ニュートリノ反応率に依存する
ニュートリノ集団振動の振る舞い

(Behaviors of Collective Neutrino Oscillations Induced by Neutrino Reactions)

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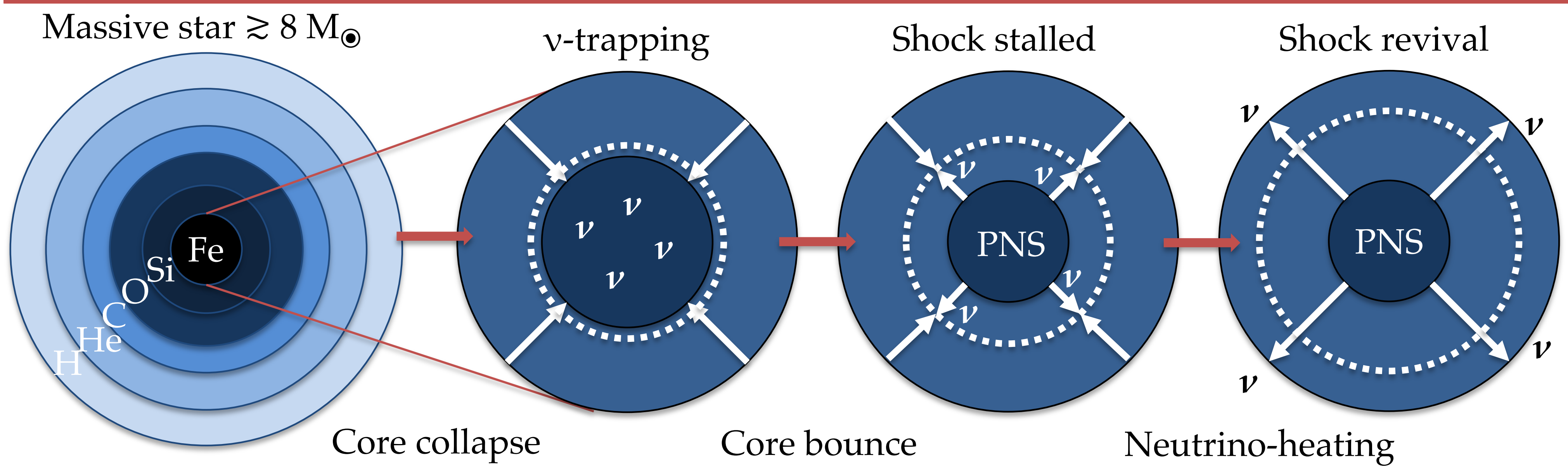
第１１回超新星ニュートリノ研究会

Komaba, Univ. of Tokyo @ Mar. 04, 2025

Roles of Neutrinos in CCSNe



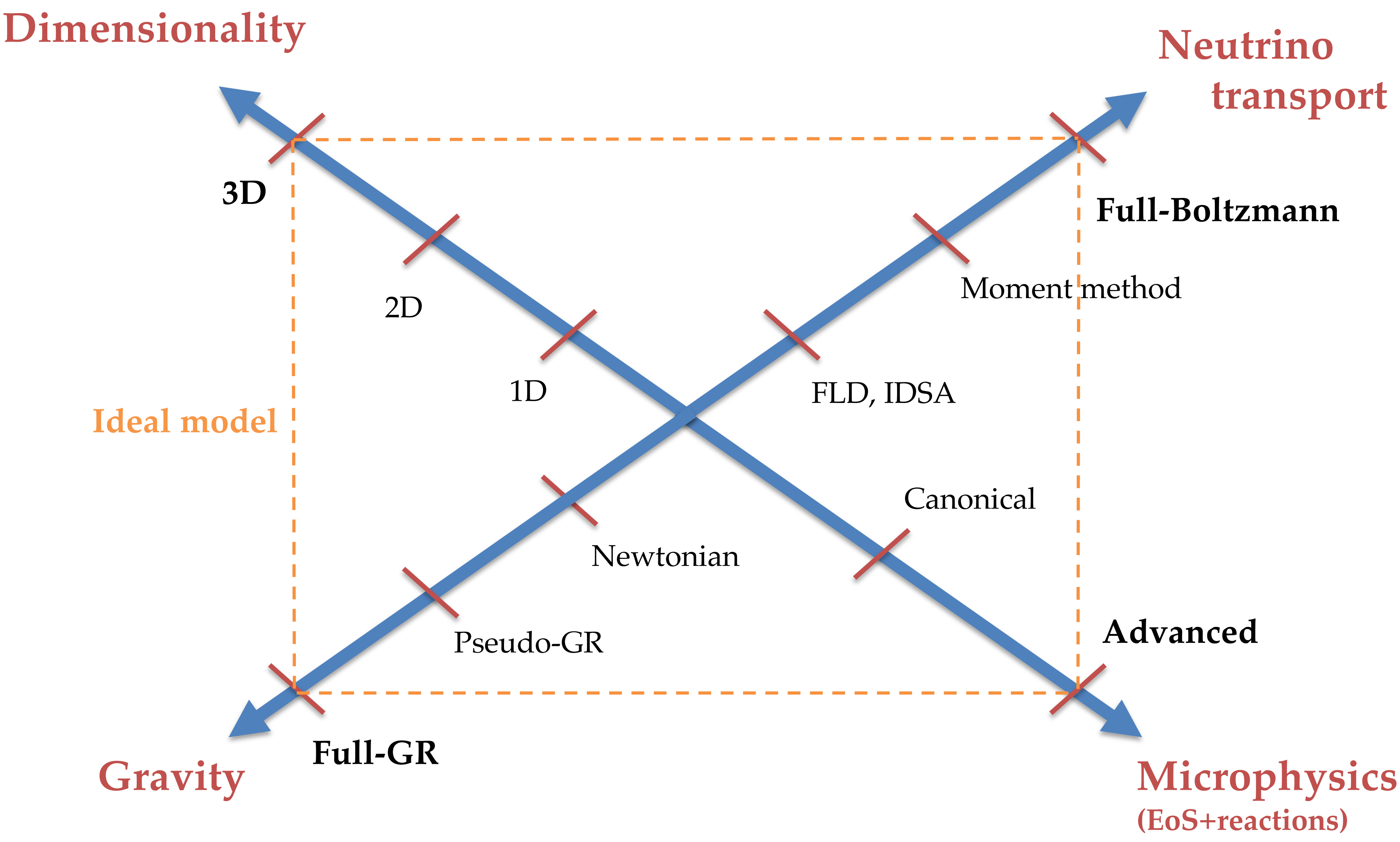
Neutrino-heating process:

- Shock wave stalls due to accreting matter and fails to explode.
- **Neutrinos transfer their energy from the hotter center to the colder stalled shock.**
- Neutrinos work as ***mediators*** because of their weakly-coupling with matter.

