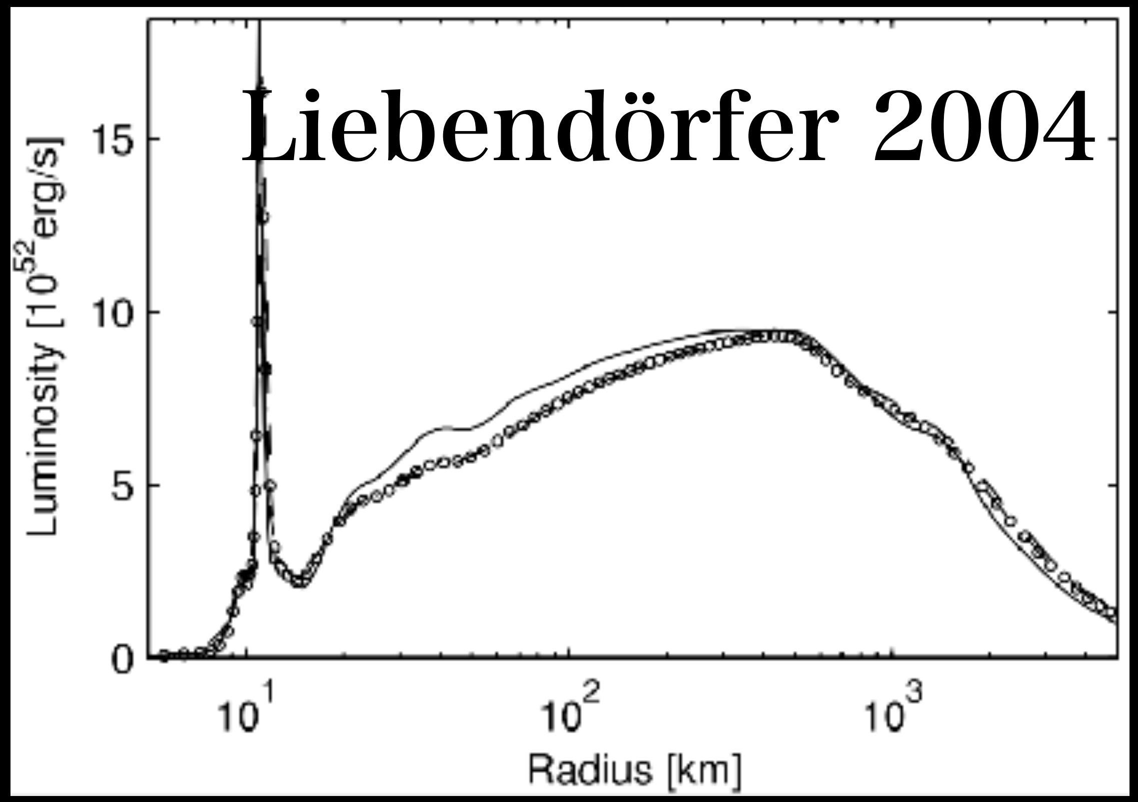
- Accurate theoretical model should be prepared in preparation for future observations.
- Unfortunately, there are still large uncertainties remaining due to numerical methods.

CCSN Simulations

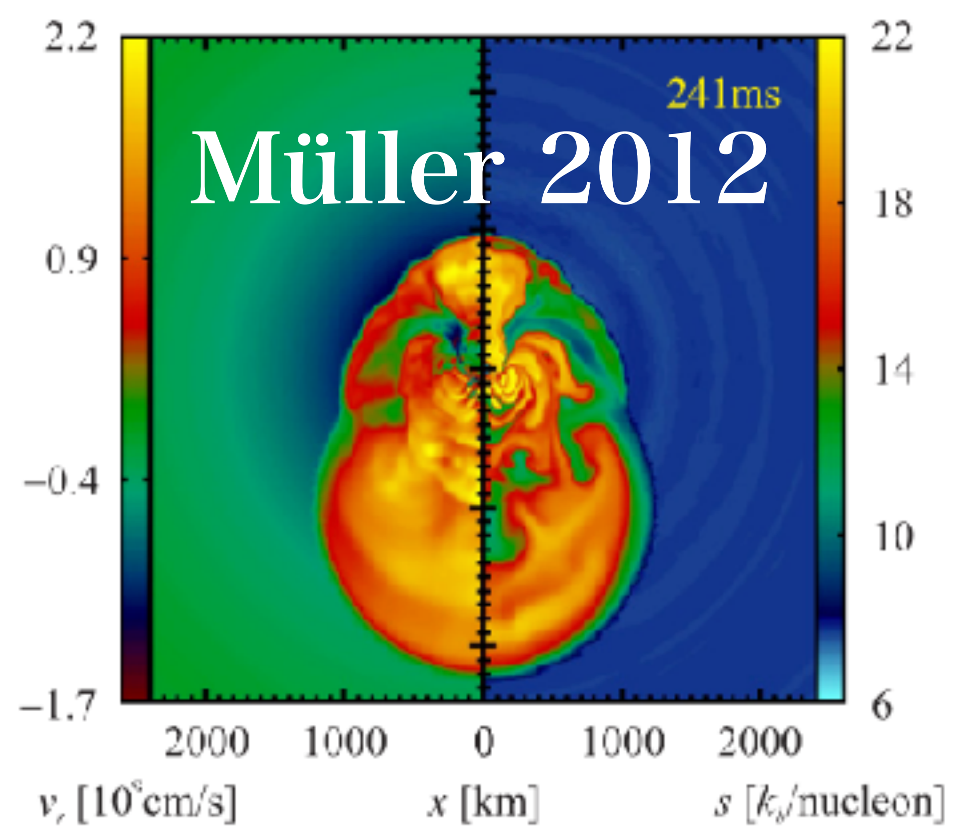
- CCSN is highly nonlinear, and requires numerical simulations to obtain the theoretical understanding.
- Thanks to the advancements of the computers, long-term CCSN simulation in 3D is feasible now.

