高解像度ボルツマン輻射輸送計算に向けて

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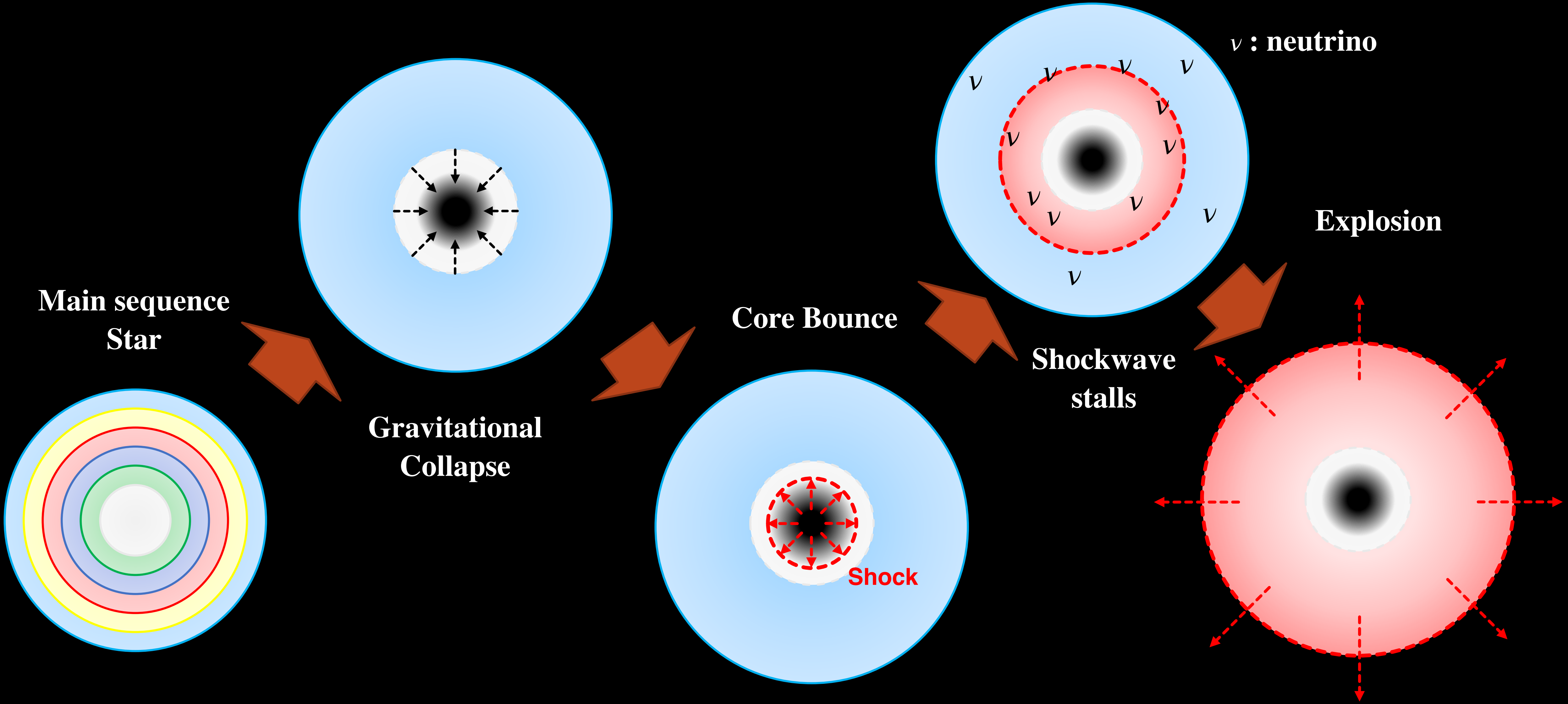
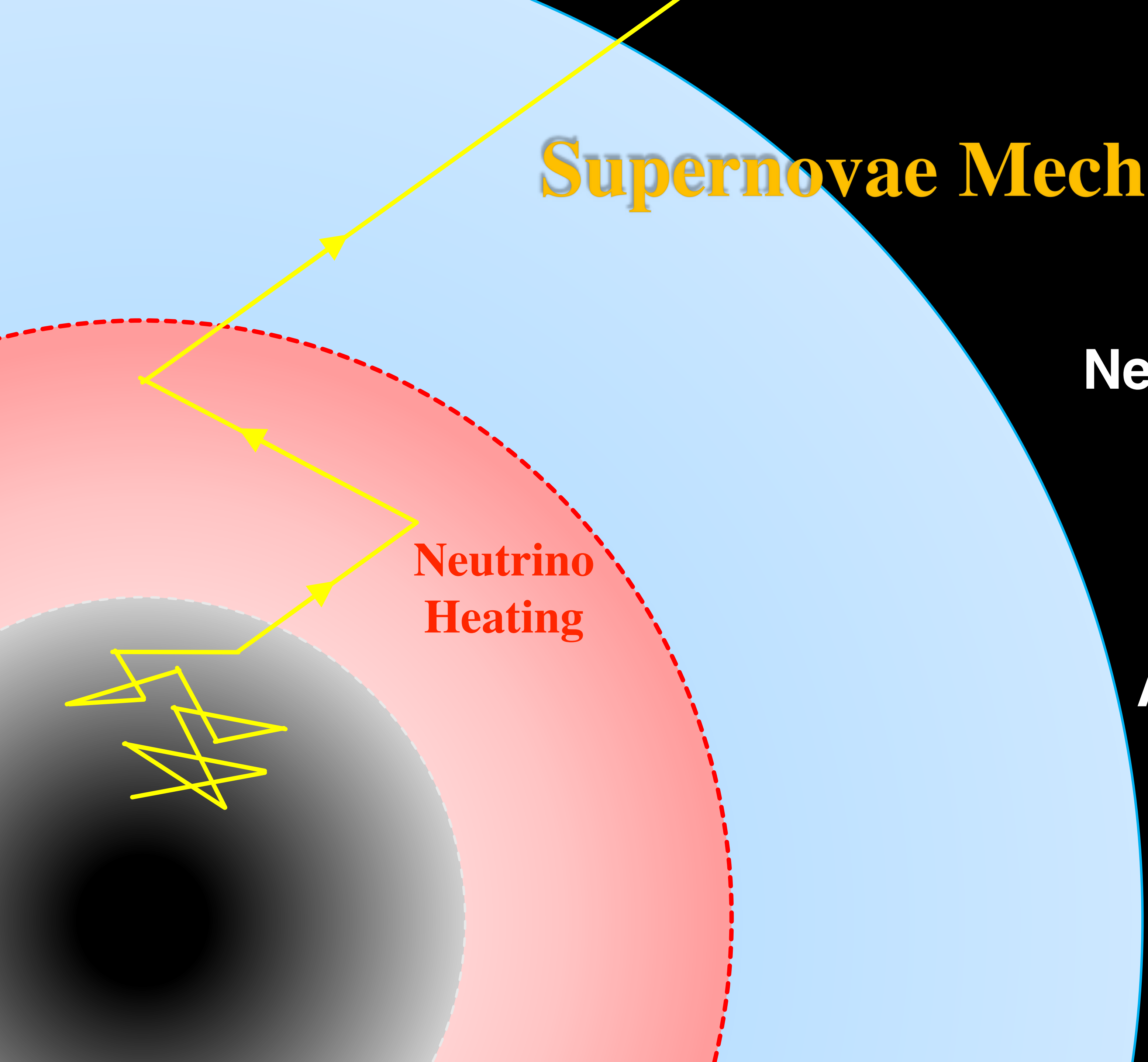
Supernovae Mechanism