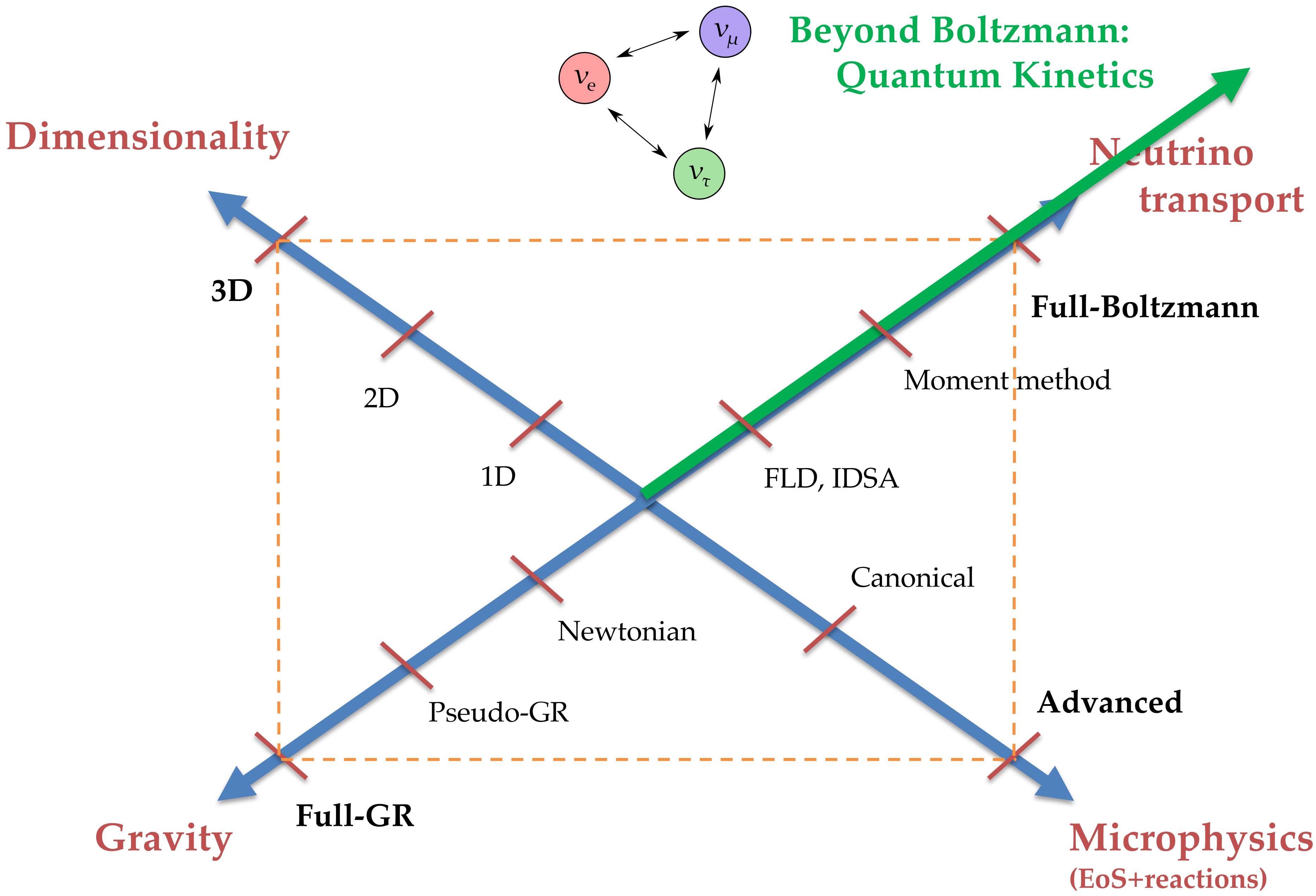
Theoretical studies/modelings on ν-transport are essential.

