March 9th in 2026

12th Supernova Neutrino Workshop

学術変革領域研究「地下稀」

第12回超新星ニュートリノ研究会

重力崩壊型超新星の三次元ボルツマン輻射流体計算による ニュートリノ集団振動の解析

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Collabolators

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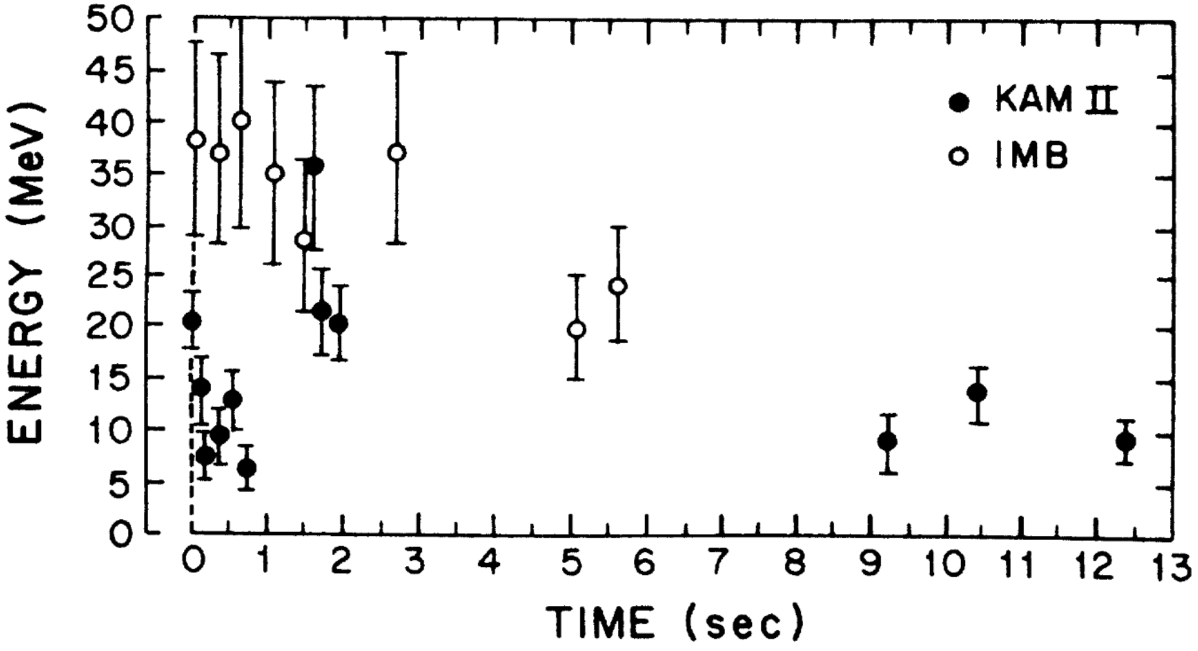
Core-Collapse Supernovae (CCSN)