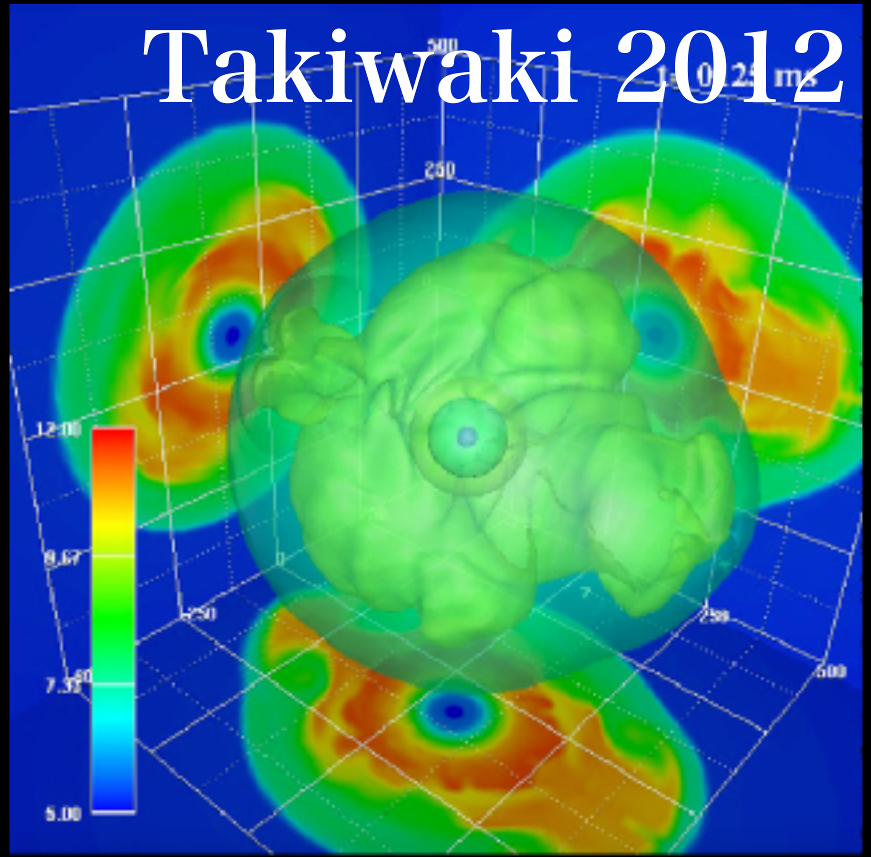
90's ~ 00's: 1D



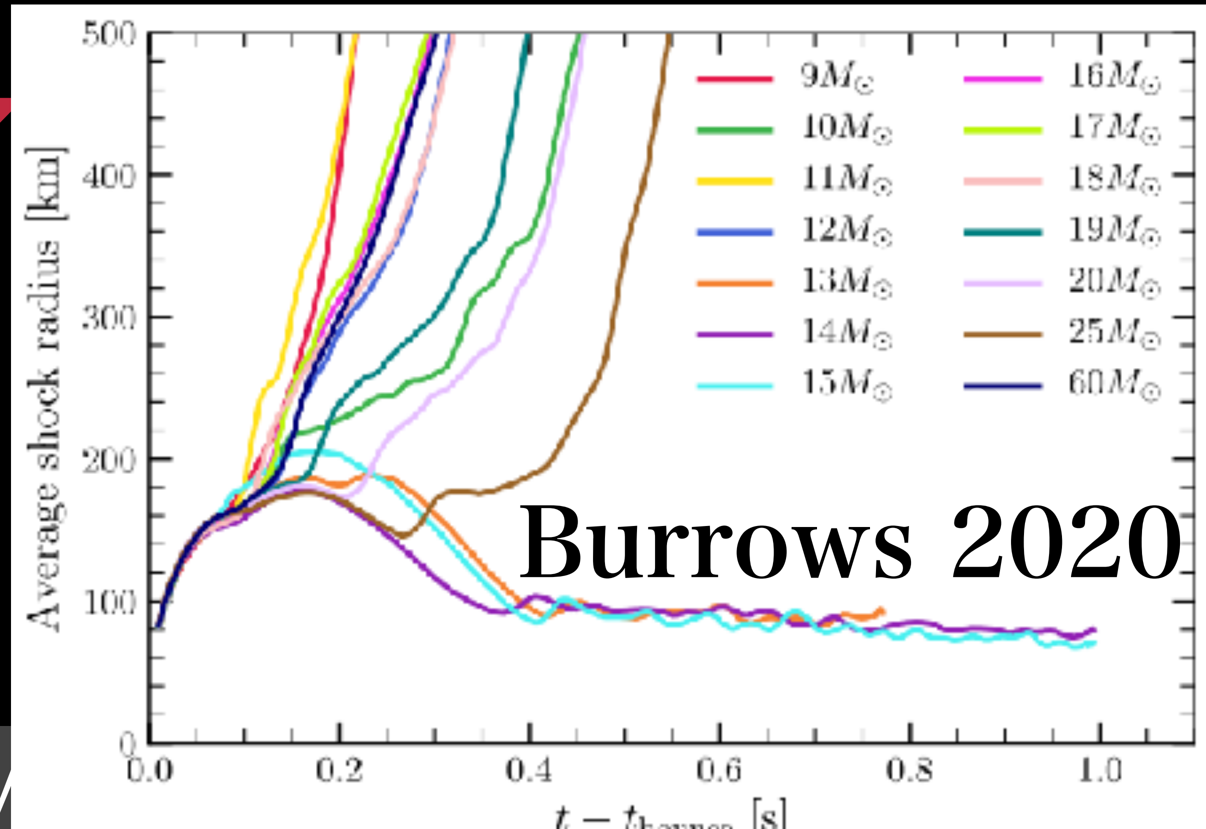


10's: 2D~3D





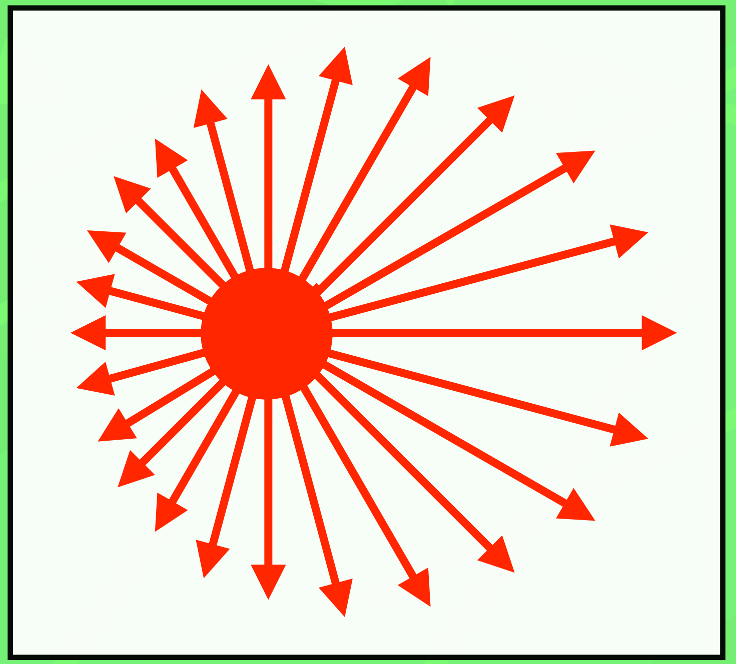
20's: 3D
(many models)
(long term)



Neutrinos inside CCSN

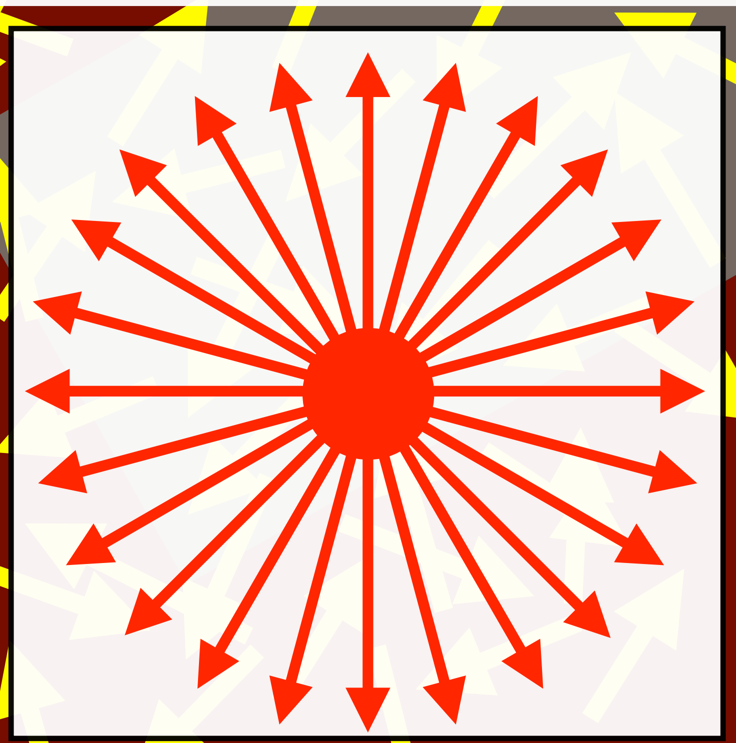
Free streaming

Intermediate: nontrivial



thermal eq. (Fermi-Dirac)

momentum: isotropic



Phase space distribution function $f(x^\mu, p^i)$

Boltzmann equation

$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} = \left[\frac{\delta f}{\delta t}\right]_{\mathrm{coll}}$$

Truncated Moment Method

Distribution Function

$$f(t, r, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$$

Boltzmann Equation

$$\frac{\partial f}{\partial t} + p^i \frac{\partial f}{\partial x^i} + \dot{p}^i \frac{\partial f}{\partial p^i} = C$$

Instead of Boltzmann transport, truncated moment method is often used.

Angular moment in momentum space

Moment eqs. (depend on higher moments)

0th $\frac{\partial E}{\partial t} = L_1(E, M_1^i, M_2^{ij})$

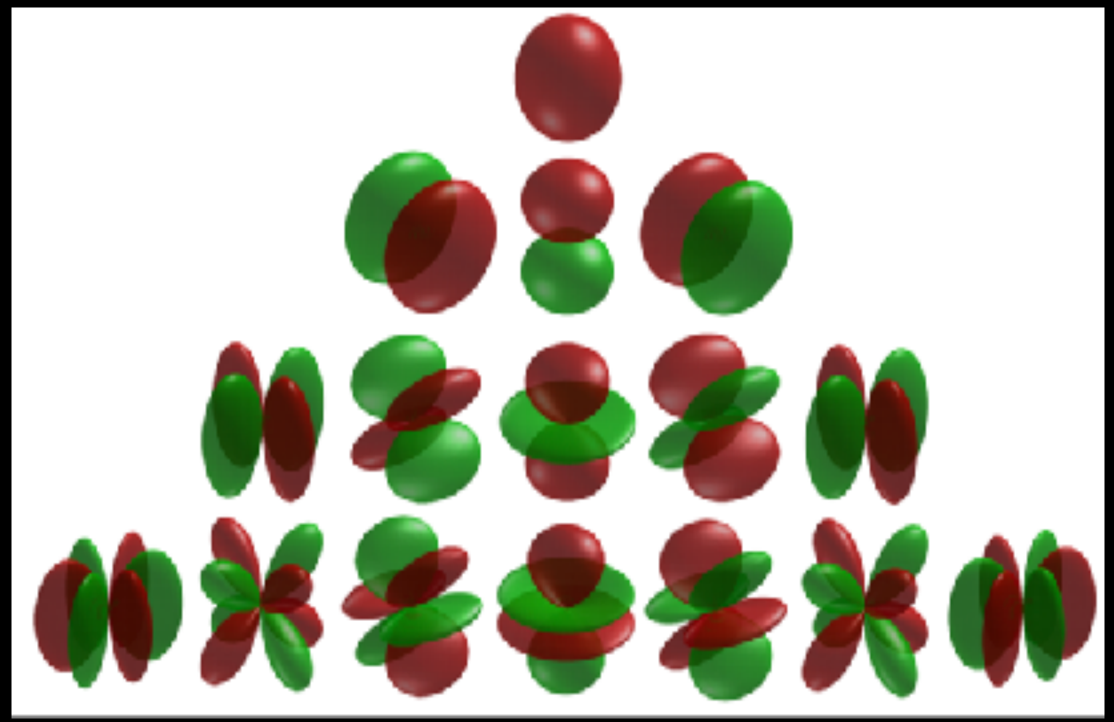
1st $\frac{\partial M_1^i}{\partial t} = L_2(E, M_1^i, M_2^{ij})$