→ ***One of the uncertainties is neutrino oscillation.***

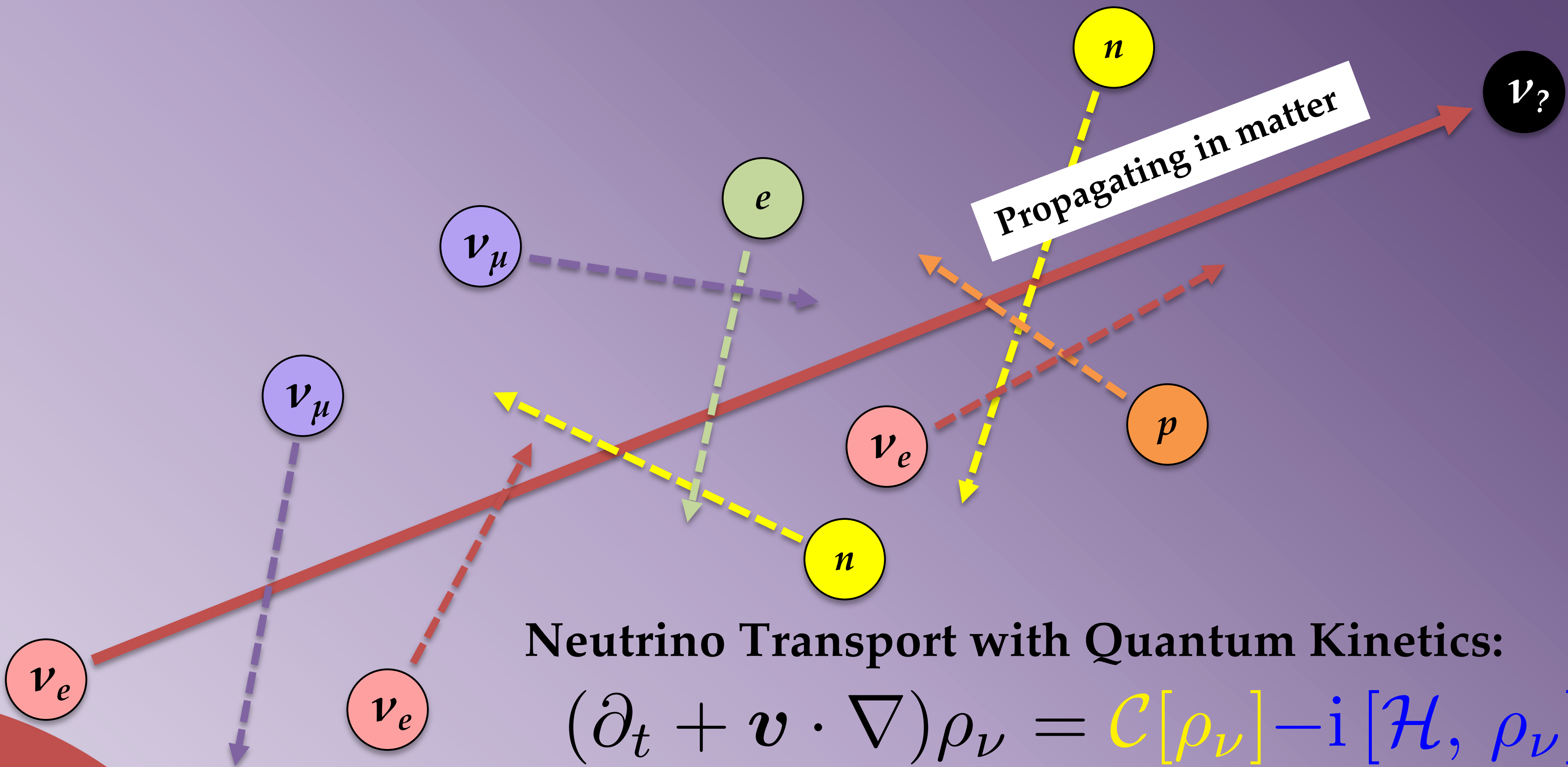
Progress in CCSN Simulation

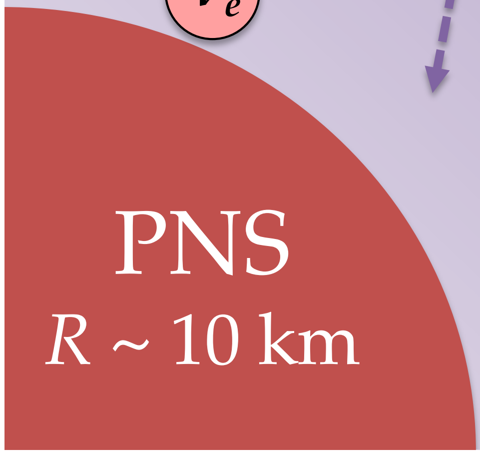


Advance in Neutrino Transport



Sea of Leptons & Nucleons



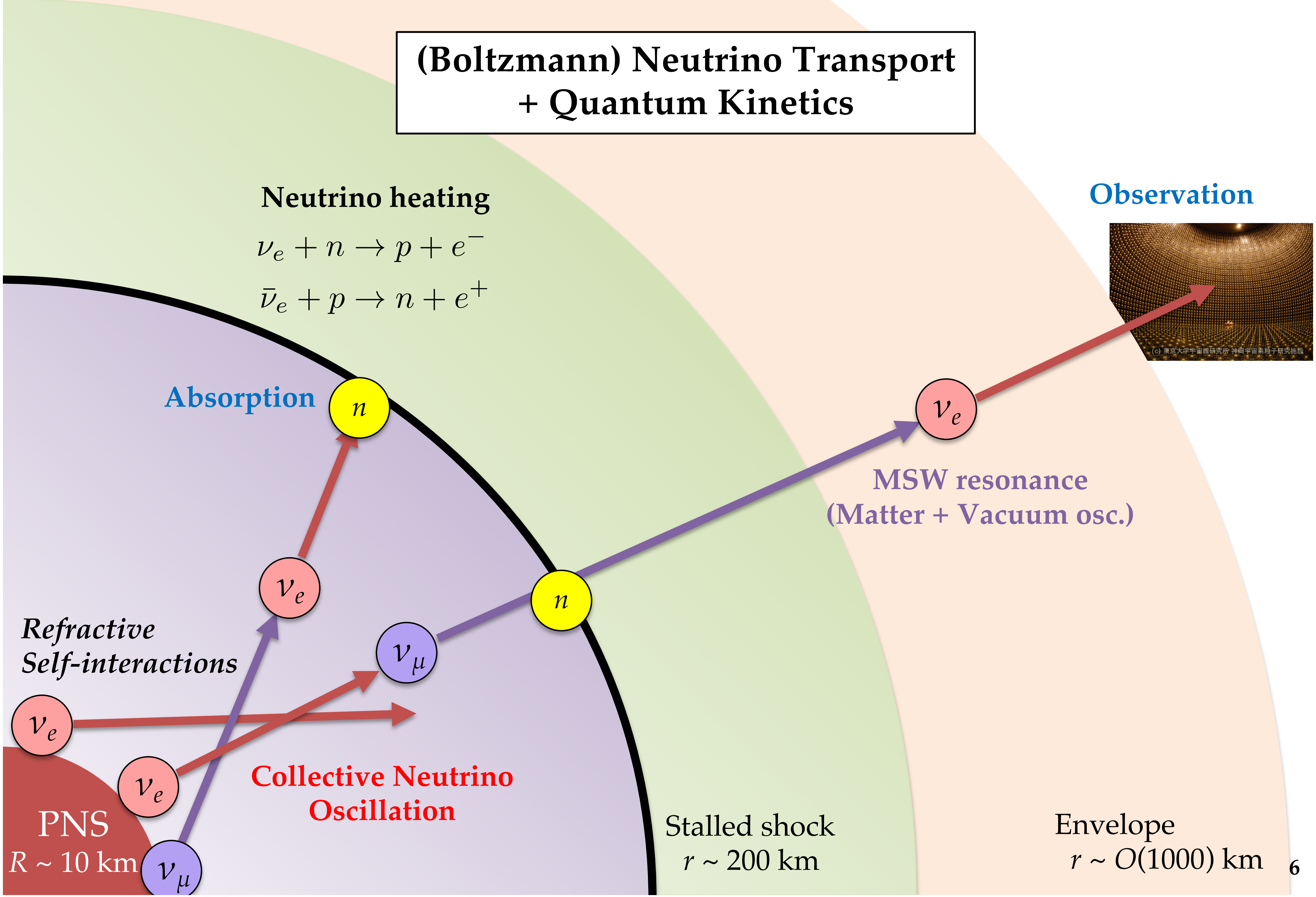


Neutrino Transport with Quantum Kinetics:

$$(\partial_t + \boldsymbol{v} \cdot \nabla)\rho_\nu = \mathcal{C}[\rho_\nu] - \mathrm{i}\,[\mathcal{H},\ \rho_\nu]$$

- Collisions $\propto (G_\mathrm{F}^2\, n_l)^{-1}$
- Refractions $\propto (G_\mathrm{F}\, n_l)^{-1}$
  → Flavor conversion (*e.g.*, MSW effects)
  → Shorter (faster) physical scale ~ 1 cm!!

Quantum Kinetic Neutrino Transport



Flavor Conversion in ν-Transport

Classical Boltzmann Equation

$$\left(p^{\mu} \frac{\partial}{\partial x^{\mu}} + \frac{\mathrm{d}p^{j}}{\mathrm{d}\tau} \frac{\partial}{\partial p^{j}} \right) f_{\nu} = \mathcal{C}[f_{\nu}]$$

$f_{\nu} \rightarrow \rho_{\nu}$

(= Include correlations between flavors)

Quantum Kinetic Equation

$$\left(p^{\mu} \frac{\partial}{\partial x^{\mu}} + \frac{\mathrm{d}p^{j}}{\mathrm{d}\tau} \frac{\partial}{\partial p^{j}} \right) \rho_{\nu} = \underline{-\mathrm{i}\left[H_{\mathrm{osc}}, \rho_{\nu} \right]} + \mathcal{C}[\rho_{\nu}]$$

Refractive effect
= Oscillation term

Neutrino density matrix (for 2-flavor) :

$$\rho_{\nu} = |\psi_{\nu}\rangle \langle \psi_{\nu}| = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix}$$

$$= \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix}$$

- $\rho^{\alpha\alpha}$: **flavor content** of α
  = *same as* f_{α}
- $\rho^{\alpha\beta}$: **flavor correlation** between α and β

Flavor Conversion in ν-Transport

Neutrino density matrix (for 2-flavor) :

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- $\rho^{\alpha\alpha}$: **flavor content** of α = *same as* f_α
- $\rho^{\alpha\beta}$: **flavor correlation** between α and β

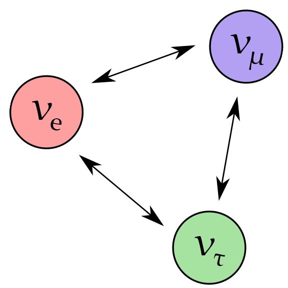
Via mixing angles,
Flavor correlation becomes perturbative.
(Less dependent on mass ordering)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\mathcal{H}_{\nu\nu} = \sqrt{2} G_{\rm F} \int {\rm d}\Gamma' \, v^\mu v'_\mu (\rho' - \bar{\rho}^{*\prime})$$

Self-interactions amplify it.

Linear stability analysis
→ Flavor instability



Significant flavor conversion

Instability by Self-Interactions

Quantum Kinetic Equation:

Self-interactions

$$(\partial_t + \boldsymbol{v}\cdot\nabla)\rho = -\mathrm{i}\,[\mathcal{H}_{\mathrm{vac}} + \mathcal{H}_{\mathrm{mat}} + \mathcal{H}_{\nu\nu},\ \rho] + \mathcal{C}_{\mathrm{col}}$$

Collisions

e.g., *Duan+ '06*

Slow flavor instability (SFI)

By energy crossing

$$\tau_{\mathrm{slow}} \sim \mathcal{O}(\sqrt{\mu\omega_{\mathrm{v}}})^{-1}$$

e.g., *Sawyer '16*

Fast flavor instability (FFI)

By anisotropy of angular dist.

$$\tau_{\mathrm{fast}} \sim \mathcal{O}(\mu^{-1})$$

$$\sim \mathcal{O}(G_{\mathrm{F}} n_{\nu})^{-1} \sim 1\,\mathrm{cm}$$

Challenging!!

e.g., *Johns '23*

Collisional flavor instability (CFI)

By disparity in collision rates

$$\tau_{\mathrm{col}} \sim \mathcal{O}(\sqrt{\mu\Gamma})^{-1} - \mathcal{O}(\Gamma)^{-1}$$

Relatively longer scale

Collisions in Quantum Regime

e.g., Emission & Absorption processes are

$$\mathcal{C}[\rho] = \begin{pmatrix} R_{\rm emi}(1-\rho_{ee}) - R_{\rm abs}\rho_{ee} & -\frac{1}{2}(R_{\rm emi}+R_{\rm abs})\rho_{ex} \\ -\frac{1}{2}(R_{\rm emi}+R_{\rm abs})\rho_{xe} & 0 \end{pmatrix}$$

$$\equiv \mathcal{C}_{\rm cls} + \mathcal{C}_{\rm qke}$$

Quantum contributions (off-diagonal parts)
- **Flavor-decohering collisions** $\sim -R_E \rho_{\rm T}$

Classical collisions (diagonal parts)
- **Changing the numbers/momenta of (anti-)neutrinos**
- **Contribution (C_{xx}) to heavy-leptonic flavors is neglected.**

→ Reactions settle down the neutrinos to **"flavor-diagonal"** distributions.
~ Oscillations are fixed on the mixed-flavor population.

