Comparison of steady-state flow calculations with M1 and FLD

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1.Objective and Background

- A code describing the cooling process of proto-neutron stars (PNSC) was created by H.Suzuki (1993) using multi-energy flux limited diffusion (MGFLD) scheme to solve Boltzmann equation approximately.
- To prepare for future SNν observations, the current code needs to be improved and updated to make it more accurate and capable of long-time calculations.
- Therefore, taking these points into account, I created a code to solve the 1D ν transport by M1 and compared it to FLD.

2.Formalism

Moment scheme

Remove the full angular dependence of the Boltzmann equation by expanding the neutrino distribution function as a series of angular moments

✓ **Flux limited diffusion (original code)**

Closes the moment expansion after the zeroth moment

✓ **M1-closure (Created Code)**

Evolves both the zeroth and first moment but assumes an analytic closure for higher moments.

Details

I. Metric

$$ds^2 = e^{2\phi(m,t)}dt^2 - e^{-2\lambda(m,t)}dm^2 - r(m,t)^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

II. Boltzmann equation (Lindquist equation)

$$D_t f_\nu + \mu D_m f_\nu - \omega\left[\mu D_m\phi + \mu^2 D_t\lambda + (1-\mu^2)\frac{U}{r}\right]\frac{\partial f_\nu}{\partial \omega} + (1-\mu^2)\left[-D_m\phi + \frac{\Gamma}{r} + \mu\left(\frac{U}{r} - D_t\lambda\right)\right]\frac{\partial f_\nu}{\partial \mu} = Coll[f_\nu]$$

$$D_t = e^{-\phi}\partial_t \quad D_m = e^{-\lambda}\partial_m \quad \Gamma = D_m r \quad U = D_t r$$

III. Equation for moments (Steady-flow for simplicity)

$$\boldsymbol{\partial_t \vec{\mathcal{U}} + \partial_m(4\pi r^2 \rho \vec{\mathcal{F}}) = \vec{\mathcal{S}}}$$

Moment

$$n_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu d\Omega$$
$$F_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu d\Omega$$
$$P_\nu = \frac{\omega^2}{(hc)^3}\int f_\nu \mu^2 d\Omega$$

Variables

$$\vec{\mathcal{U}} = \frac{1}{\rho}(n_\nu, F_\nu) \qquad \vec{\mathcal{F}} = \frac{1}{\rho}(F_\nu, P_\nu)$$
$$\vec{\mathcal{S}} = \frac{1}{\rho}\left(S^0, S^1 - 4\pi r c \rho e^{\phi}(n_\nu - P_\nu)\partial_m(re^{-\phi})\right)$$

Closure : Levermore, ME, etc.

IV. Neutrino-Matter reaction

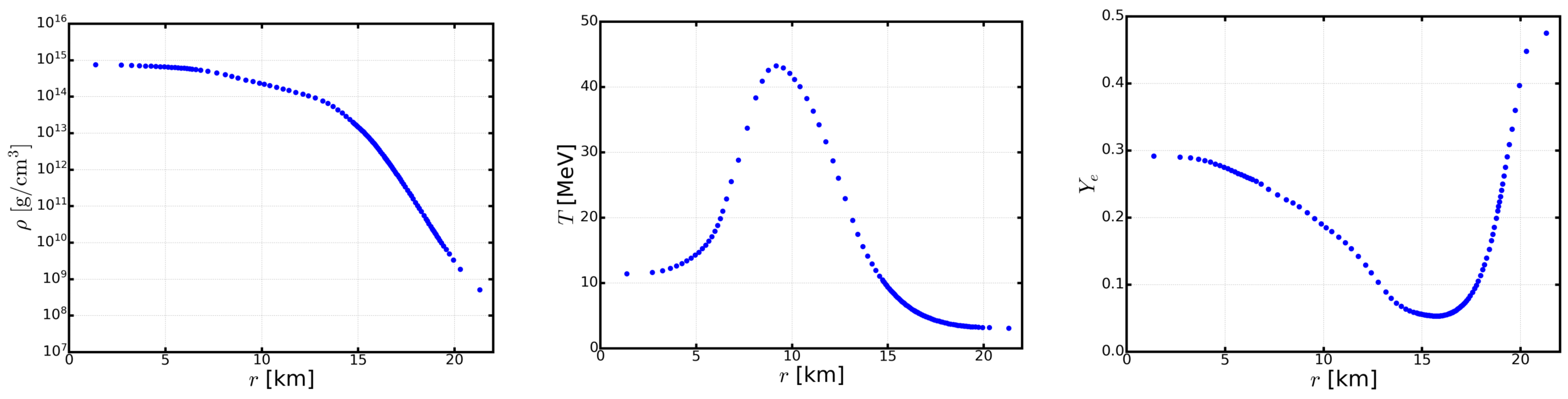
NuLib (O'Connor 2015)