Supernovae Mechanism

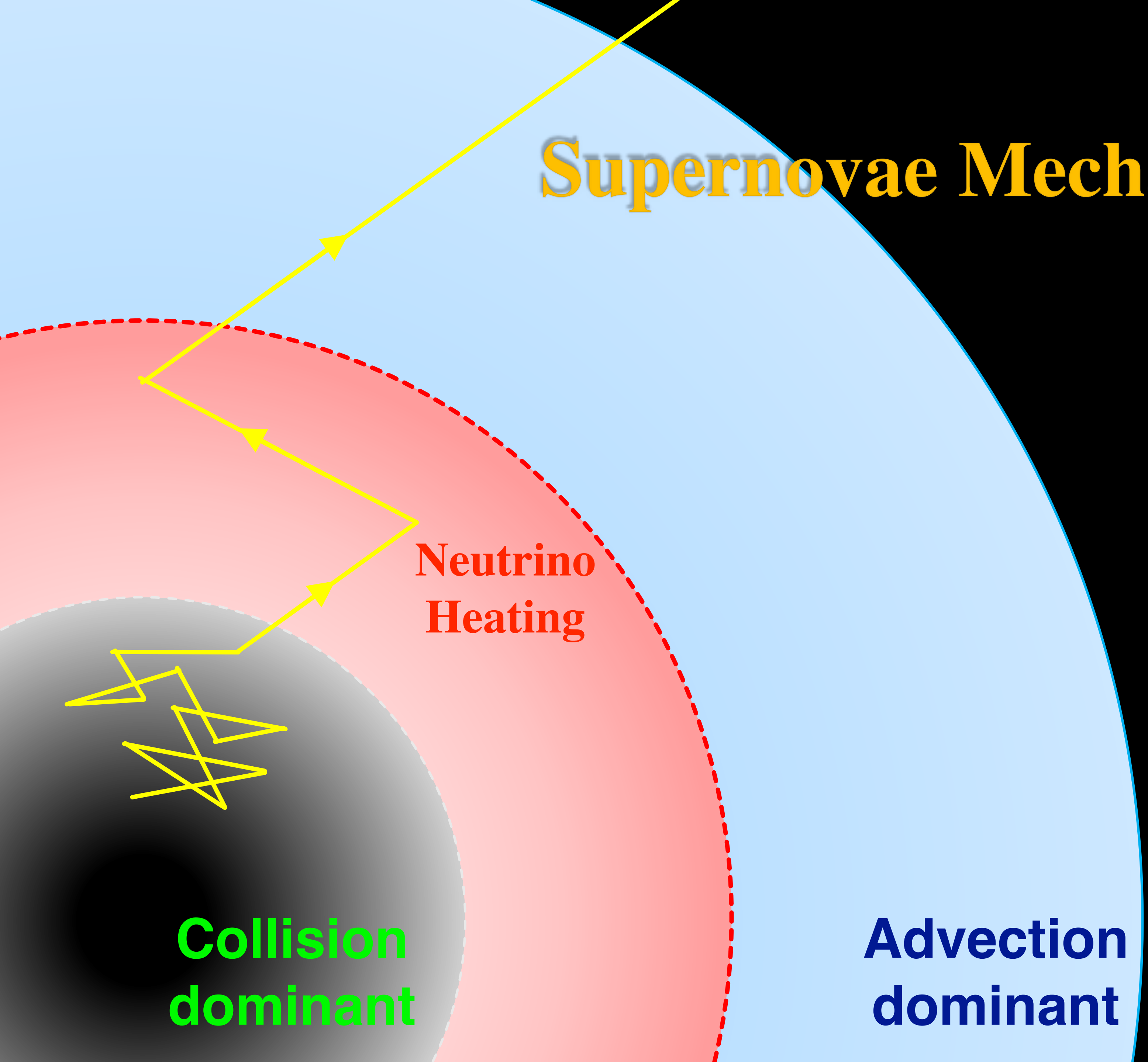


Neutrino Heating Mechanism

⇩

Revival the shockwave
And achieve **the explosion**

Supernovae Mechanism



Neutrino Transfer

Advection

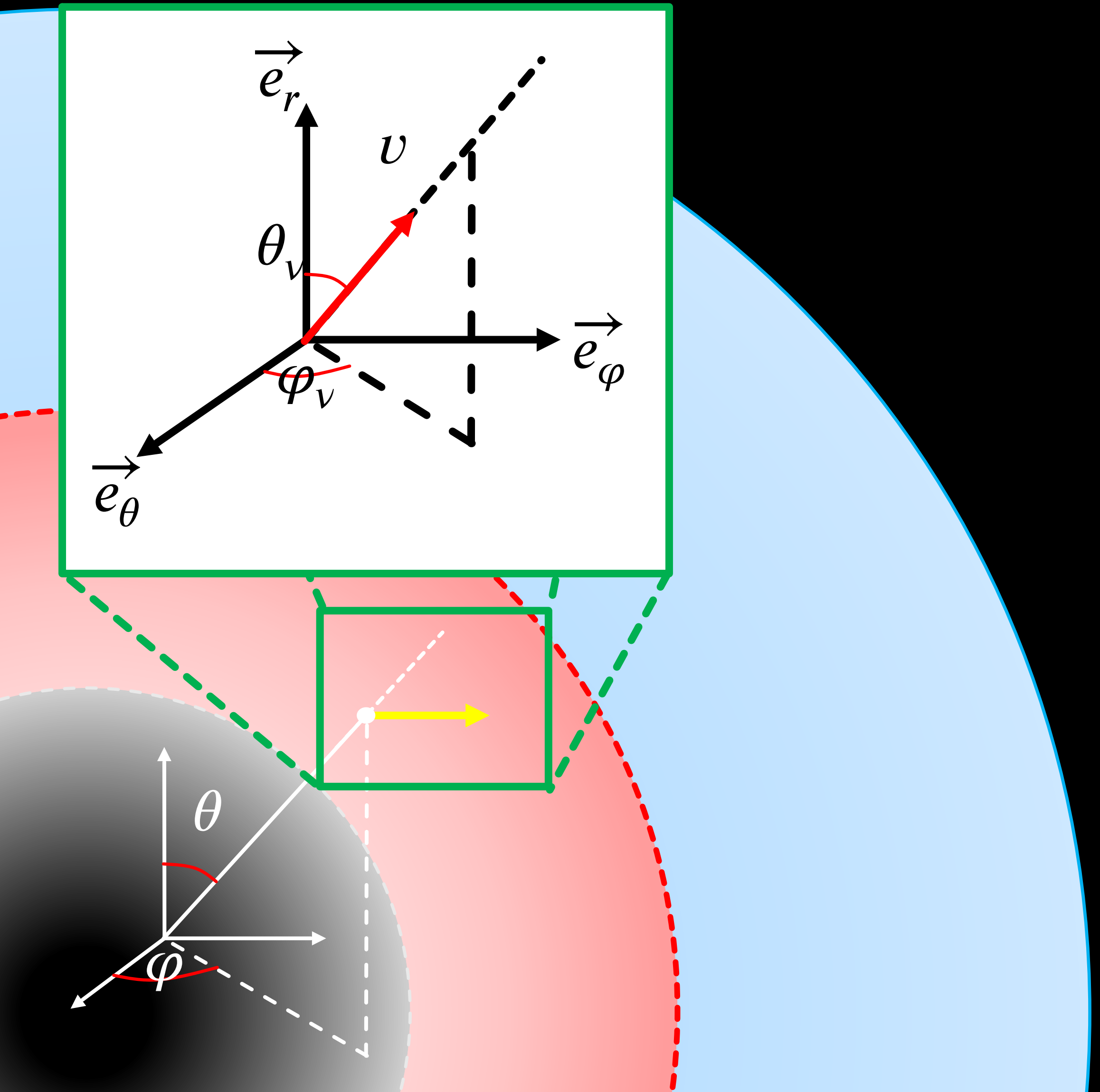
+

Collision
(ex, scattering,
emission,
absorption, etc…)

$$\Omega = (\theta, \phi)$$

$$\Omega_\nu = (\theta_\nu, \phi_\nu)$$

Boltzmann Equation



Boltzmann Eq.

$$\frac{1}{c}\frac{\partial f(r, \Omega, \epsilon_\nu, \Omega_\nu)}{\partial t} + \frac{\partial f(r, \Omega, \epsilon_\nu, \Omega_\nu)}{\partial s} = \frac{1}{c}\left[\frac{\partial f(r, \Omega, \epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{coll}$$