→ ***Flavor Instability?***

Collisional Flavor Instability

$$\partial_t \rho = -\mathrm{i}\left[\mathcal{H}_{\nu\nu}, \rho\right] - R_E \rho_{\mathrm{T}}$$

Self-interactions: $\mathcal{H}_{\nu\nu} = \sqrt{2} G_{\mathrm{F}} \int \mathrm{d}\Gamma' \, v^{\mu} v'_{\mu} (\rho' - \bar{\rho}^{*\prime})$

Couplings

$$\partial_t \bar{\rho} = -\mathrm{i}\left[\bar{\mathcal{H}}_{\nu\nu}, \bar{\rho}\right] - \bar{R}_E \bar{\rho}_{\mathrm{T}}$$

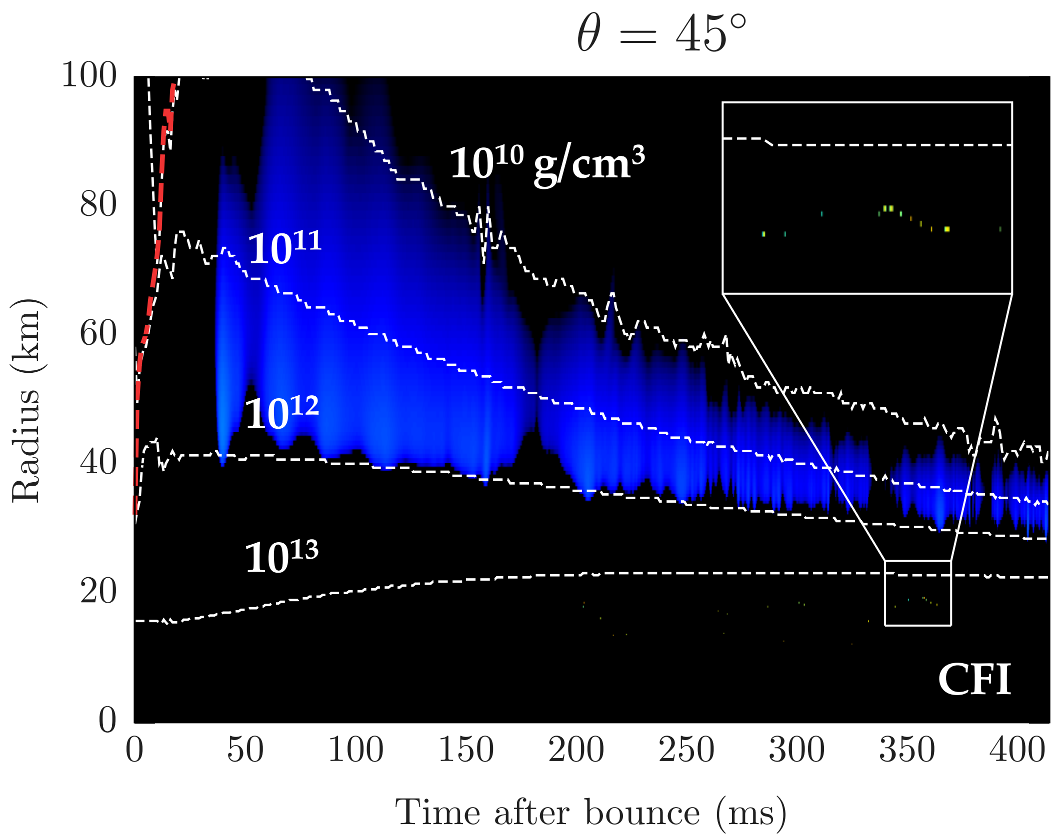
→ When the entire system has ***the disparity in collision rates*** between neutrinos and antineutrinos, flavor instability can appear. *Johns PRL '23*

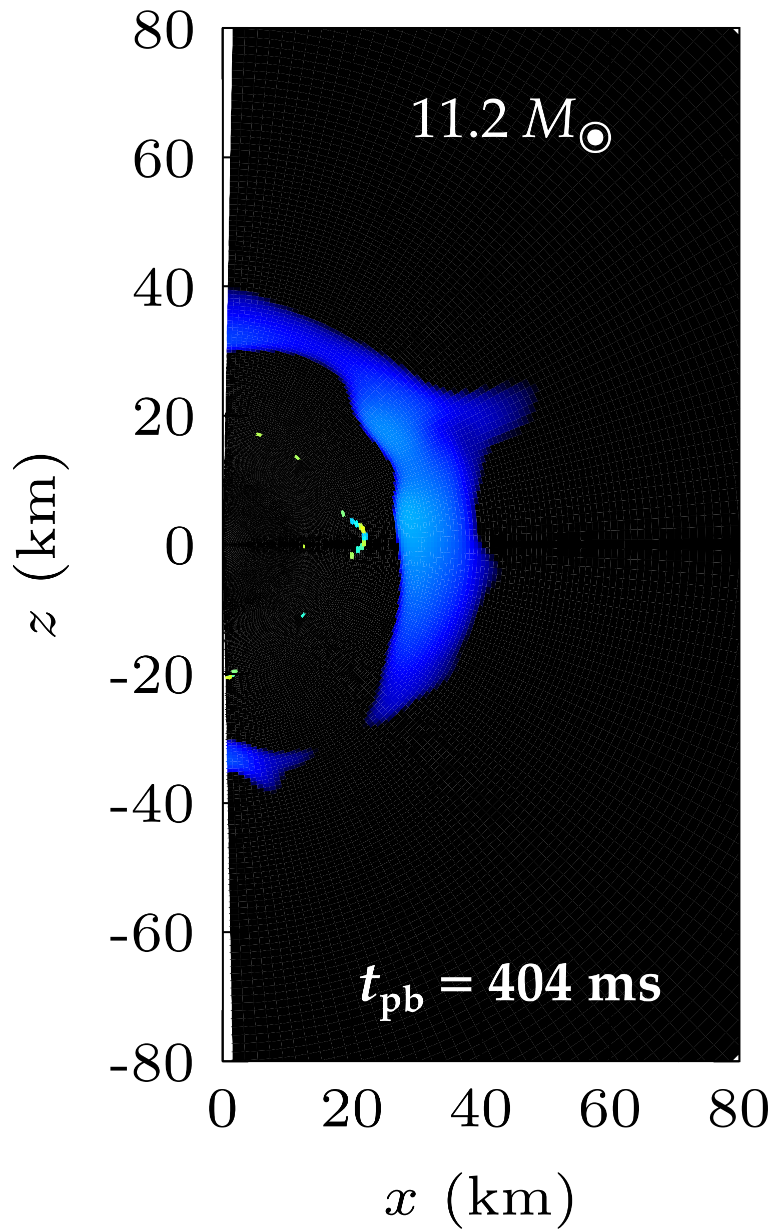
$$\mathcal{H}_{\nu\nu} \to \int \mathrm{d}\Gamma (-R_E \rho_{ex} + \bar{R}_E \bar{\rho}_{ex}) \propto -\langle R_E \rangle + \langle \bar{R}_E \rangle$$

→ Flavor correlation can grow!!
(But relatively slower)

Search of Flavor Instability

In the 2D-model, flavor instability (CFI) appears at broad radii.