Absorption and emission	$\nu_e + n \rightarrow e^- + p$ $\overline{\nu_e} + p \rightarrow e^+ + n$	$\nu_e + A \rightarrow e^- + A$
Elastic scattering	$\nu + n \rightarrow \nu + n$ $\nu + p \rightarrow \nu + p$	$\nu + A \rightarrow \nu + A$ $(\nu + \alpha \rightarrow \nu + \alpha)$
Thermal pair production/annihilation	$e^+ + e^- \rightarrow \nu + \bar{\nu}$	
Inelastic scattering	$\nu + e \rightarrow \nu + e$	

3.Numerical situation

✓ To do the numerical simulations of steady flow, I imposed the following situation.
✓ These profile obtained from the result of the PNSC calculation (original code).

■ after 600 msec of shock revival



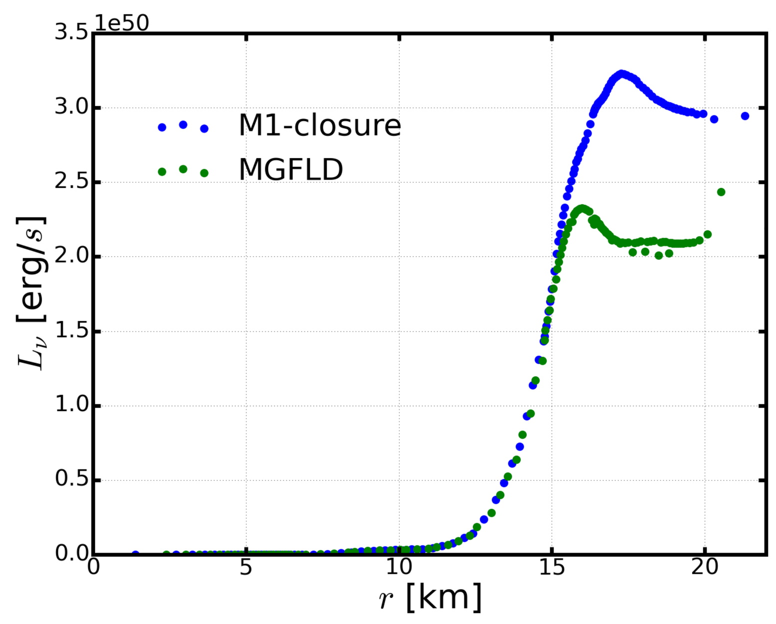
4.Calculation results

✓ Compare M1 and FLD in steady flow with a fixed distribution of fluid after 600 msec of proto-neutron star cooling.
✓ Compare the results of different closures.