Boltzmann Equation：

6D(space + momentum space) time evolution equation

It costs many computational resources to solve Boltzmann equation

Boltzmann Equation

Advection term　　　Collision term

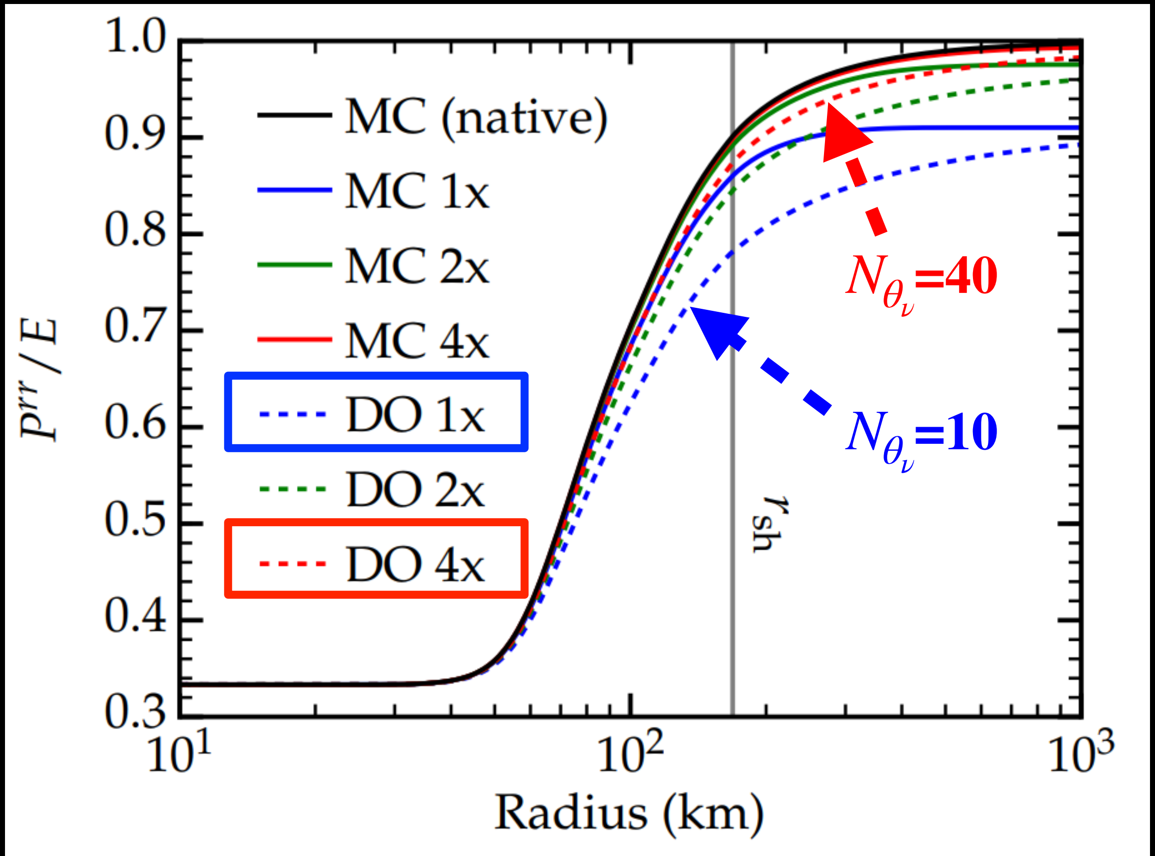
Boltzmann Eq. $$\frac{1}{c}\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial t}+\frac{\partial f(r,\mathit{\Omega},\epsilon_\nu,\Omega_\nu)}{\partial s}=\frac{1}{c}\left[\frac{\partial f(r,\mathit{\Omega},\epsilon_\nu,\Omega_\nu)}{\partial t}\right]_{coll}$$

Advection term — **Neutrino propagation**

Collision term — **Neutrino-matter interactions**

Resolution Dependence

The rr component of the lab-frame energy-integrated Eddington tensor



S. Richers et. al. 2017

If the θ_ν resolution is 4 times higher than current, Eddington tensor P^{rr} differs by 10%.

As the radius is bigger, the error due to resolution is bigger

The resolution of the advection is not sufficient

Solving the Boltzmann Equation

Advection term Collision term

Boltzmann Eq. $$\frac{1}{c}\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial t}+\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial s}=\frac{1}{c}\left[\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial t}\right]_{coll}$$

Explicit Implicit

Problem

- **Advection term** should be solved with **high angle resolution**
- If **Collision term** is solved with high resolution, **a significant increase** in costs is necessary.

Idea

Only the Advection term is solved with **high angular resolution**

The resolution of the collision term is kept.

→ **Dual Resolution Prescription**

Method : Dual Resolution Prescription

Advection : $f^*\left(\epsilon_\nu, \Omega_\nu\right) = f^n(\epsilon_\nu, \Omega_\nu) - c\Delta t \ S(\epsilon_\nu, \Omega_\nu, t(r, \Omega))$

High Resolution

Collision : $f^{n+1}\left(\epsilon_\nu, \Omega_\nu\right) = f^*(\epsilon_\nu, \Omega_\nu) - \Delta t \left[\frac{\partial f^{n+1}(\epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{coll}$

Low Resolution

Method : Dual Resolution Prescription

Advection : $f^*\left(\epsilon_\nu, \Omega_\nu\right) = f^n(\epsilon_\nu, \Omega_\nu) - c\Delta t\ S(\epsilon_\nu, \Omega_\nu, t(r, \Omega))$ — **High Resolution**

Resolution Conversion : $f_{N_{fine}} \mapsto f_{N_{rough}}$

Collision : $f^{n+1}\left(\epsilon_\nu, \Omega_\nu\right) = f^*(\epsilon_\nu, \Omega_\nu) - \Delta t \left[\frac{\partial f^{n+1}(\epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{coll}$ — **Low Resolution**

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

Method : Dual Resolution Prescription

Resolution Conversion : $f_{N_{fine}} \mapsto f_{N_{rough}}$

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

Method : Dual Resolution Prescription

High Resolution

Resolution Conversion : $f_{N_{fine}} \mapsto f_{N_{rough}}$

trivial

Low Resolution

Low Resolution

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

Non-trivial

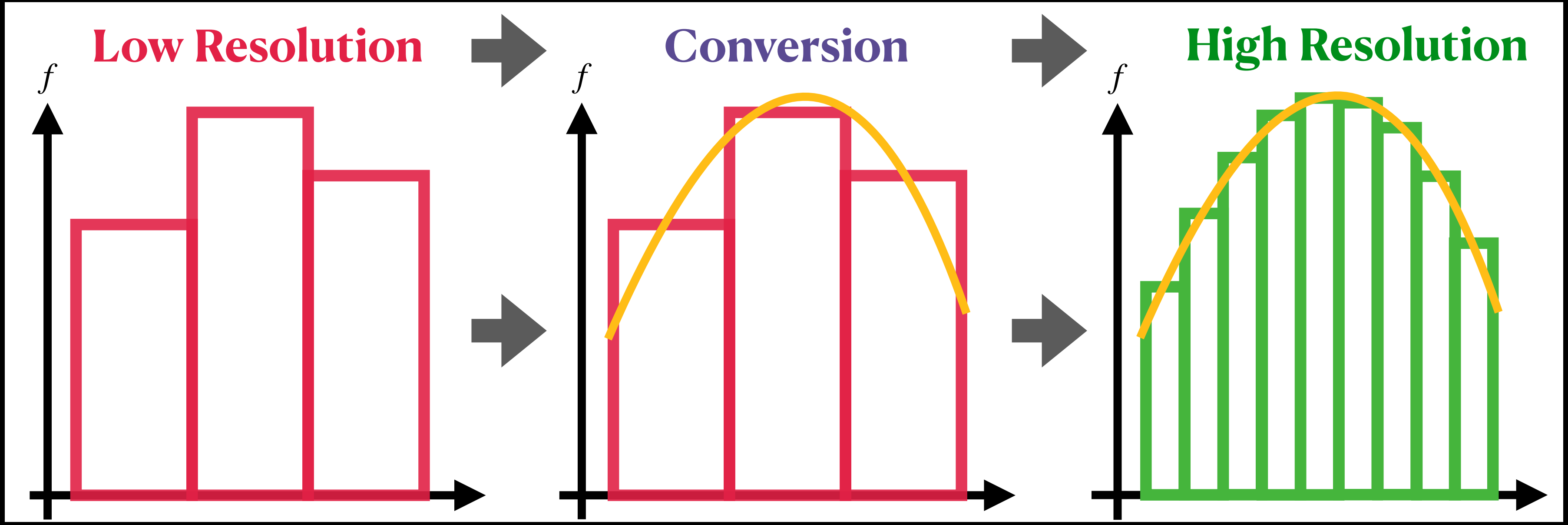
High Resolution

Method : Dual Resolution Prescription

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

Non-trivial

The distribution function is assumed to be expressed by the polynomial



Method : Dual Resolution Prescription

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

The distribution function is assumed to be expressed by the polynomial

$$f_{\mathrm{appr}}(\mu = \cos\theta_\nu) = \sum_{i=0}^{N_{\theta_\nu}^{\mathrm{poly}}} a_n \mu^n$$