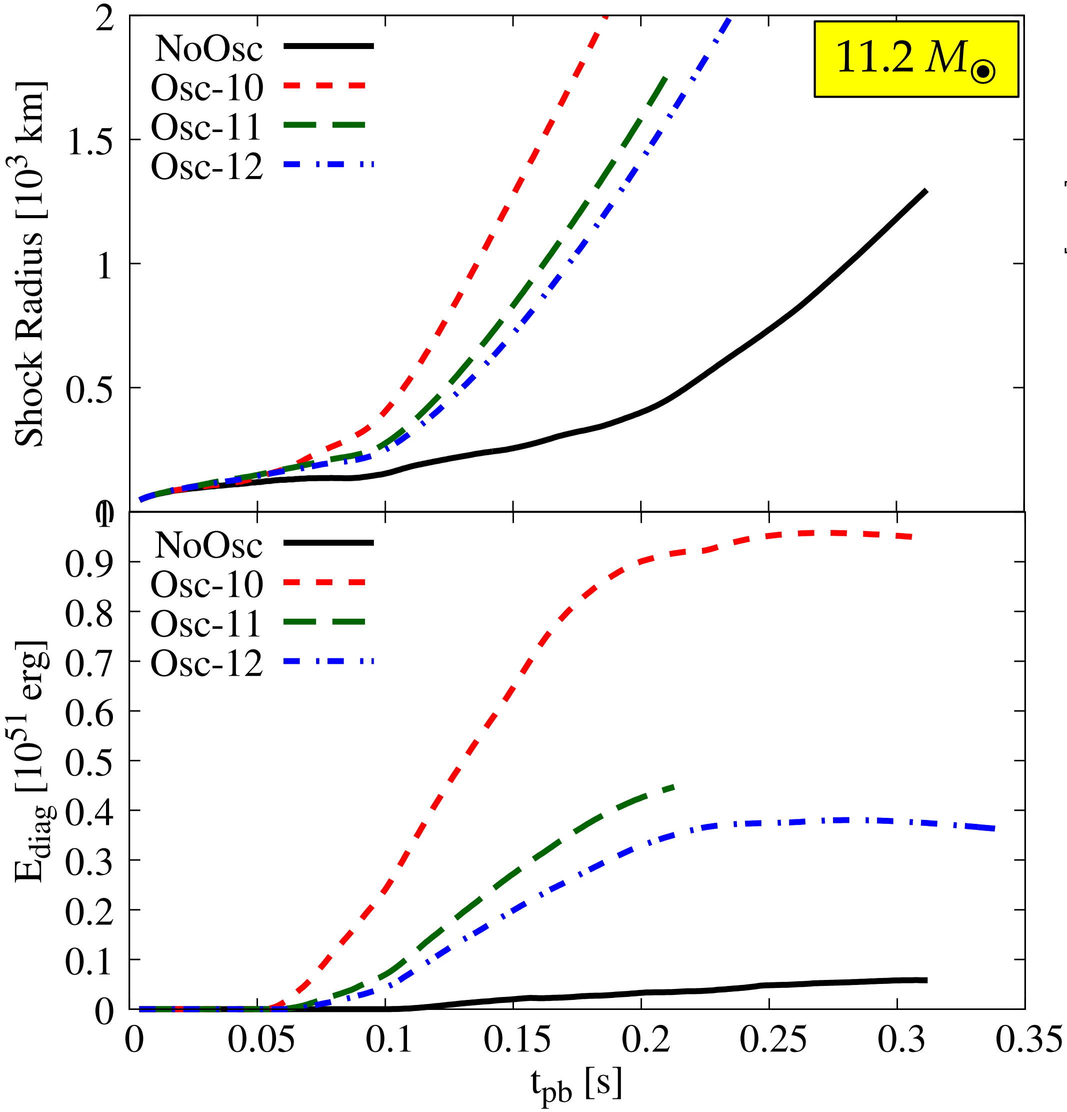




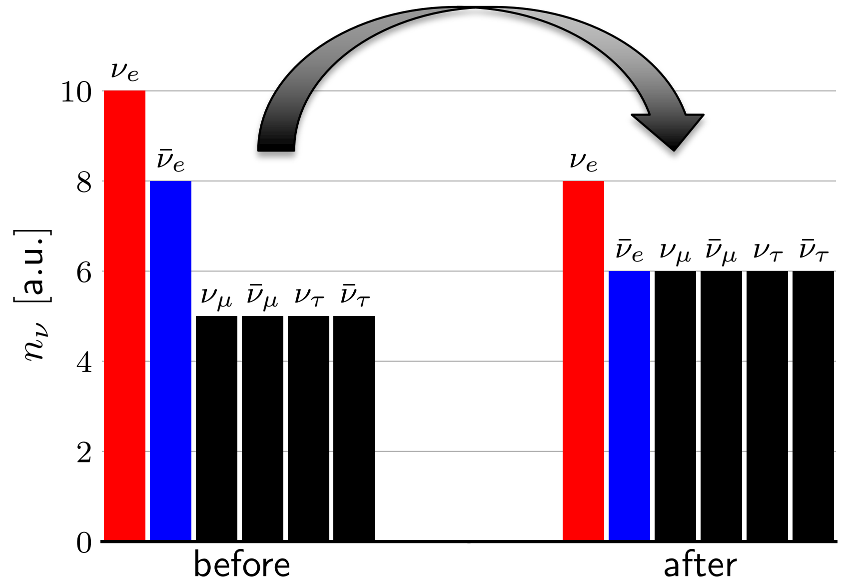
Akaho, MZ+ PRD '24

Phenomenological Approach

K. Mori+ PASJ '25. 3D-CCSN simulations



Flavor conversion



Equipartition

Assumptions:

1. Below a critical density, $\rho < \rho_{\rm crit}$.
2. Equipartition with conservations.

This may overestimate but can change the shock dynamics.

Need more accurate FC theory.

→ ***Direct computation!!***

Strategy for Quantum Transport

Quantum Kinetic Neutrino Transport

~ $O(cm)$

Separately considered among physical scales

~ $O(km)$

Quantum Kinetics
[Collective Neutrino Oscillation]

+

Classical Neutrino Transport
[Boltzmann Equation]

Independently investigated

Sub-grid modeling

1. **Slow Instability**
2. **Fast Instability**
3. ***Collisional Instability***

Back into ``classical” transport
with relaxation scheme

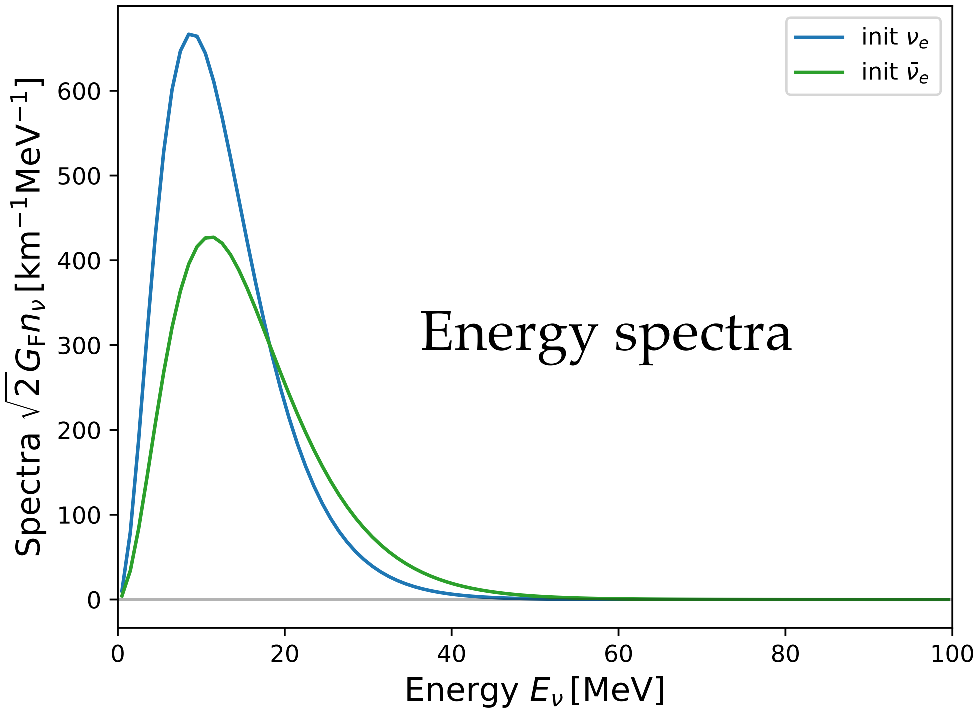
Setting QKE

$$\partial_t \rho = -\mathrm{i}\,[\mathcal{H}_{\nu\nu}, \rho] - R_E \rho_\mathrm{T}$$

Flavor-decohering collisions: $$\overset{(-)}{R}_\nu(E_\nu) = \overset{(-)}{R}_0 \left(\frac{E_\nu}{10\,\mathrm{MeV}}\right)^2$$