Truncation

Part of momentum space information is lost

2nd $\frac{\partial M_2^{ij}}{\partial t} = L_2(E, M_1^i, M_2^{ij}, M_3^{ijk})$

⋮

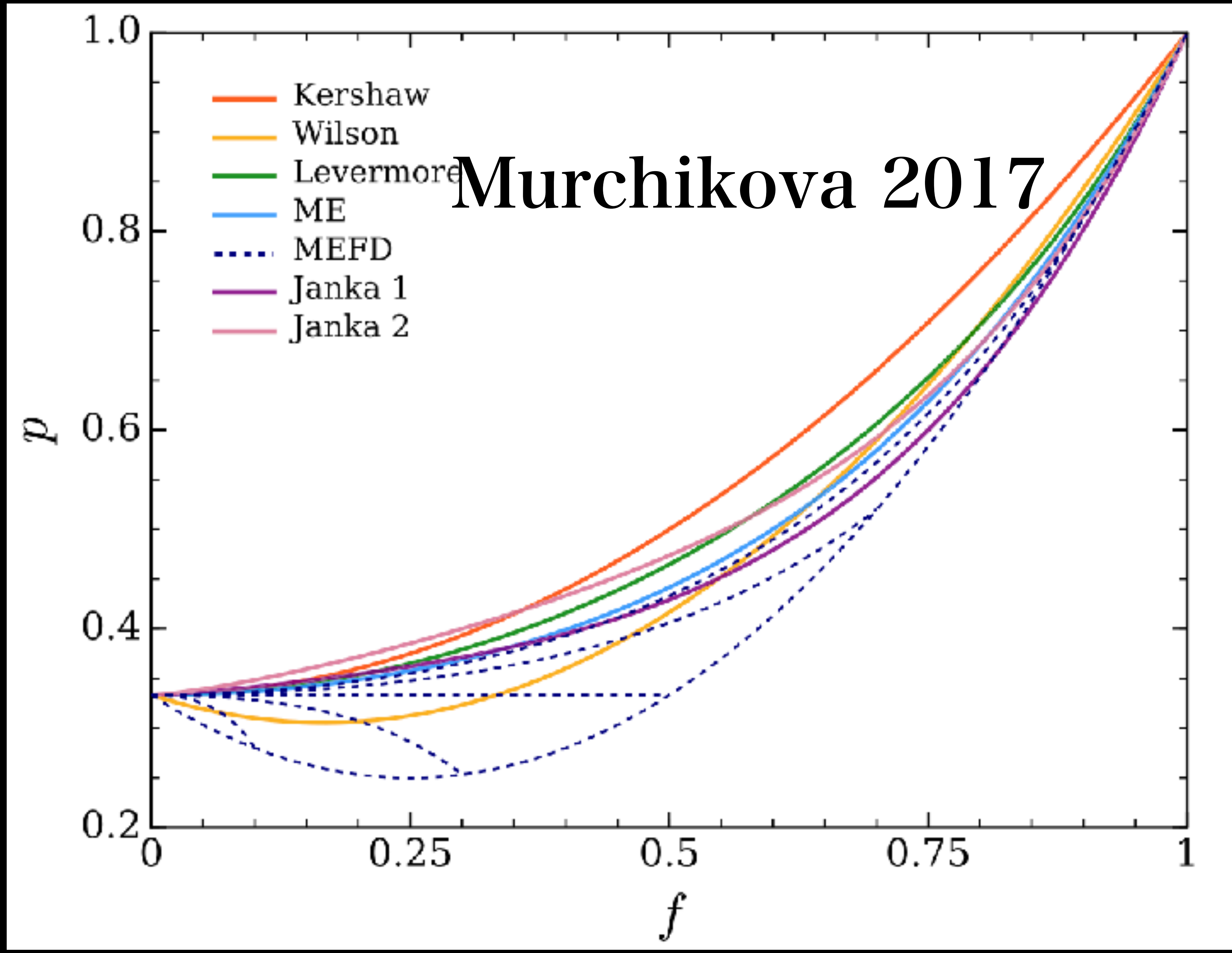


Analytical Closure

Assume closure relation to calculate 2nd moments only from 0th and 1st moments

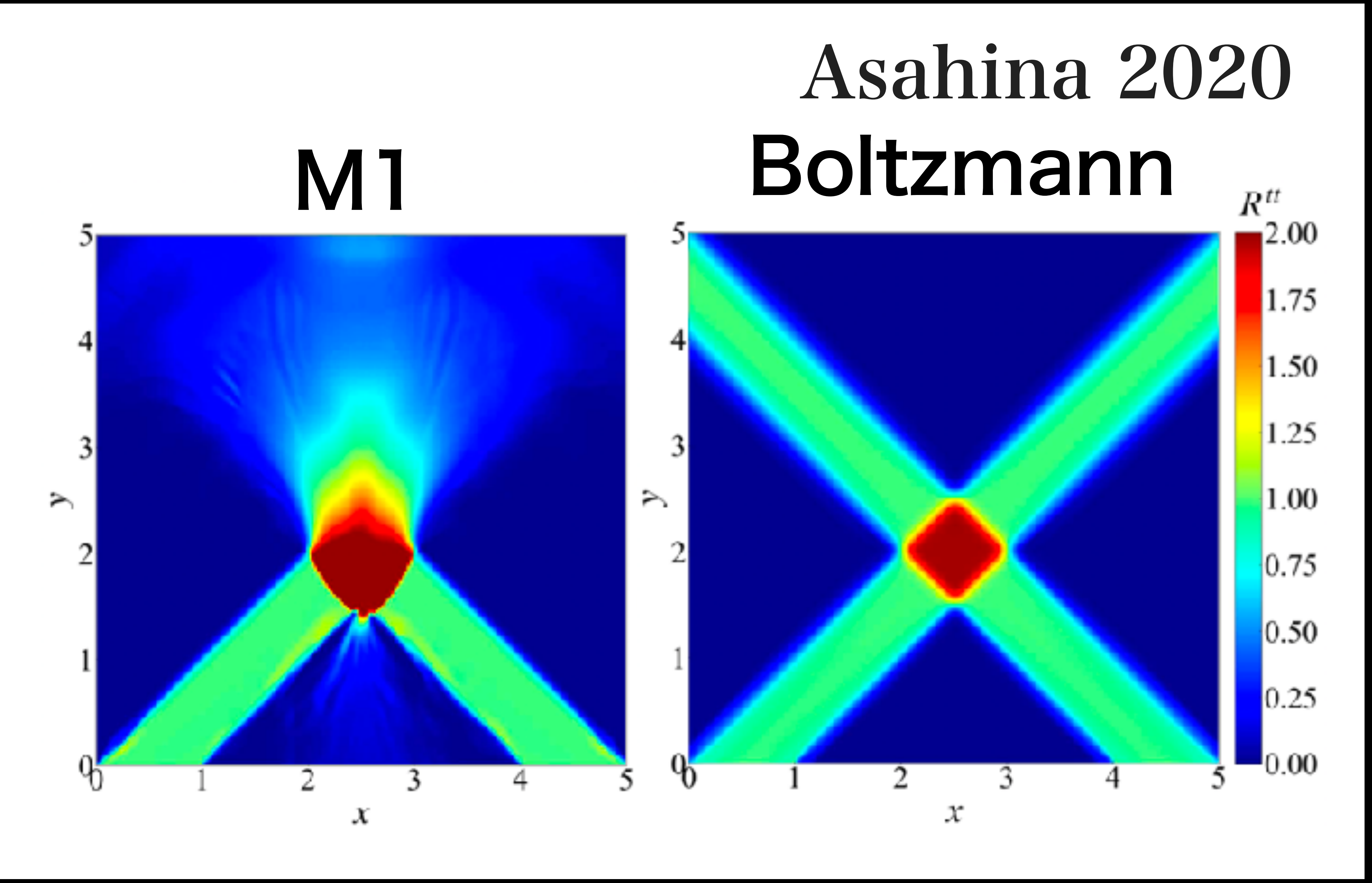
$$P^{ij}_{M1} = \frac{3p-1}{2} P^{ij}_{thin} + \frac{3(1-p)}{2} P^{ij}_{thick}$$

Eddington factor



Flux factor (function of 0th and 1st moment)

Moment method fails to solve ray crossing test



Boltzmann Radiation-hydro Simulation Project

Sumiyoshi 2012

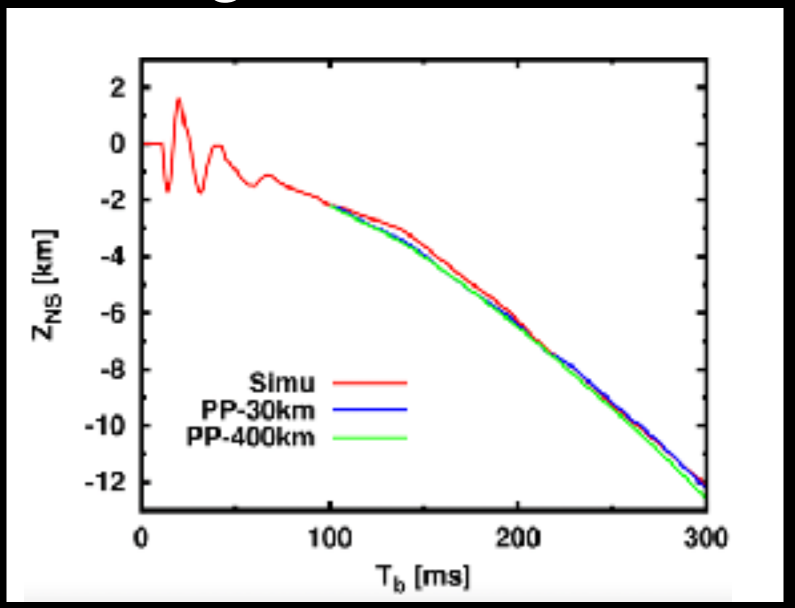
Nagakura 2014
Nagakura 2017
Nagakura 2019