$N_{\theta_\nu}^{\mathrm{poly}}$ **: The order of polynomial**

The distribution function in low mesh is

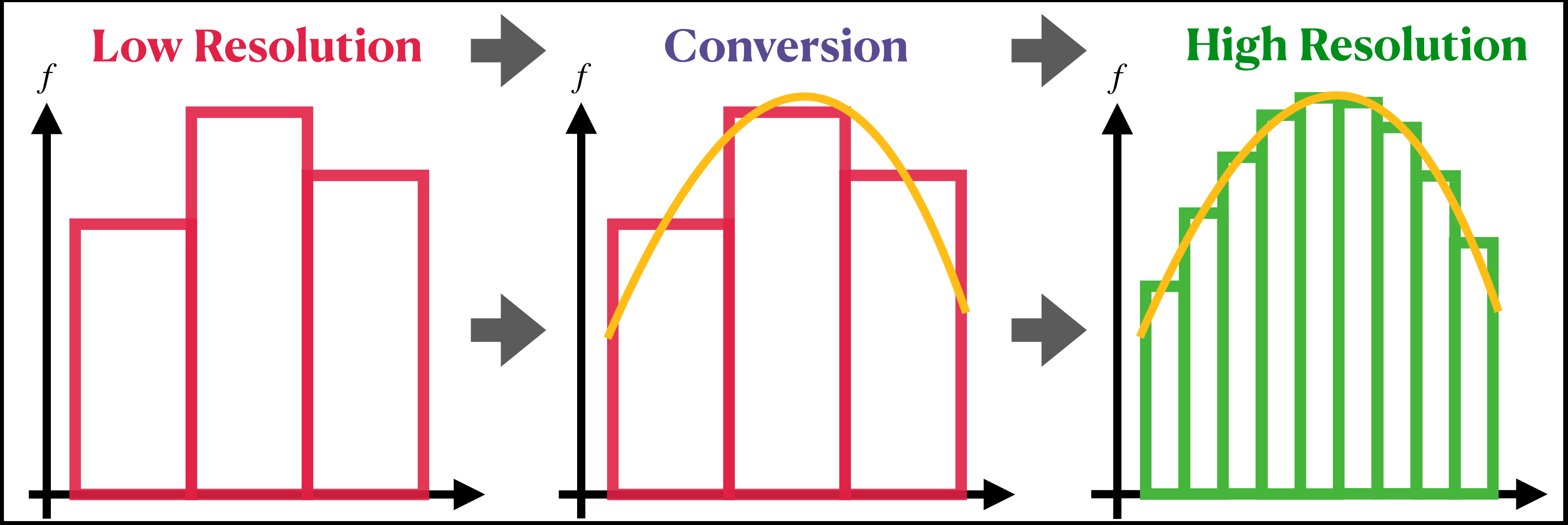
$$f_{\mathrm{high}}(\mu_i = \cos\theta_{\nu,i})(\mu_I - \mu_{I-1}) = \int_{\mu_{I-1}}^{\mu_I} d\mu f_{\mathrm{appr}}(\mu)$$

$$= \sum_{n=0}^{N_{\theta_\nu}^{\mathrm{poly}}} \frac{a_n}{n+1}(\mu_I^{n+1} - \mu_{I-1}^{n+1})$$

Method : Dual Resolution Prescription

Resolution Conversion : $f_{N_{rough}} \mapsto f_{N_{fine}}$

The distribution function is assumed to be expressed by the polynomial



How accurate is this conversion?

Methods

In the study, we perform the pilot study in order to test the effectiveness of this dual resolution prescription

In dual resolution prescription

the advection term ⟷ rigorous expression in the equations

we should study
the resolution dependence of the collision term!

Method : Advection term

Boltzmann Eq. **Advection term**

$$\frac{1}{c}\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial t}+\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial s}=\frac{1}{c}\left[\frac{\partial f(r,\Omega,\epsilon_\nu,\Omega_\nu)}{\partial t}\right]_{coll}$$

the spatial dependence is eliminated

$$\frac{1}{c}\frac{\partial f(\epsilon_\nu,\Omega_\nu)}{\partial t}=\frac{1}{c}\left[\frac{\partial f(\epsilon_\nu,\Omega_\nu)}{\partial t}\right]_{\rm coll}+S(\epsilon_\nu,\Omega_\nu,t(r,\Omega))$$

Source term

Methods

we should study the resolution dependence of the collision term!

Advection term is replaced by the artificial source term

→ Only 3D(momentum) remains

As the artificial source term,

2 types of the source terms were carried out:

Steady state Test & Time evolution Test

Method : Advection term

Steady state Test : resolution dependence

$$S_{\mathrm{steady}} = -\frac{1}{c}\left[\frac{\partial f_{\mathrm{ref}}(\epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{coll}$$

$$\frac{1}{c}\frac{\partial f(\epsilon_\nu, \Omega_\nu)}{\partial t} = \frac{1}{c}\left[\frac{\partial f(\epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{\mathrm{coll}} + S_{\mathrm{steady}}(\epsilon_\nu, \Omega_\nu, t(r, \Omega))$$

In this case, the source term and collision term are canceled

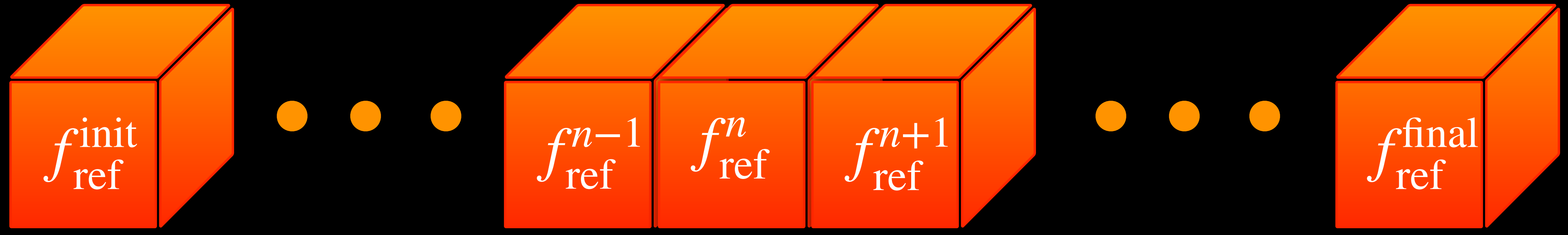
The source term reproduce the reference distribution function if the resolutions are reference ones

Method : Advection term

Time evolution Test : resolution and interpolation dependence

$$S^n_{\text{time}} = \frac{f^{n+1}_{\text{ref}} - f^n_{\text{ref}}}{\delta t} - \frac{1}{c}\left[\frac{\partial f^{n+1}_{\text{ref}}(\epsilon_\nu, \Omega_\nu)}{\partial t}\right]_{coll}$$

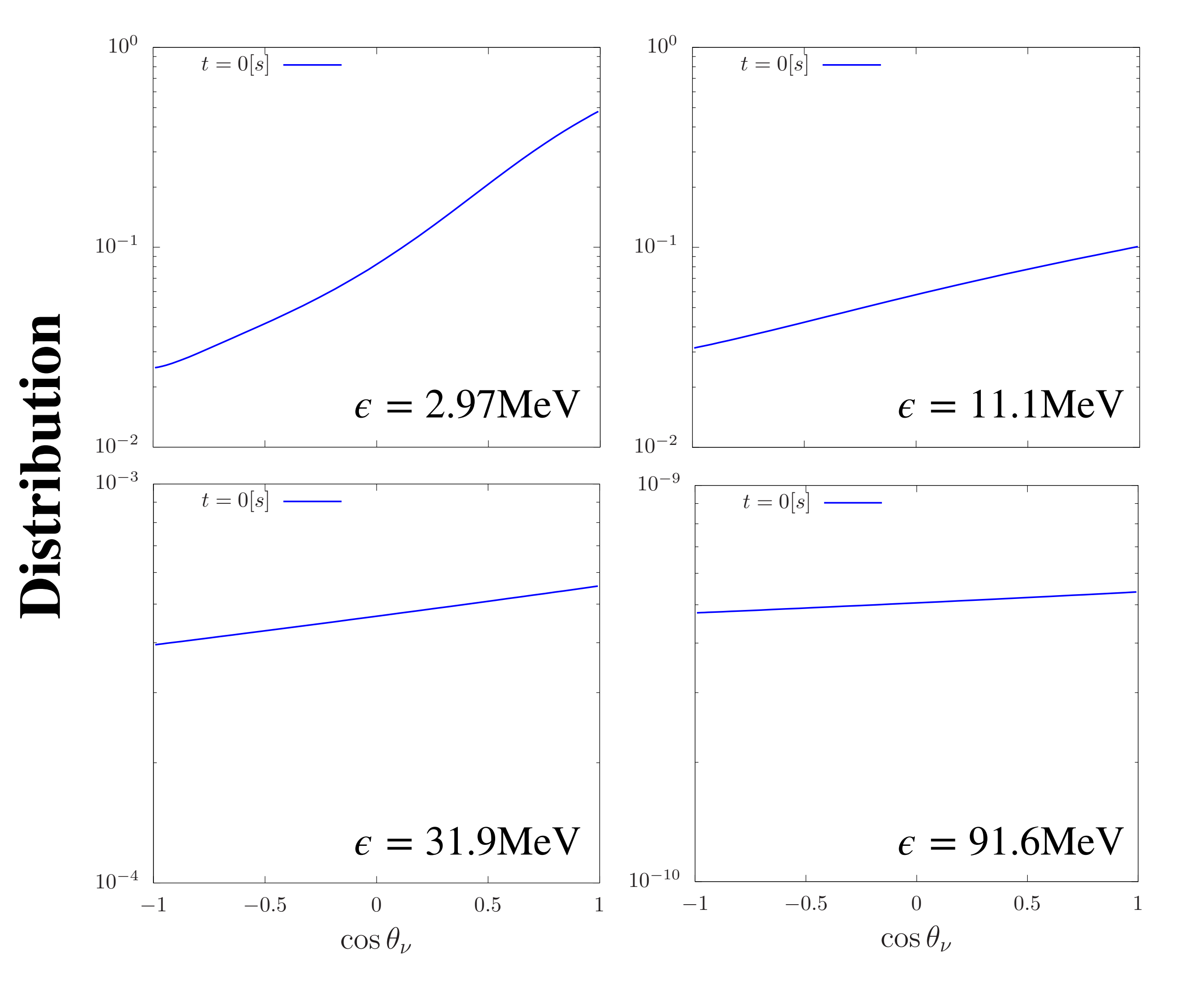
The source term reproduce the reference distribution function if the resolutions are reference ones