$R_0 = 1\,\mathrm{km}^{-1}$

$\bar{R}_0$ for anti-neutrinos from 0 to $2\,\mathrm{km}^{-1}$

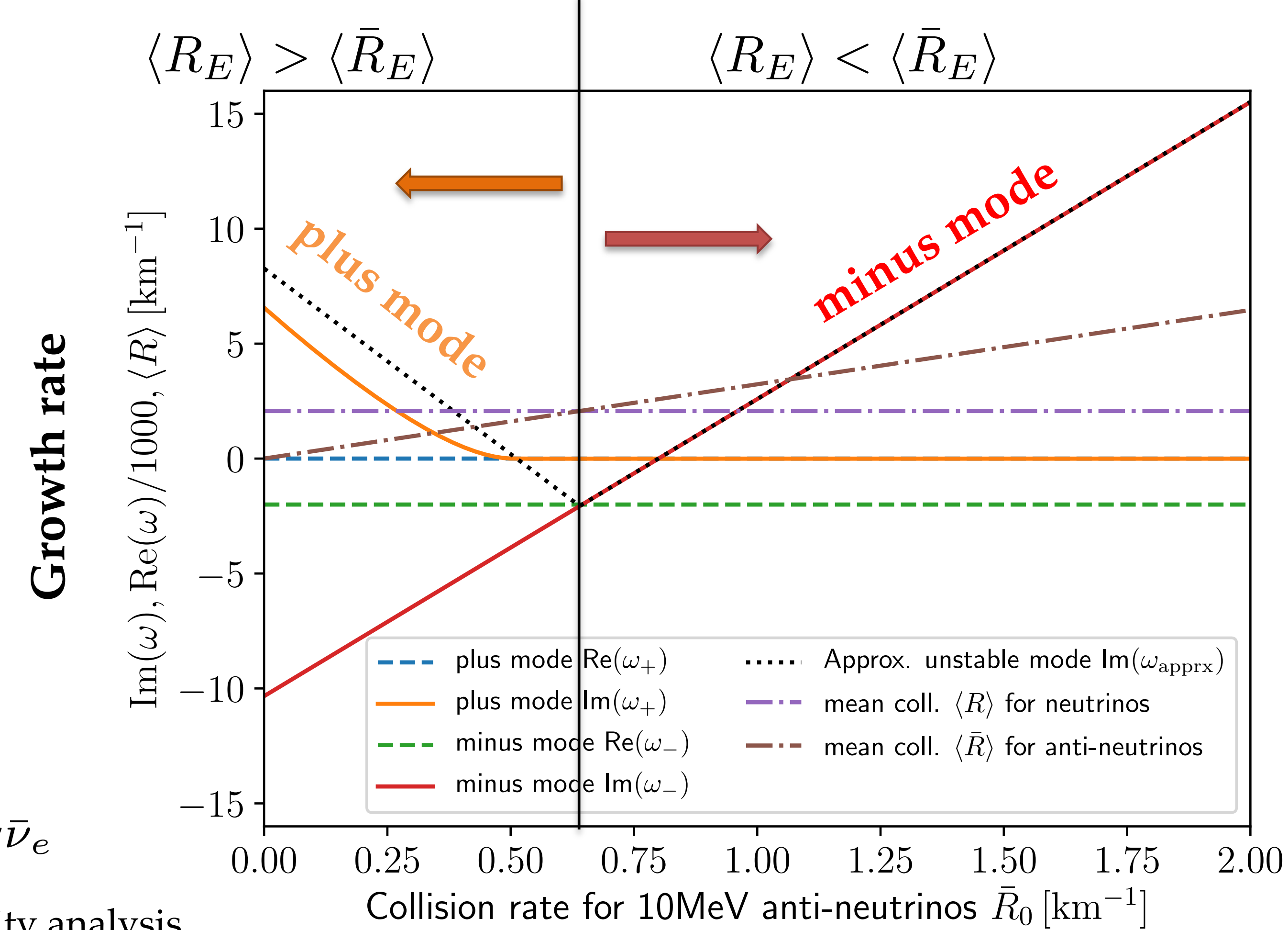


At $\bar{R}_0 \sim 0.64\ \mathrm{km}^{-1}$, $\langle R \rangle \sim \langle \bar{R} \rangle$

= No disparity

MZ arXiv:2502.09260

Collisionally Unstable Modes



$$n_{\nu_e} > n_{\bar{\nu}_e}$$

In linear stability analysis

$$\rho_{ex} \propto \tilde{Q}_E \exp[-i\omega t]$$

plus mode

$$\omega \pm -A - i\gamma \pm \left(|A| + i\frac{G\alpha}{A} \right)$$

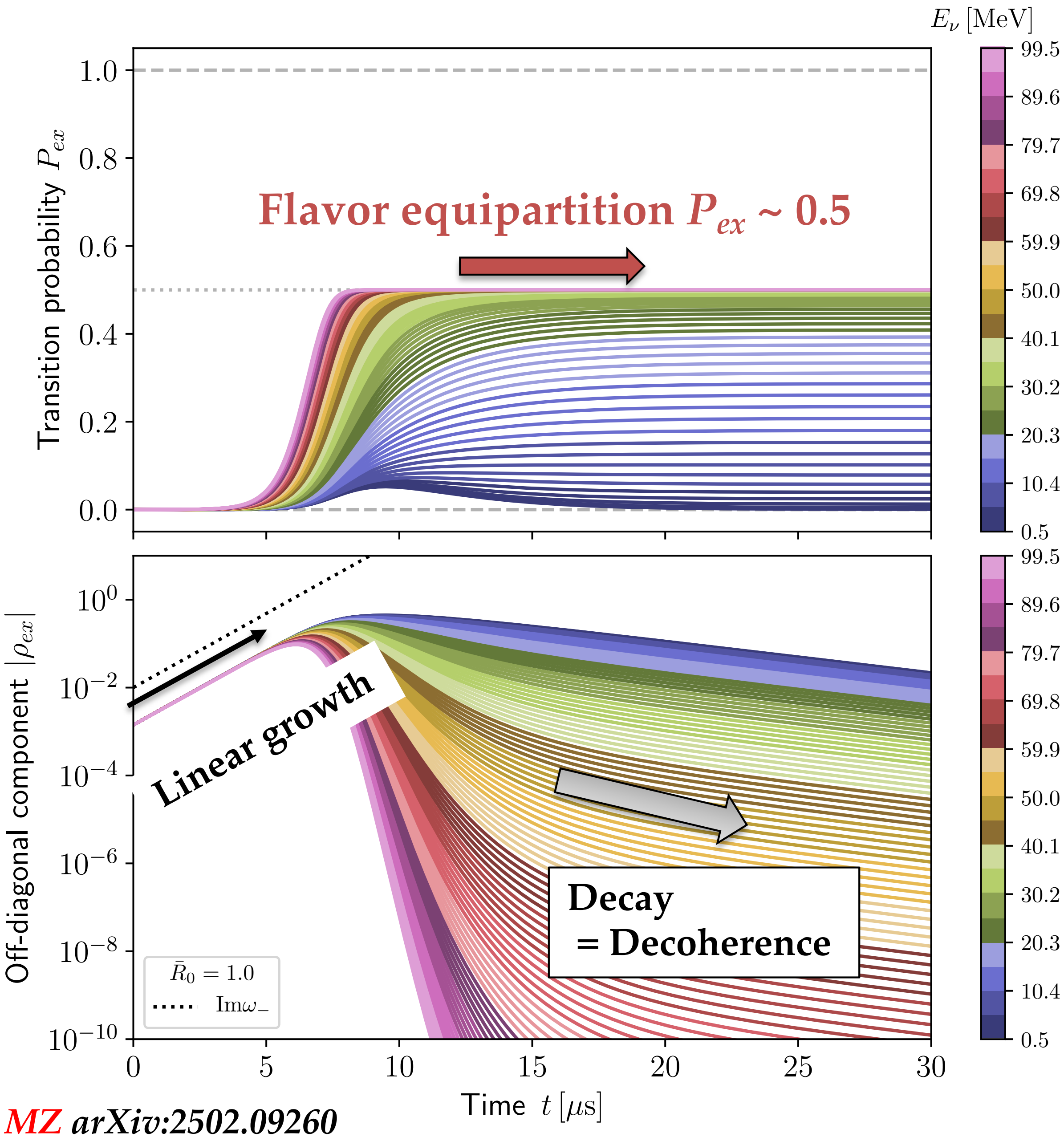
minus mode

$$\gamma = \frac{R + \bar{R}}{2}, \quad \alpha = \frac{R - \bar{R}}{2},$$

$$G = \frac{\mathfrak{g} + \bar{\mathfrak{g}}}{2}, \quad A = \frac{\mathfrak{g} - \bar{\mathfrak{g}}}{2},$$