Akaho 2021
Akaho 2023

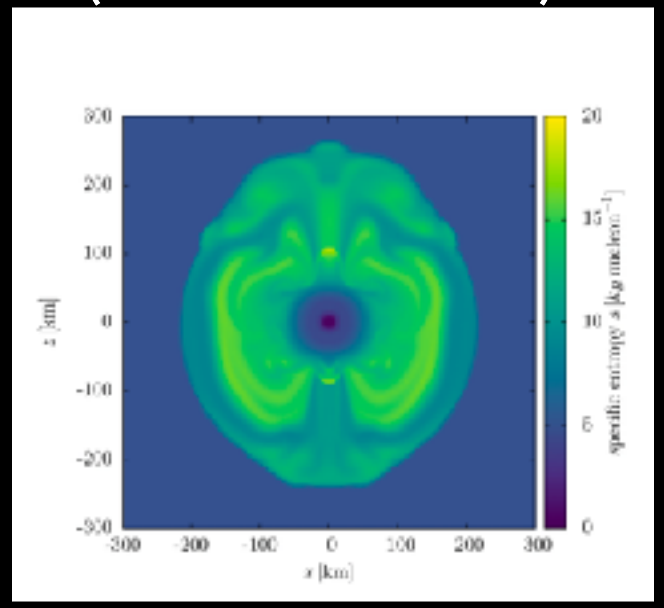
Harada in prep.

Boltzmann solver → **SR Boltzmann + Newtonian hydro** → **GR Boltzmann + GR hydro + 1D metric** → **GR Boltzmann + GR hydro + Numerical Relativity**

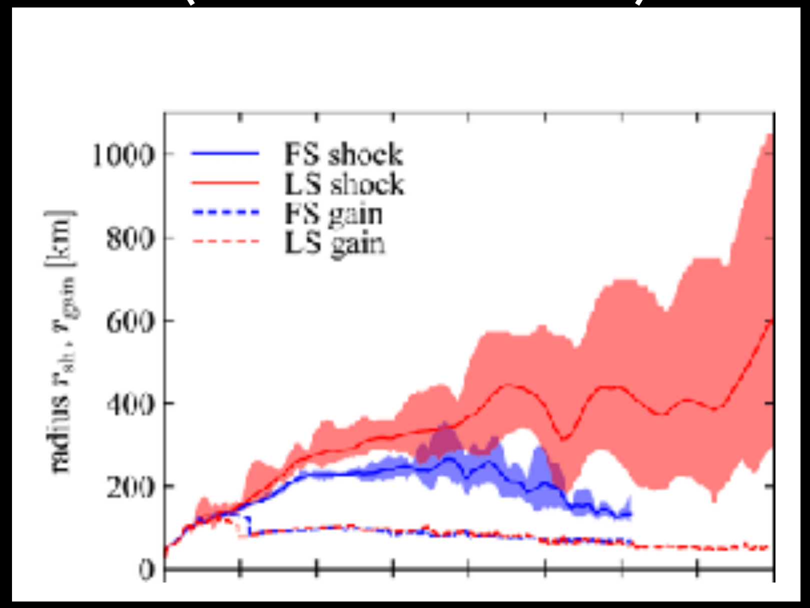
PNS kick
(Nagakura 2019)



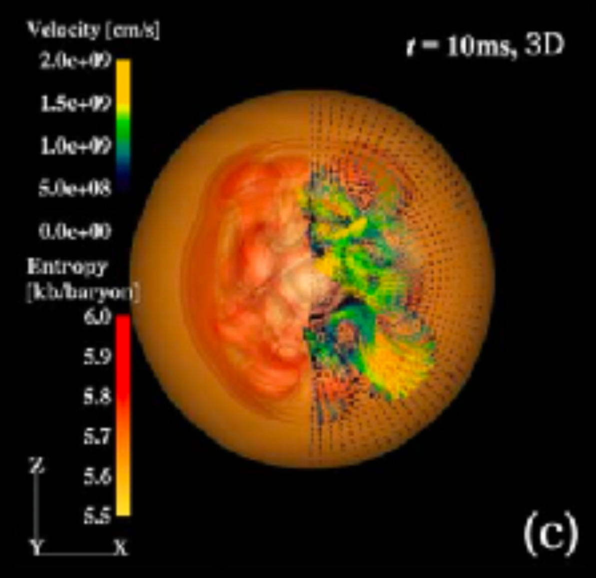
Rotating progenitor
(Harada 2019)



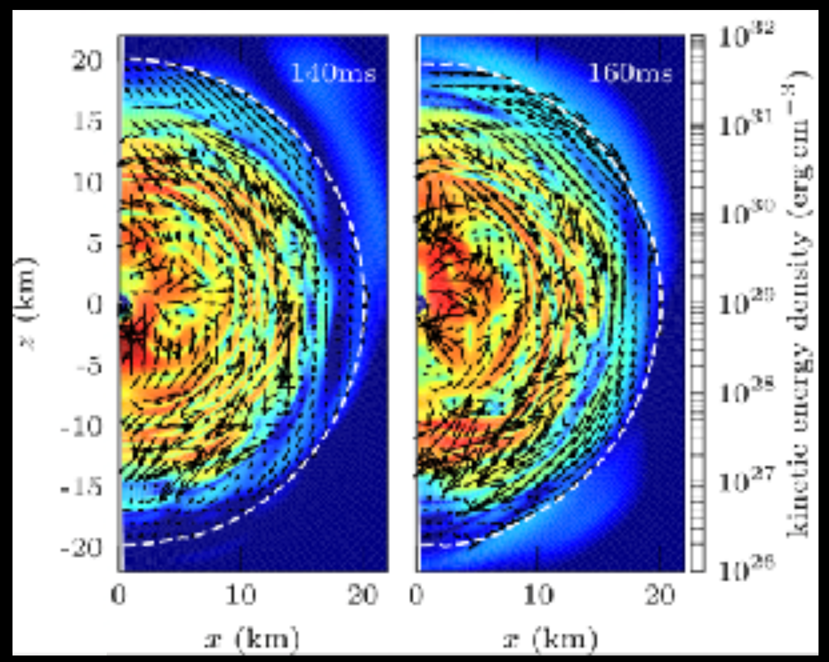
EOS dependence
(Harada 2020)



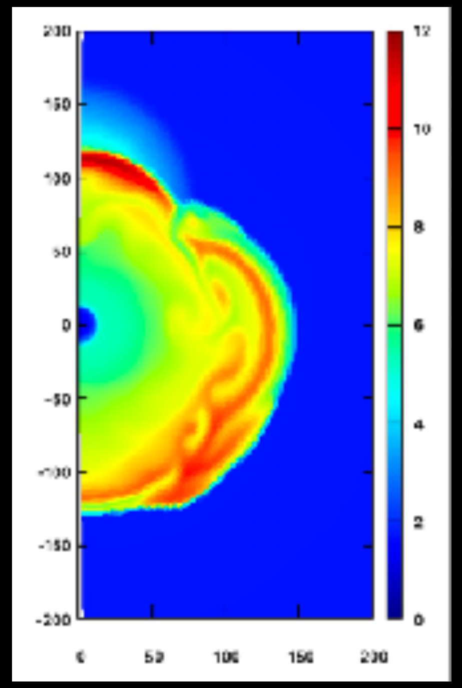
3D simulation
(Iwakami 2020)



PNS convection
(Akaho 2023)



GR CCSN simulation
(Akaho in prep.)