$r \sim 60\text{km}$
$\rho \sim 10^{12}\text{g/cc}$

$r \sim 100\text{km}$
$\rho \sim 10^{10}\text{g/cc}$

Result : Steady state Test in θ_ν



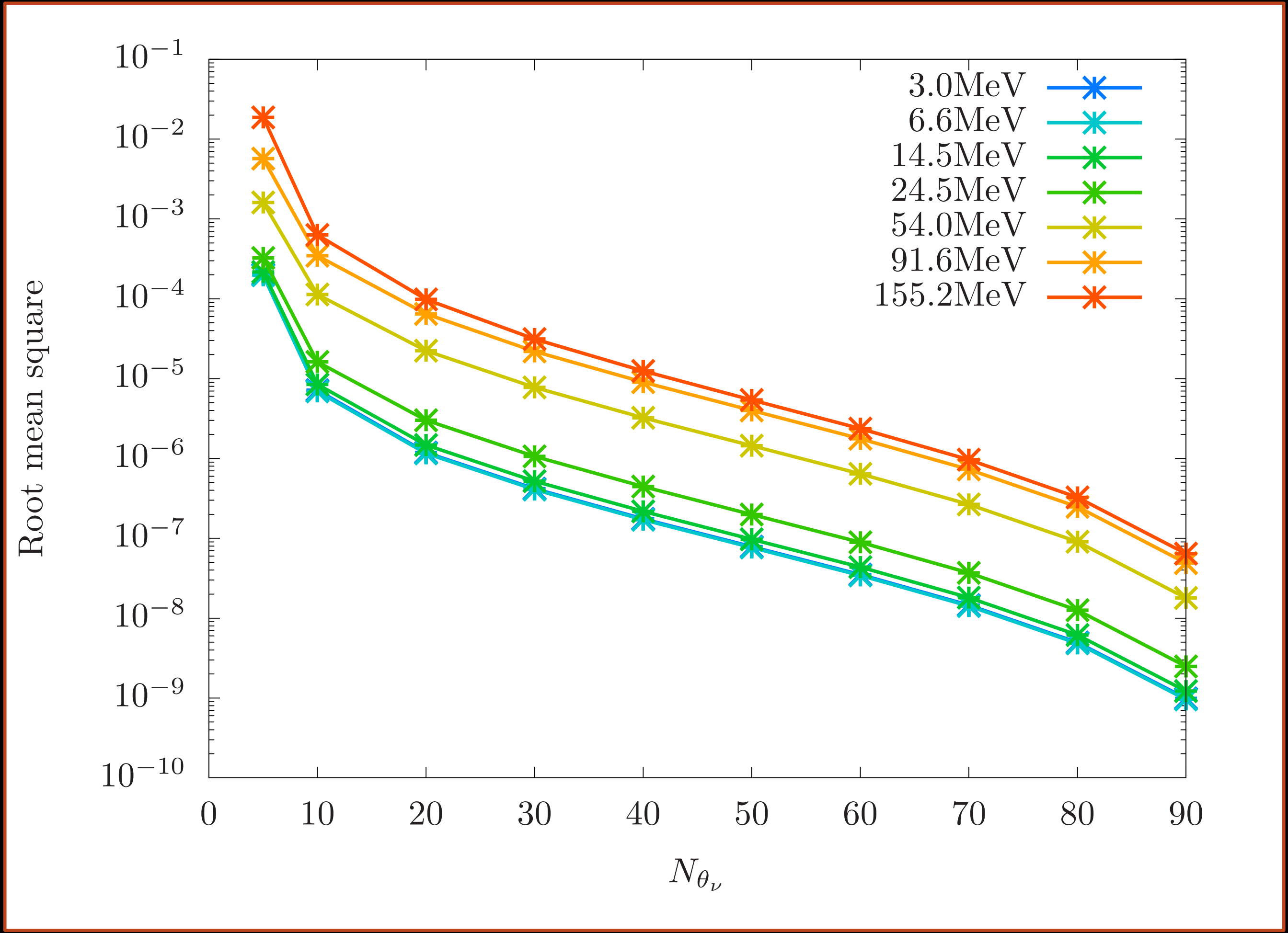
Reference Resolution :

$N_\epsilon = 20$

$N_{\theta_\nu} = 100$

$N_{\phi_\nu} = 6$

Result : Steady state Test in θ_ν



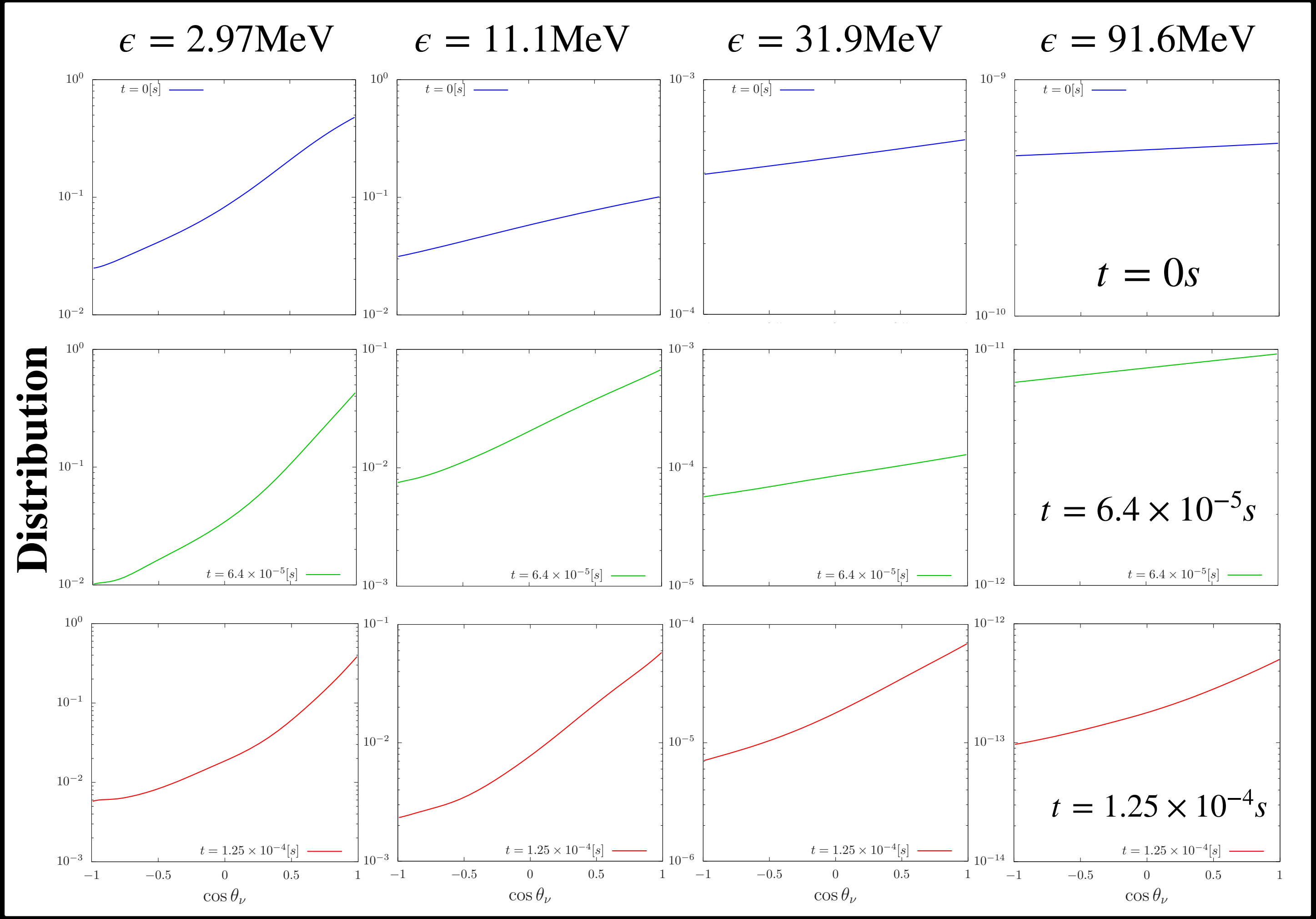
Reference Resolution : $N_{\theta_\nu} = 100$