MZ arXiv:2502.09260

Evolution of Minus Mode



$$n_{\nu_e} > n_{\bar{\nu}_e}$$
$$\langle R_E \rangle < \langle \bar{R}_E \rangle$$
} *Twisted Hierarchy*

$$(\bar{R}_E = R_E) \quad Y_e \sim 0.5$$

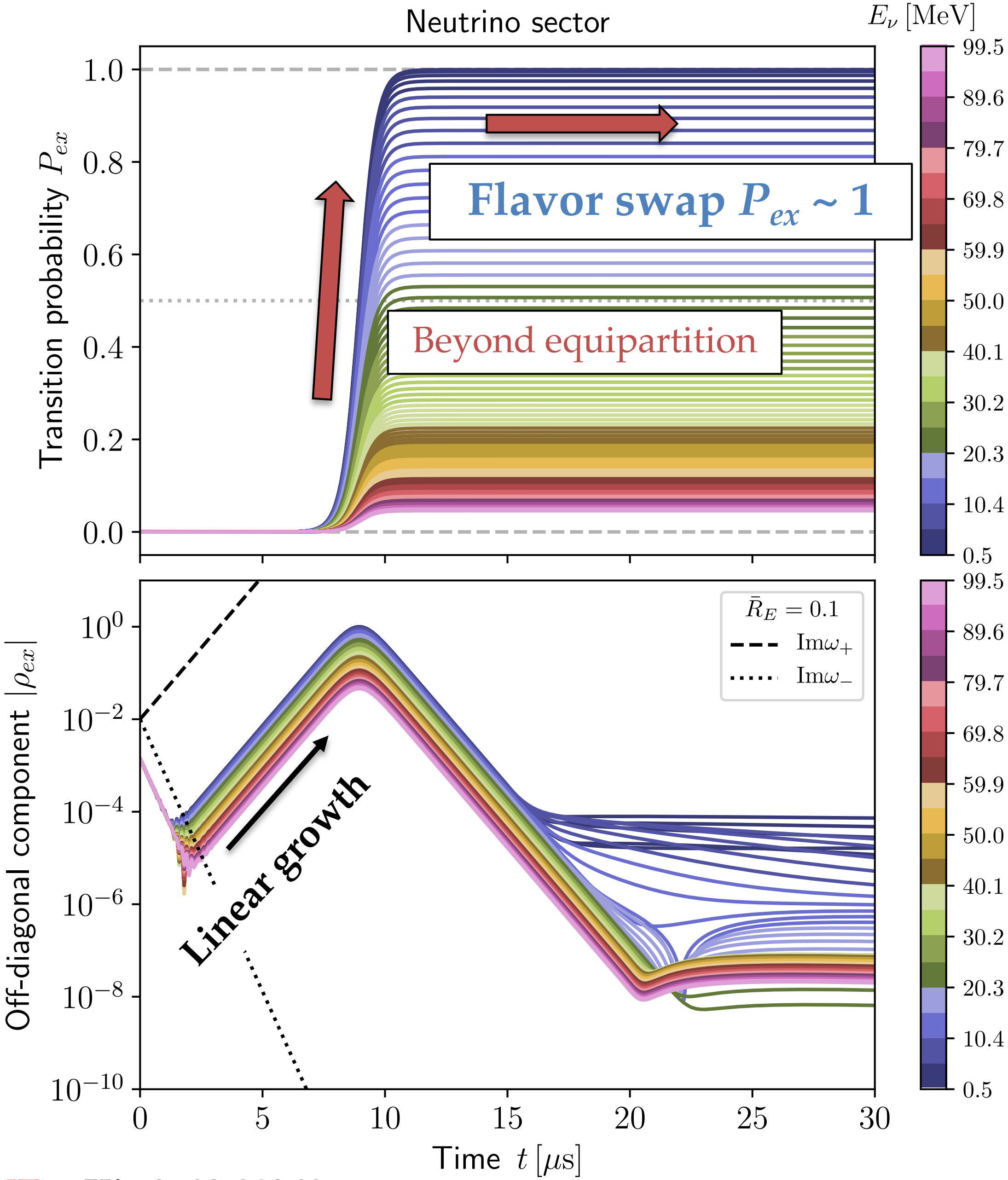
ν_e = ν_μ **Equipartition**

Energy-dependent flavor evolution.

Collision decoheres the system.
→ Neutrino reaches a steady state.

MZ arXiv:2502.09260

Evolution of Plus Mode



$$n_{\nu_e} > n_{\bar{\nu}_e}$$
$$\langle R_E \rangle > \langle \bar{R}_E \rangle$$
$$(\bar{R}_E = 0.1 R_E)$$

Same Hierarchy

$Y_e \sim 0.1$

$\nu_e \leftrightarrow \nu_\mu$ **Swap**

Rich spectral diversity!

Not just equipartition

MZ arXiv:2502.09260

Eigenvector of Unstable Modes

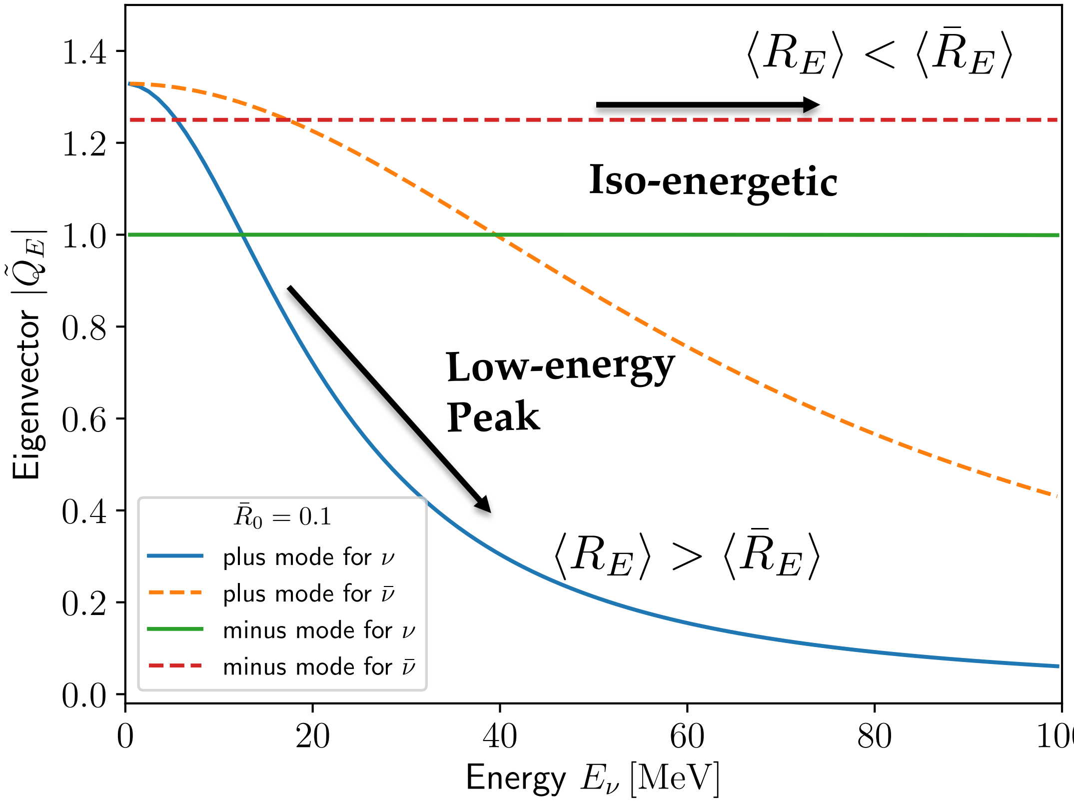
Plane-wave ansatz in linear stability analysis:

$$\rho_{ex} \propto \tilde{Q}_E \exp[-i\omega t]$$

Grows or Damps or Oscillates

Amplitude = Spectral structure

$$\left|\tilde{Q}_E\right| = (\rho_{ee} - \rho_{xx}) \frac{1}{\sqrt{\omega_r^2 + (\omega_i + R_E)^2}}$$