GR Boltzmann Neutrino Radiation Hydrodynamics Code

Boltzmann & hydrodynamics equations are solved together to simulate CCSN

Boltzmann equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\Bigg|_{q_i}\left[\left(e^\mu_{(0)}+\sum_{i=1}^{3} l_i e^\mu_{(i)}\right)\sqrt{-g}f\right]-\frac{1}{\epsilon^2}\frac{\partial}{\partial \epsilon}\left(\epsilon^3 f\omega_{(0)}\right)$$
$$+\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}\left(\sin\theta_\nu f\omega_{(\theta_\nu)}\right)-\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}\left(f\omega_{(\phi_\nu)}\right)=S_{\rm rad}$$

Neutrino-matter interactions

Emission/Absorption

$e^- + p \leftrightarrow \nu_e + n$

$e^+ + n \leftrightarrow \bar{\nu}_e + p$

$e^- + A \leftrightarrow \nu_e + A'$

Scattering

$\nu + N \leftrightarrow \nu + N$

$\nu + A \leftrightarrow \nu + A$

$\nu + e^- \leftrightarrow \nu + e^-$

Pair

$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$

$N + N \leftrightarrow N + N + \nu + \bar{\nu}$

Hydrodynamics equation

$$\partial_t \rho_* + \partial_j(\rho_* v^j) = 0$$
$$\partial_t S_i + \partial_j(S_i v^j + \alpha\sqrt{\gamma}P\delta_i{}^j) = -S_0\partial_i\alpha + S_j\partial_i\beta^j - \frac{1}{2}\alpha\sqrt{\gamma}S_{jk}\partial_i\gamma^{jk} - \alpha\sqrt{\gamma}G_i$$
$$\partial_t(S_0-\rho_*) + \partial_k((S_0-\rho_*)v^k + \sqrt{\gamma}P(v^k+\beta^k)) = \alpha\sqrt{\gamma}S^{ij}K_{ij} - S_iD^i\alpha + \alpha\sqrt{\gamma}n^\mu G_\mu$$

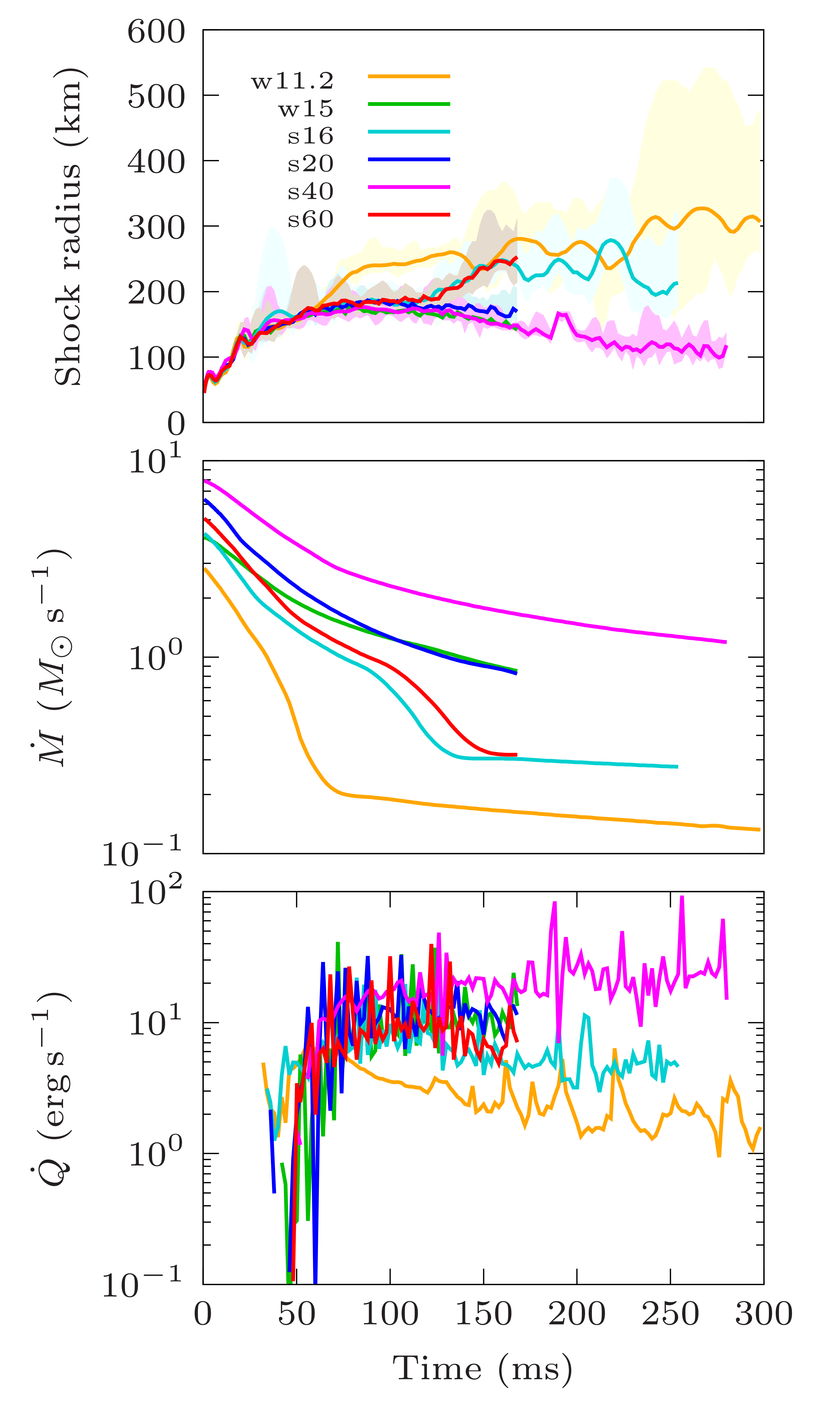
Spacetime metric

(1D assumption with radial gauge polar slicing)

$$g_{\mu\nu} = {\rm diag}\left[-e^{2\Phi(t,r)}, \left(1-2m(t,r)/r\right)^{-1}, r^2, r^2\sin^2\theta\right]$$
$$\frac{\partial m}{\partial r} = 4\pi r^2(\rho h W^2 - P)$$
$$\frac{\partial \Phi}{\partial r} = \left(1-\frac{2m(t,r)}{r}\right)^{-1}\left(\frac{m(t,r)}{r^2}+4\pi r(\rho h v^2 + P)\right)$$

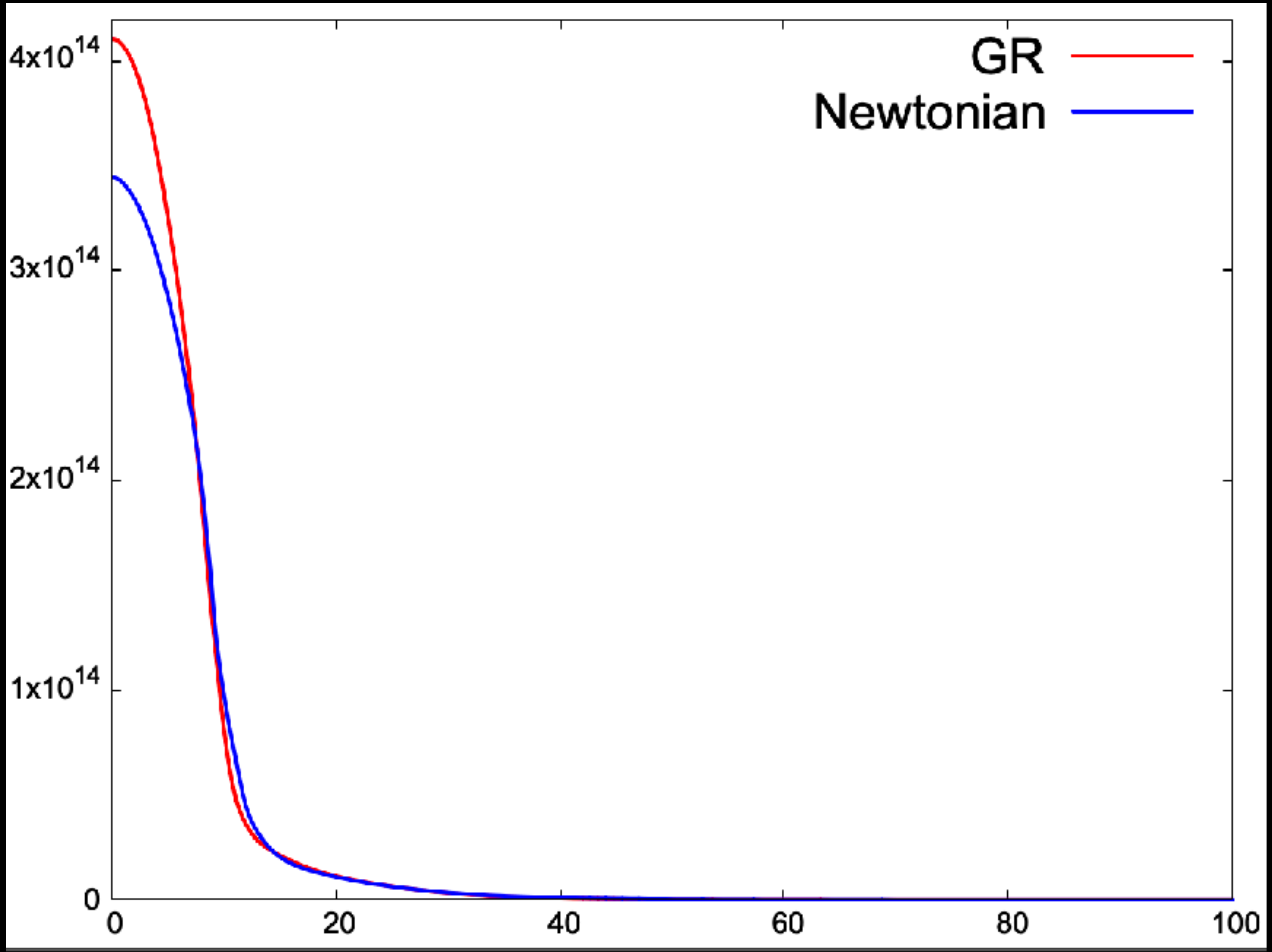
GR Simulations

- $M = 11.2M_\odot$ (Woosley (2002))
- $M = 15M_\odot$ (Woosley (2002))
- $M = 16M_\odot$ (Sukhbold (2018))
- $M = 20M_\odot$ (Sukhbold (2018))
- $M = 40M_\odot$ (Sukhbold (2018))
- $M = 60M_\odot$ (Sukhbold (2018))