Error : $\left| \dfrac{f_{N_{\theta_\nu}} - f_{N_{\theta_\nu}=100}}{f_{N_{\theta_\nu}=100}} \right|$

If the resolution is current, (N_{θ_ν}=10), the accuracy (RMS $\sim 10^{-4}$) is acceptable

Result : Time Evolution in θ_ν

Reference distribution



Reference Resolution :

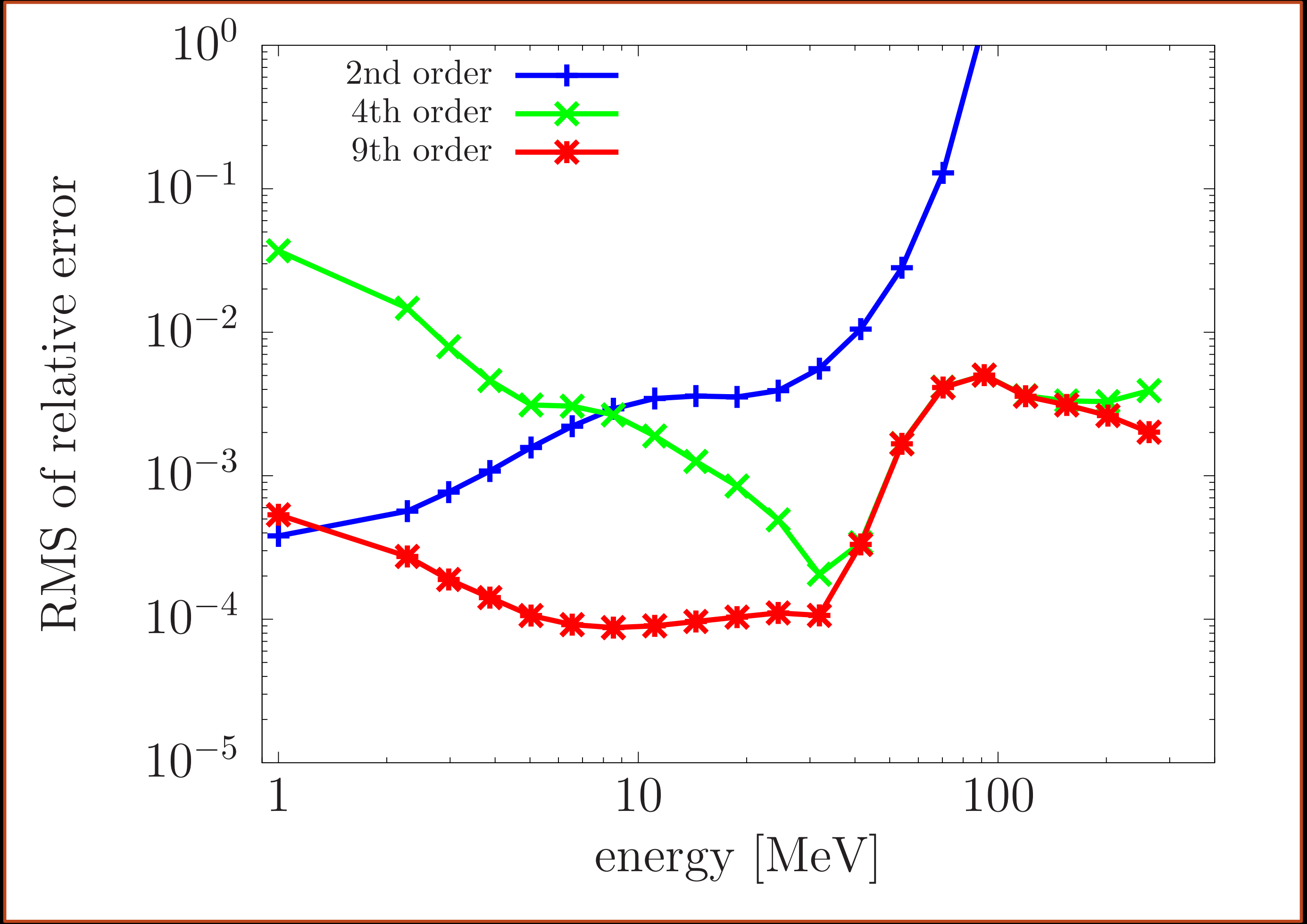
$N_\epsilon = 20$

$N_{\theta_\nu} = 40$

$N_{\phi_\nu} = 6$

Result : Time Evolution in θ_ν

The interpolation dependence



Reference Resolution : $N_{\theta_\nu} = 40$

Interpolation : Polynomial

Error : $\left| \dfrac{f_{N_{\theta_\nu}} - f_{N_{\theta_\nu}=40}}{f_{N_{\theta_\nu}=40}} \right|$