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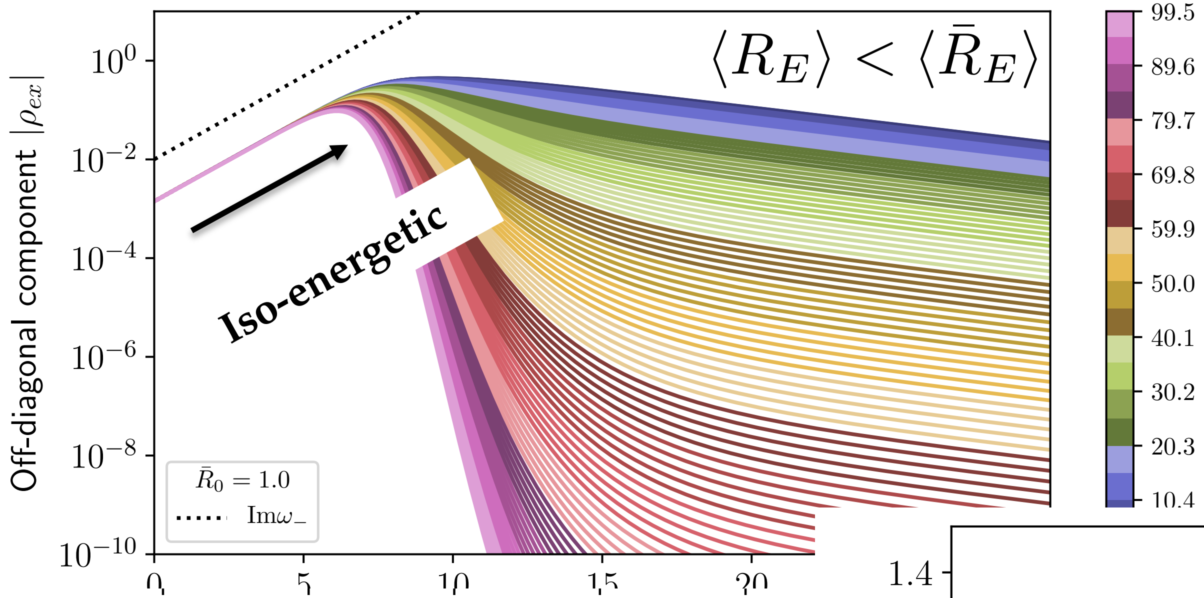
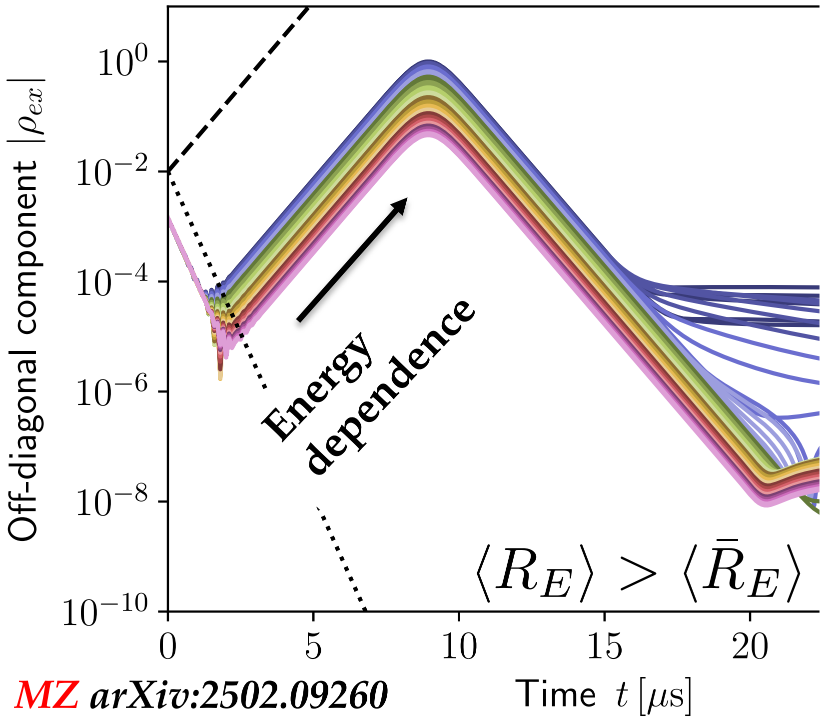
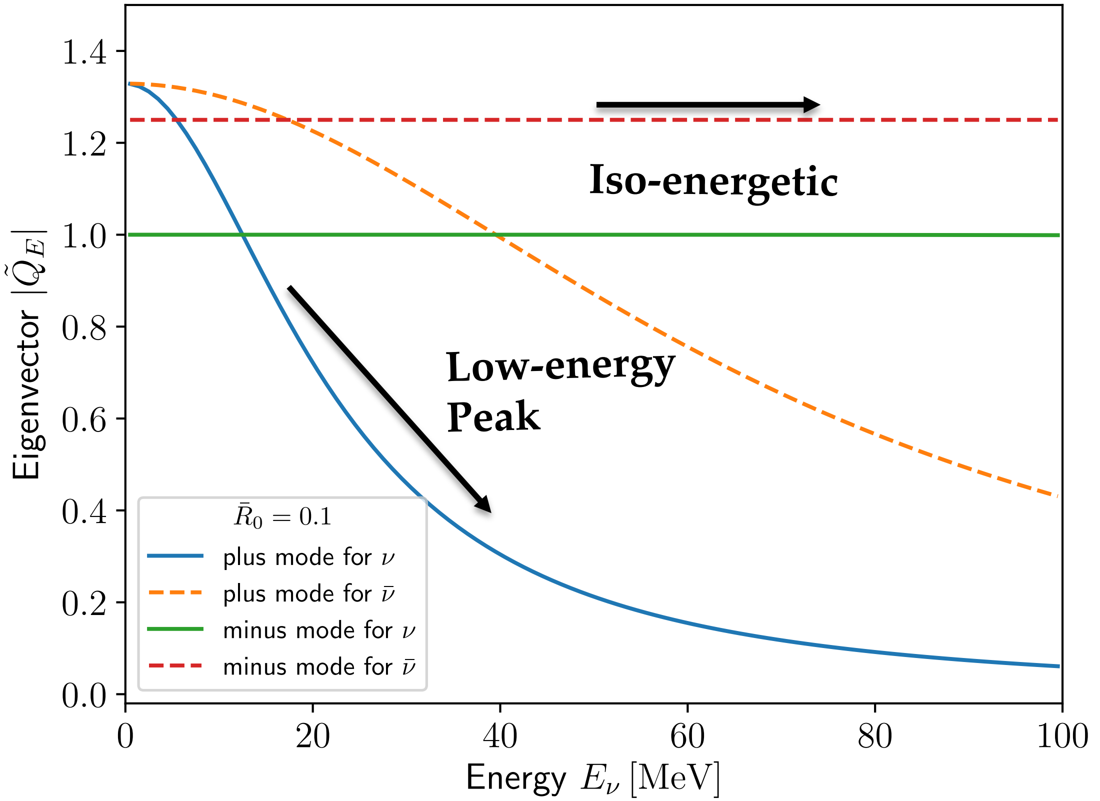
Eigenvector of Unstable Modes



$$\left|\tilde{Q}_E\right| = (\rho_{ee} - \rho_{xx}) \frac{1}{\sqrt{\omega_r^2 + (\omega_i + R_E)^2}}$$

$$\rho_{ex} \propto \tilde{Q}_E \exp[-\mathrm{i}\omega t]$$





***MZ* arXiv:2502.09260**

Back to Classical Transport

$$(\partial_t + \boldsymbol{v}\cdot\nabla)\rho = \underline{-\mathrm{i}\,[\mathcal{H}_{\rm osc},\,\rho]} + \underline{\mathcal{C}_{\rm col}}$$

$$\equiv \mathcal{C}_{\rm cls} + \underline{\mathcal{C}_{\rm qke}}$$

Relaxation-Time Approximation

c.f., Nagakura, MZ+ PRD '24

$$(\partial_t + \boldsymbol{v}\cdot\nabla)\rho = \sum_{\rm inst.} \frac{1}{\tau}(\rho - \boxed{\rho^{\rm a}}) + \underline{\mathcal{C}_{\rm cls}}$$

Supply the number density.

For all flavor instabilities

Growth rate

Modeling of Asymptotic states

We'll obtain more accurate ν-transport implementing Quantum Kinetics w/o direct computation.

Summary

1. Collision-induced flavor conversion can occur at deeper radii.

2. Dominant unstable mode depends on the magnitude relation in number density & collision rate.

3. Asymptotic behavior (state) with energy dependence is determined by the corresponding unstable mode.
    1. Flavor equipartition or swap

4. But there are still some assumptions. More accurate demonstration is required with realistic models.