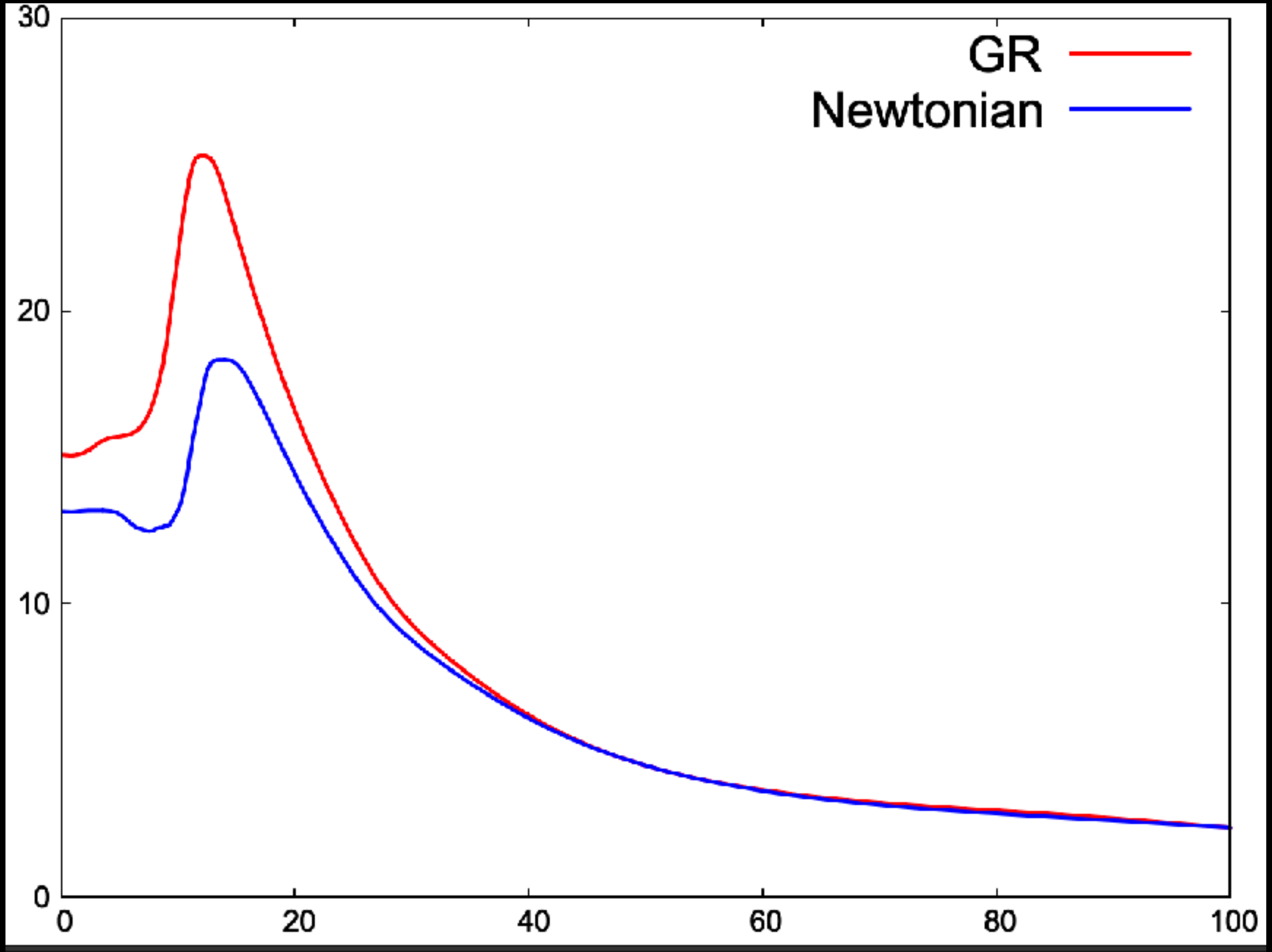


Comparison between Newtonian and GR

Density



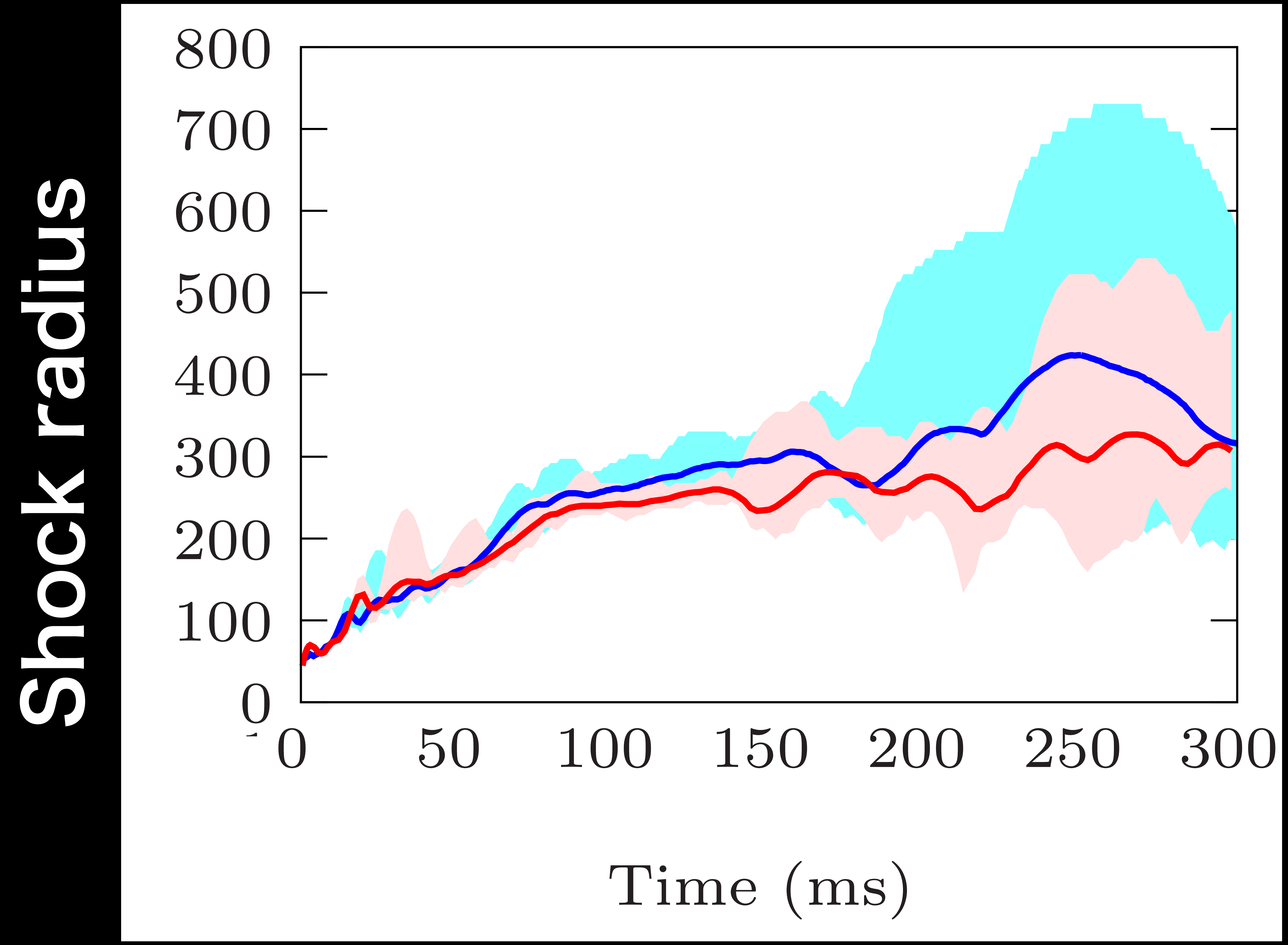
Temperature

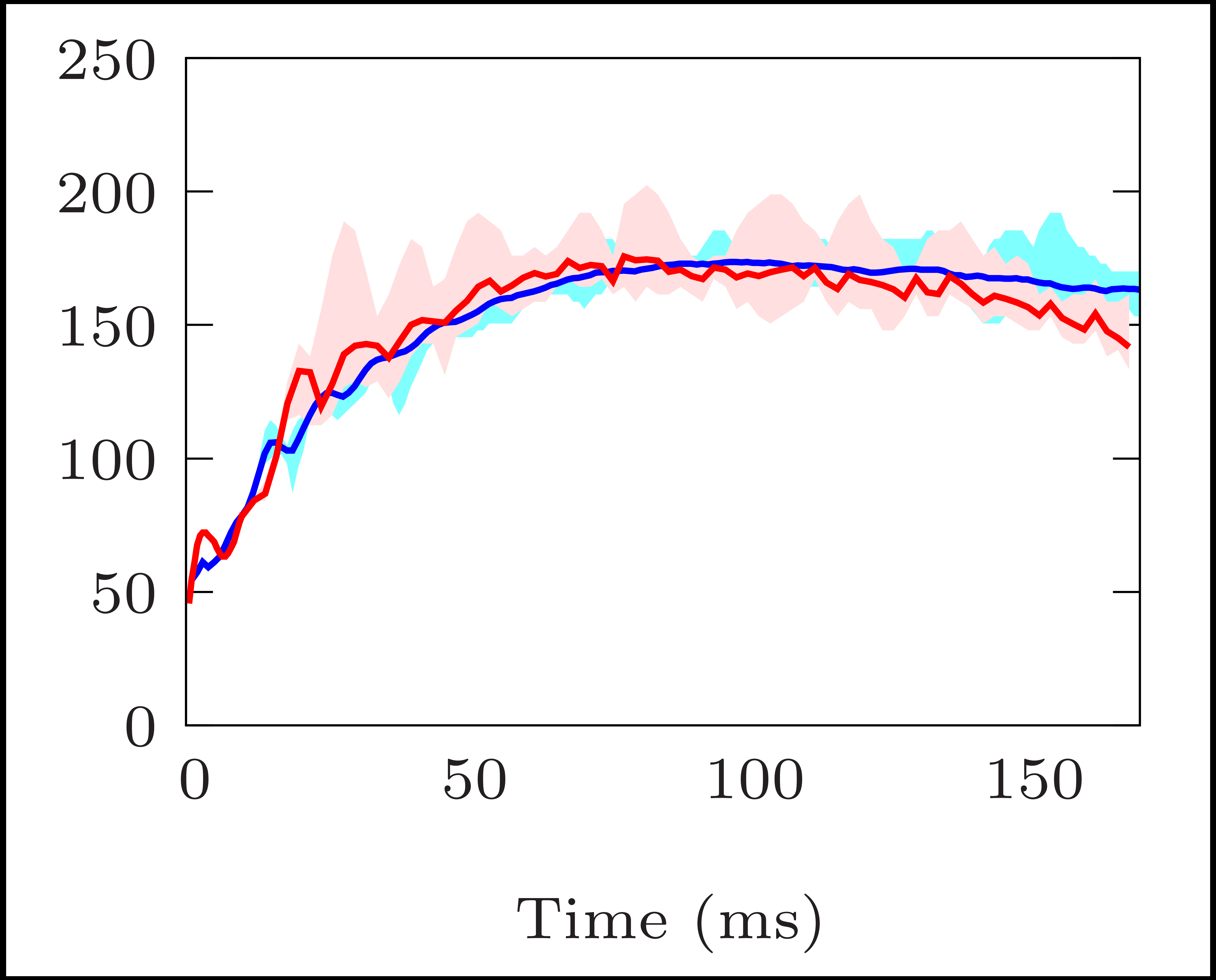


Comparison between Newtonian and GR

Woosley $M = 11.2M_{\odot}$

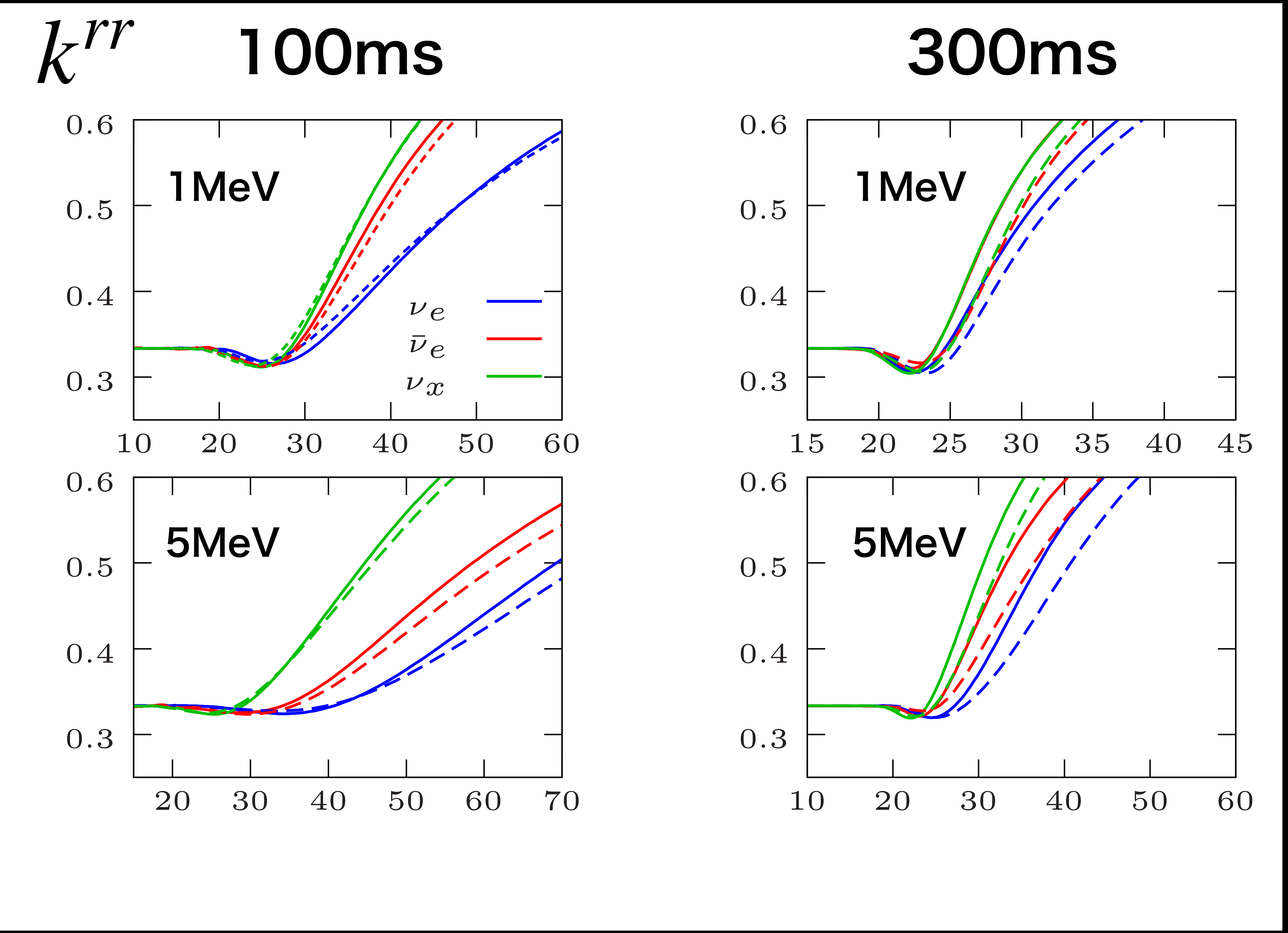