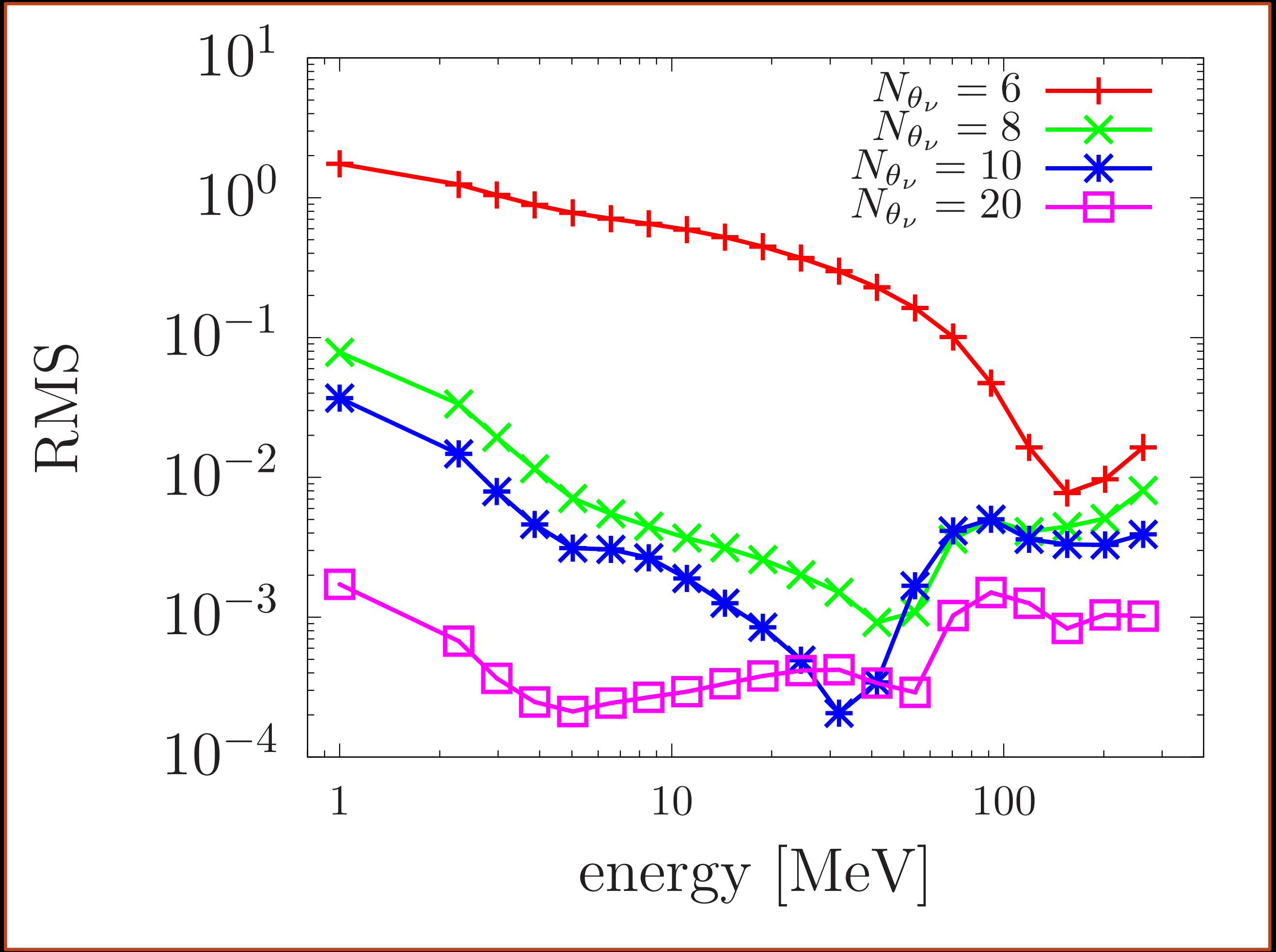
Higher order of polynomial

⇓

Smaller relative error

Result : Time Evolution in θ_ν

The resolution dependence



Reference Resolution : $N_{\theta_\nu} = 40$

Interpolation : 4th-order Polynomial

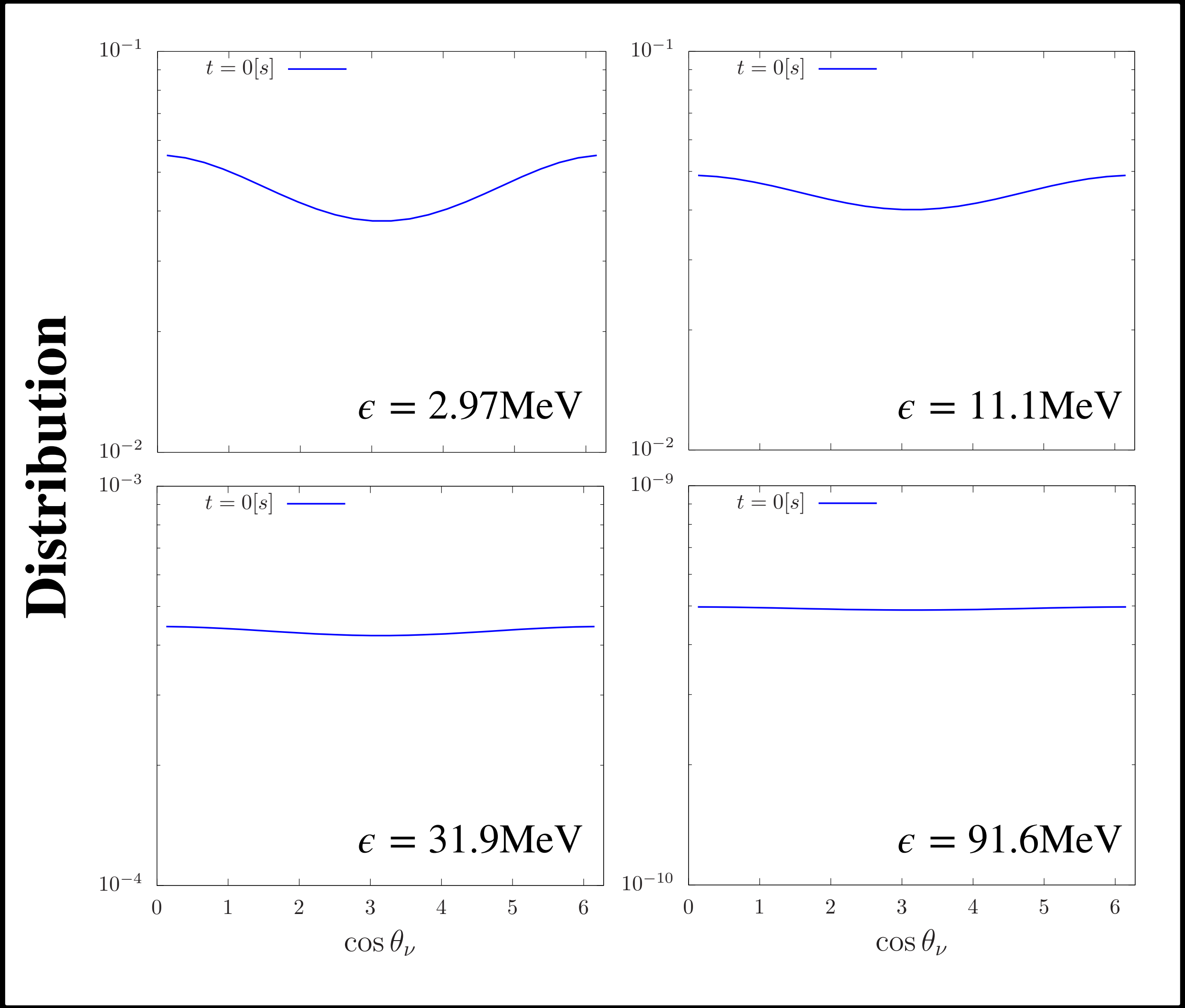
Error : $\left| \dfrac{f_{N_{\theta_\nu}} - f_{N_{\theta_\nu}=40}}{f_{N_{\theta_\nu}=40}} \right|$

Higher resolution

↓

Smaller relative error

Result : Steady state Test in ϕ_μ



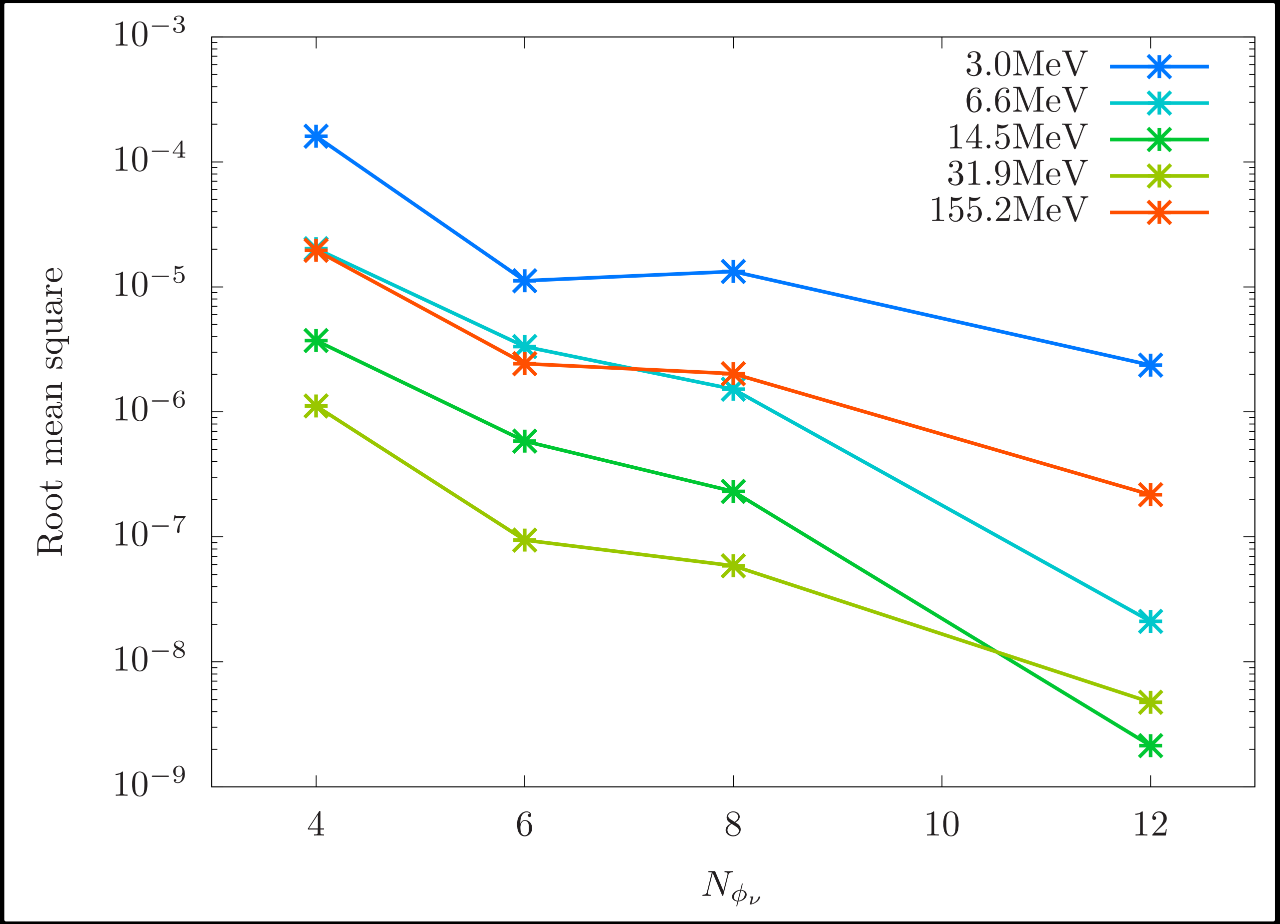
Reference Resolution :

$N_\epsilon = 20$

$N_{\theta_\nu} = 10$

$N_{\phi_\nu} = 24$

Result : Steady state Test in ϕ_μ



Reference Resolution :