Details

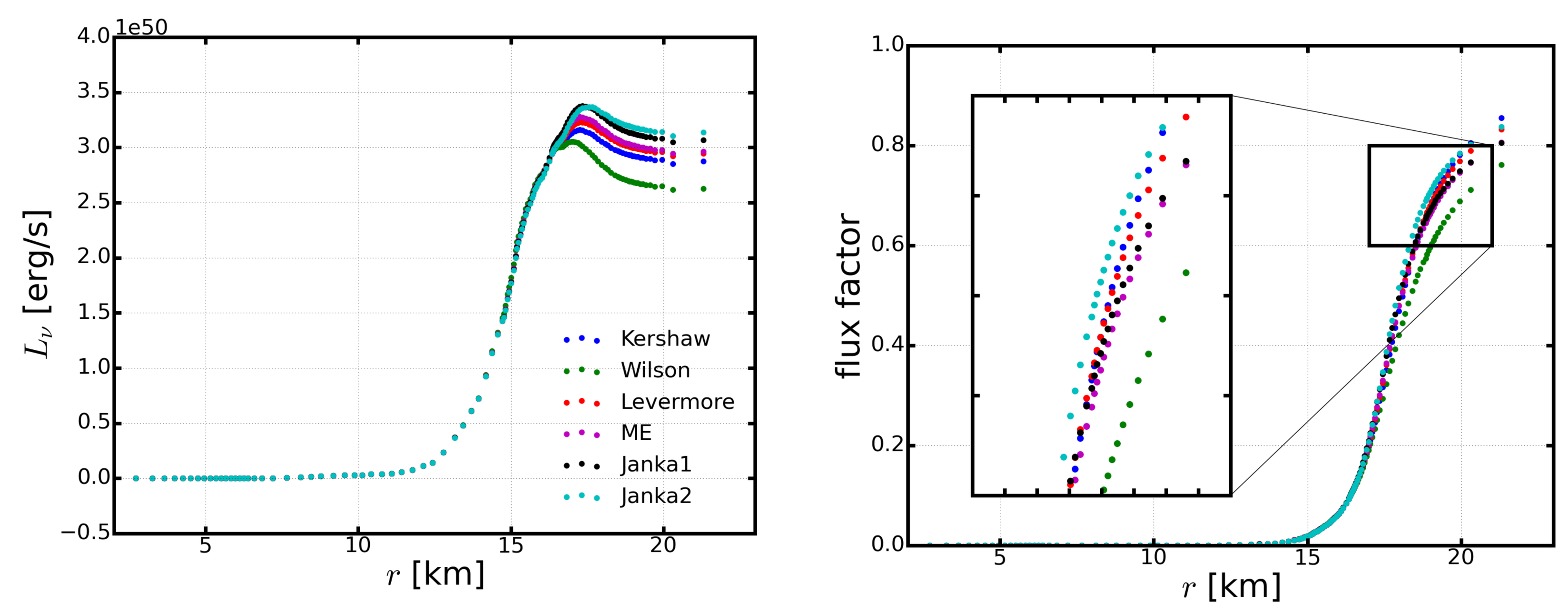
I. Compare M1 and MGFLD

Luminosity ($\frac{dL_\nu}{dE_\nu}\Delta E_\nu$, observed as a neutrino of 16 MeV at infinity)



- Near the center, the neutrino is in **thermal equilibrium**, so the results for FLD and M1 are the same.
- The difference between FLD and M1 appears **near the neutrino sphere** (~17 km). The difference is up to **35%** (around average energy).
- The higher the energy, the larger the neutrino sphere and the smaller the difference between FLD and M1.

II. Compare with different closures



- Each closure behaves differently near the surface.
- The difference is up to **15%** (based on the minimum).
- It is impossible to know which closure is better without comparing it to the exact solution.

4. Conclusion/Future Plans

- Neutrino luminosity were found to be larger for M1 than for FLD.
- However, the result for M1 is also an **approximation**, and the actual value may be somewhere between M1 and FLD.
- Therefore, we would like to discuss the behavior of M1 by **comparing** the more accurate or direct integration solution of the Boltzmann equation (of course, due to numerical cost, long time calculation is impossible) with M1 in the future.