Woosley $M = 15M_{\odot}$

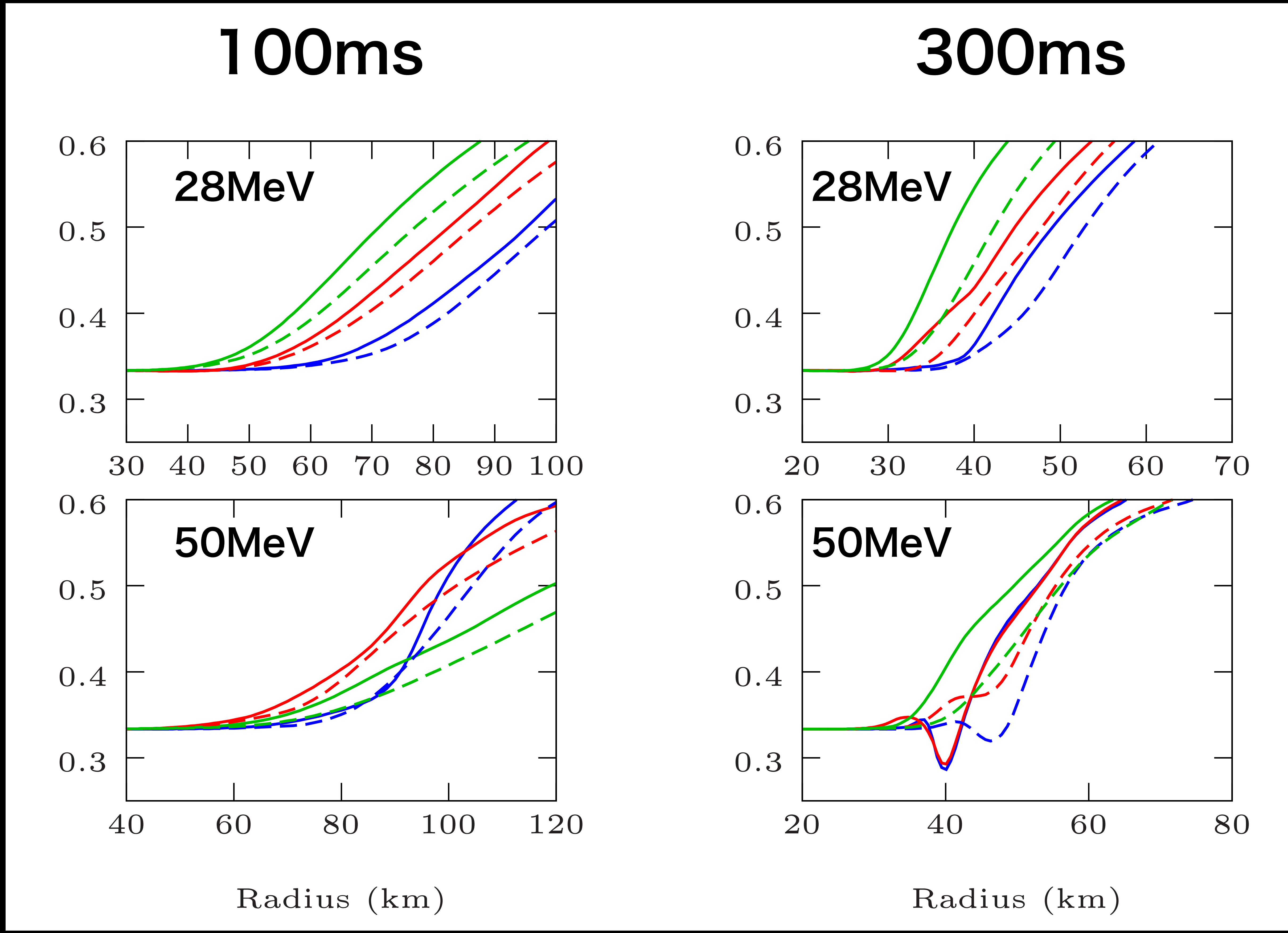




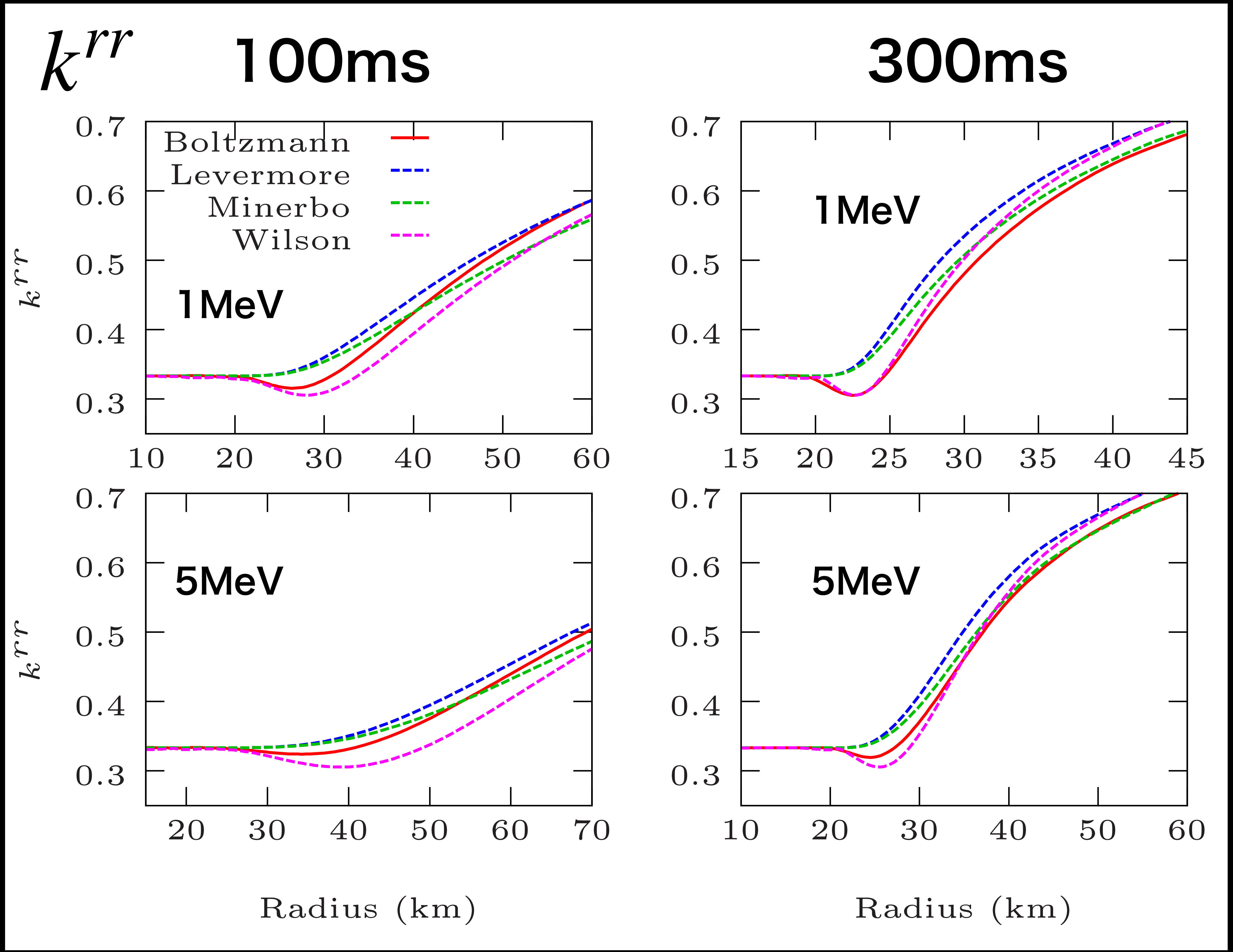
Eddington factor

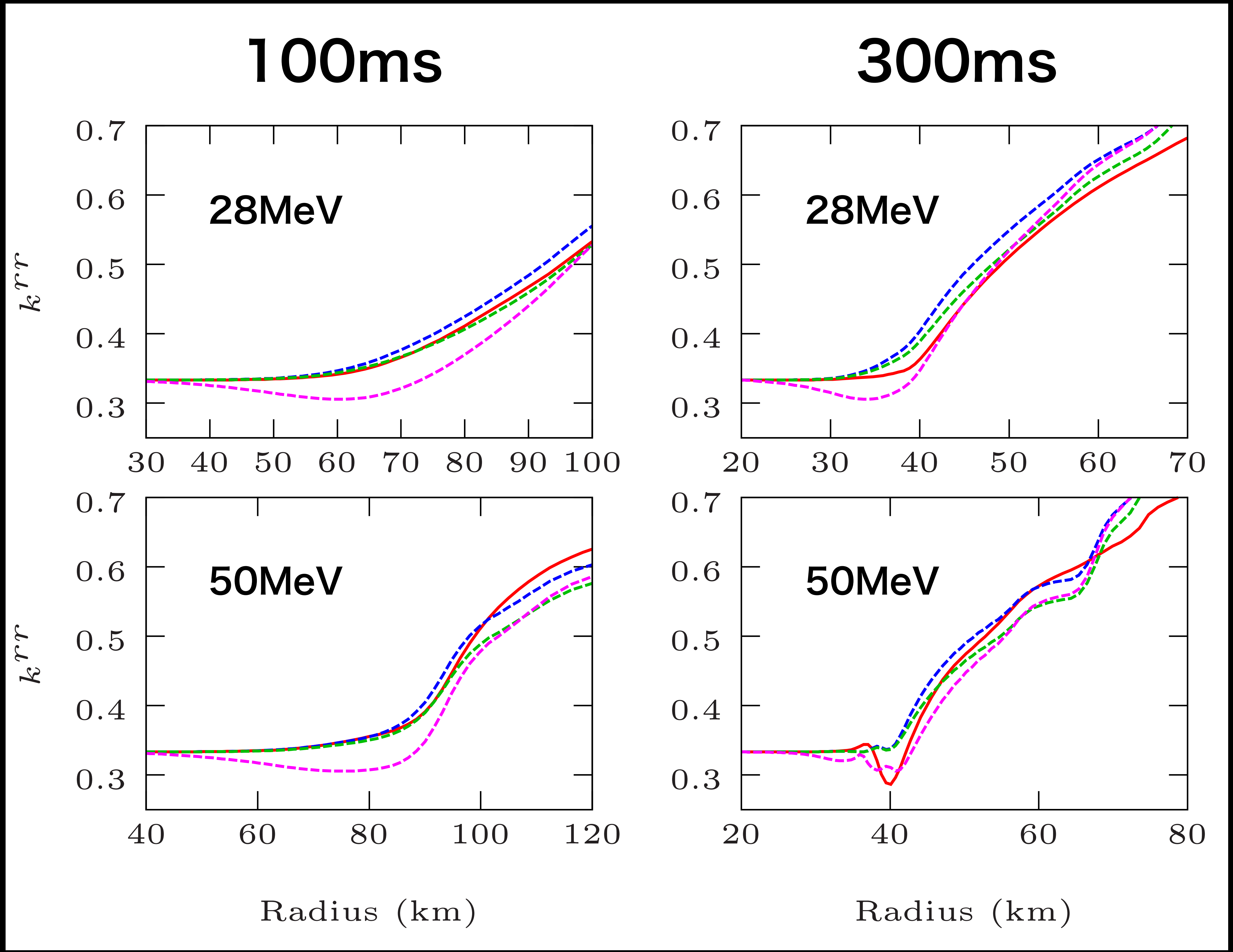
$k^{ij} \equiv P^{ij}/E$ 2nd moment devided by 0th moment





Comparison with closure relations





Summary

- 2D GR Boltzmann simulations of CCSNe have been performed.
- Comparisons with Newtonian counterparts were made.

Future Prospects

- Long-term systematic GR simulations in 2D
- GR simulation in 3D
- Machine learning Eddington tensor using GR simulations
- Applying AI Eddington tensor to actual moment calculations