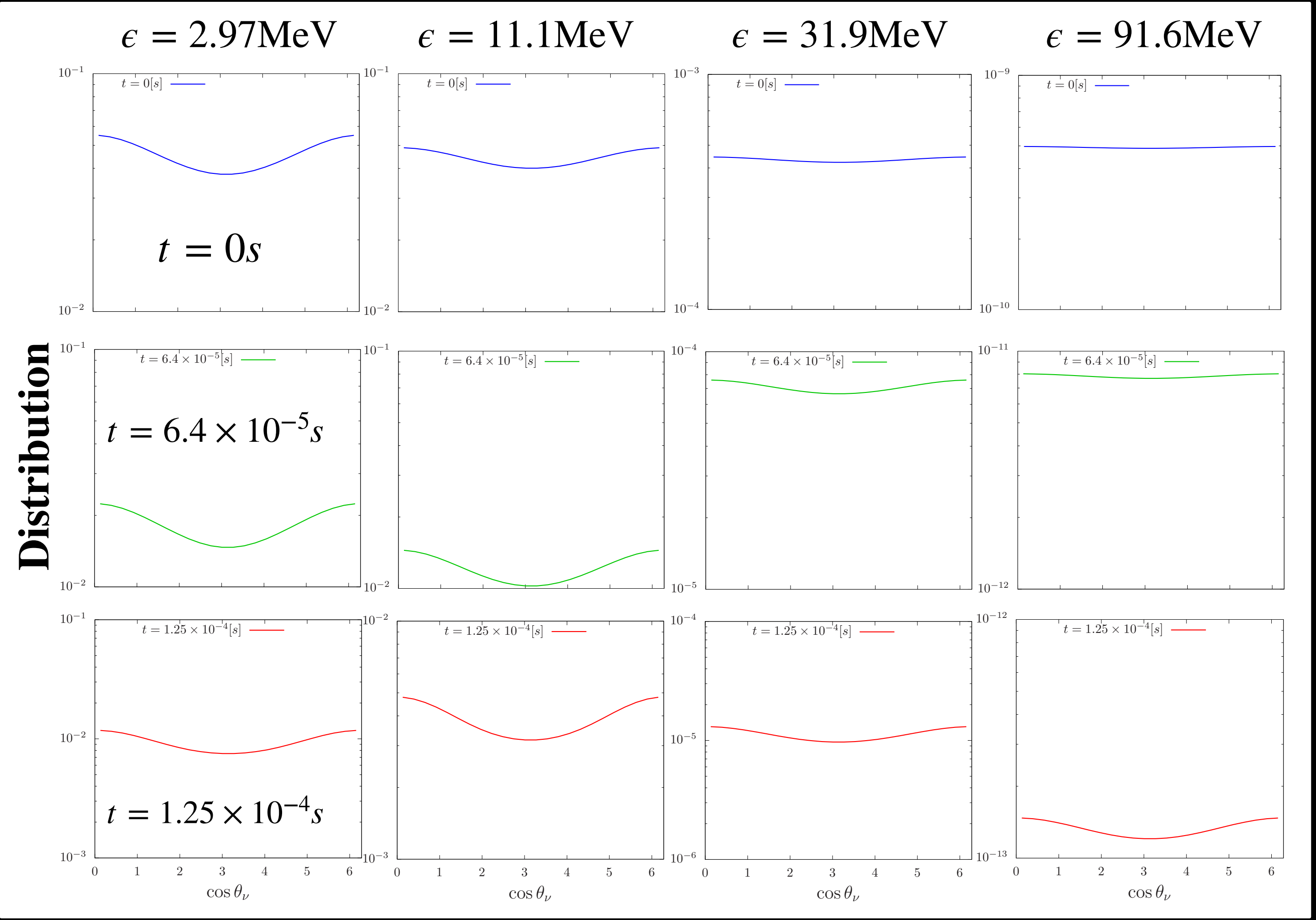
$$N_\epsilon = 20$$
$$N_{\theta_\nu} = 10$$
$$N_{\phi_\nu} = 24$$

Error : $\left| \dfrac{f_{N_{\phi_\nu}} - f_{N_{\phi_\nu}=24}}{f_{N_{\phi_\nu}=24}} \right|$

If the resolution is current, (N_{ϕ_ν}=6), the accuracy (RMS $\lesssim 10^{-4}$) is acceptable

Result : Time Evolution in ϕ_μ

Reference Distribution

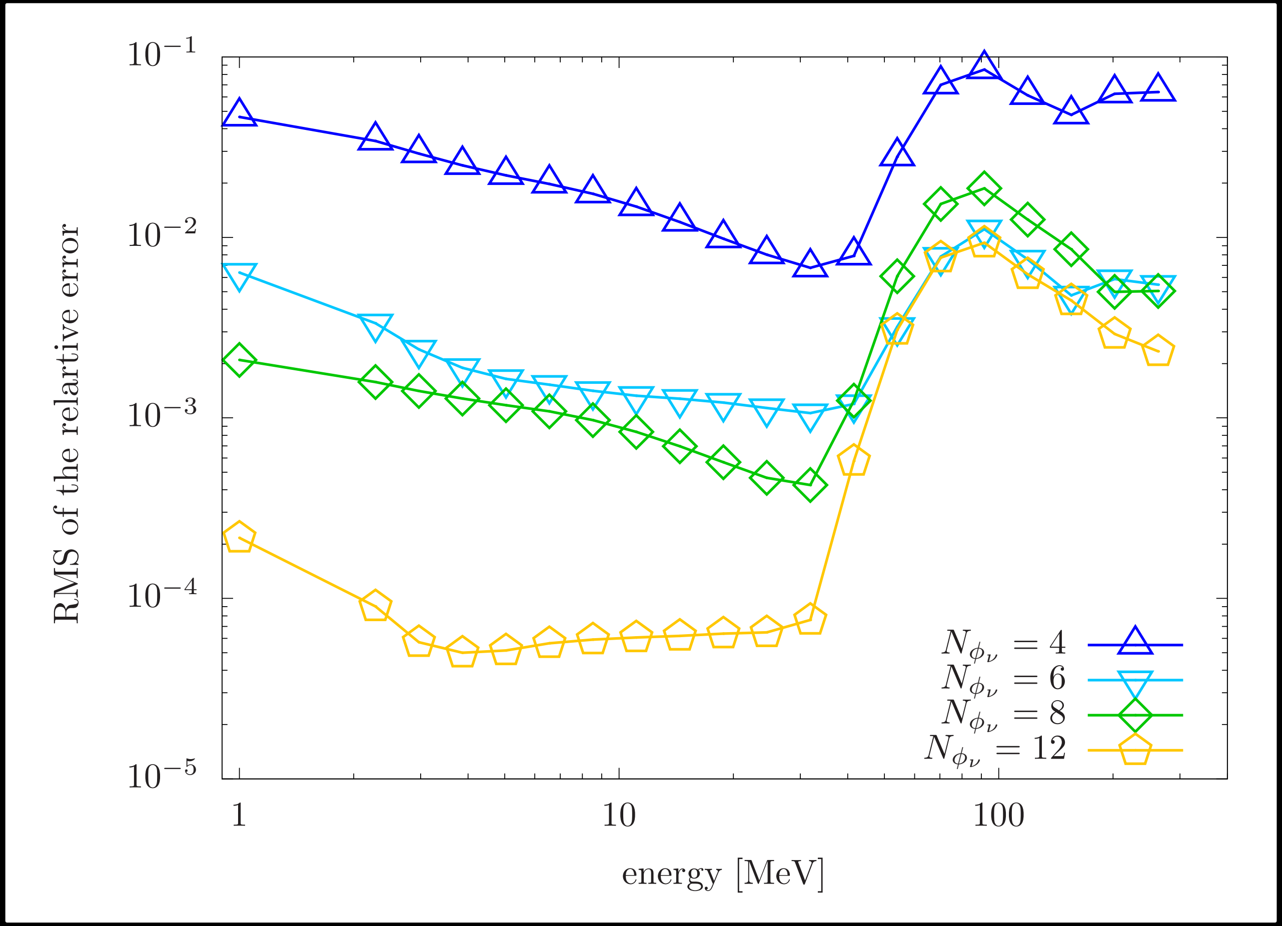


Reference Resolution :

$$N_\epsilon = 20$$
$$N_{\theta_\nu} = 10$$
$$N_{\phi_\nu} = 24$$

Result : Time Evolution in ϕ_μ

The resolution dependence



Reference Resolution :

$$N_\epsilon = 20$$
$$N_{\theta_\nu} = 10$$
$$N_{\phi_\nu} = 24$$

Interpolation : 4th-order Polynomial

Error : $$\left| \frac{f_{N_{\phi_\nu}} - f_{N_{\phi_\nu}=24}}{f_{N_{\phi_\nu}=24}} \right|$$

Higher resolution

↓

Smaller relative error

Summary

- We perform the pilot study with dual resolution prescription and study the momentum angle resolution dependence.
- When $N_{\theta_\nu} = 10$ ($N_{\phi_\nu} = 6$) with 9th (4th) order polynomial, the error ($\lesssim 10^{-2}$) is acceptable and the dual resolution prescription is valid.

Future Work

- We will implement the Dual Resolution Prescription into the Boltzmann code and reveal the neutrino behavior with the high resolution calculation.