Explosions of massive stars at the end of life

Suddenly brighten up and darken in a few days

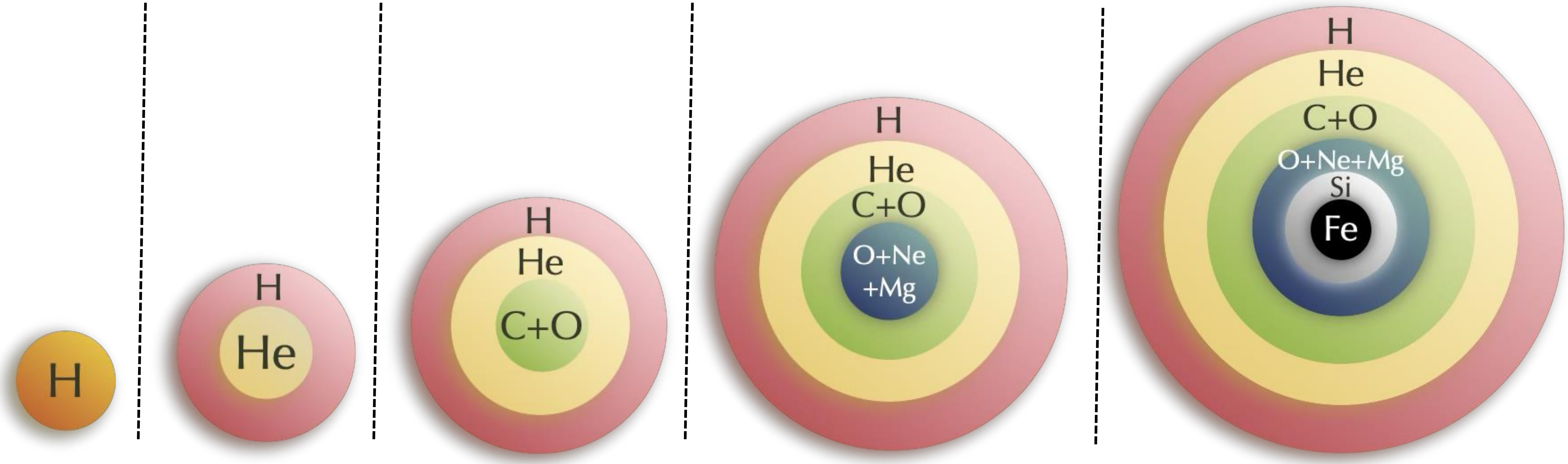
Observation in the neutrino burst in SN1987A



K. Hirata, et al. (1988)

Final State of A Single Star

$M < 0.08\ M_\odot$ $0.45\ M_\odot$ $8\ M_\odot$ $10\ M_\odot < M$



Explosion Scenario

- **Mass of a star > $10M_{\odot}$ ⇒ Red Supergiant**

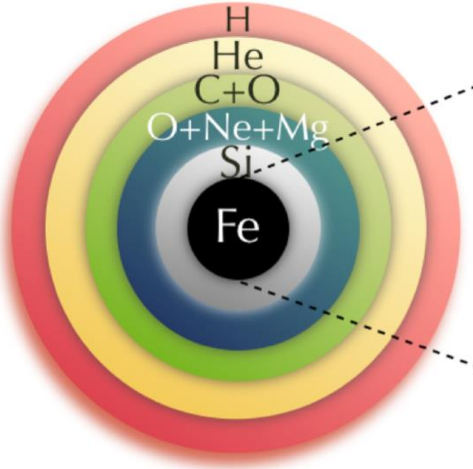
  Nuclear fusion reactions **H→He→C→ · · · →Fe**

- **Temperature > 10^9K ⇒ Gravitational collapse**

  Photo disintegration reaction: **$^{56}Fe \rightarrow 13\ ^{4}He + 4\ n$**

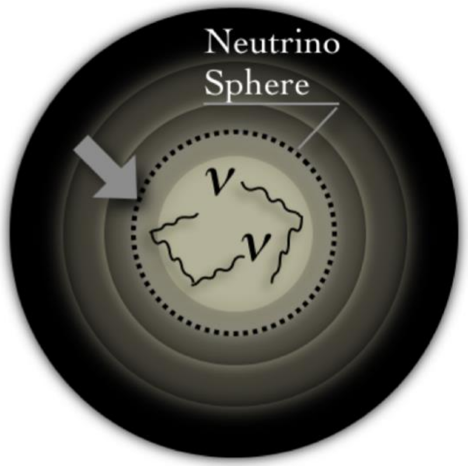
  Electron capture reaction: **$p + e^- \rightarrow n + \nu_e$**

- **Density > 10^{10}g/cm^{-3} ⇒ Neutrino Trapping**



Formation of an iron core

Lowering of central pressure



Mean free pass ~ Core radius

Prompt Explosion

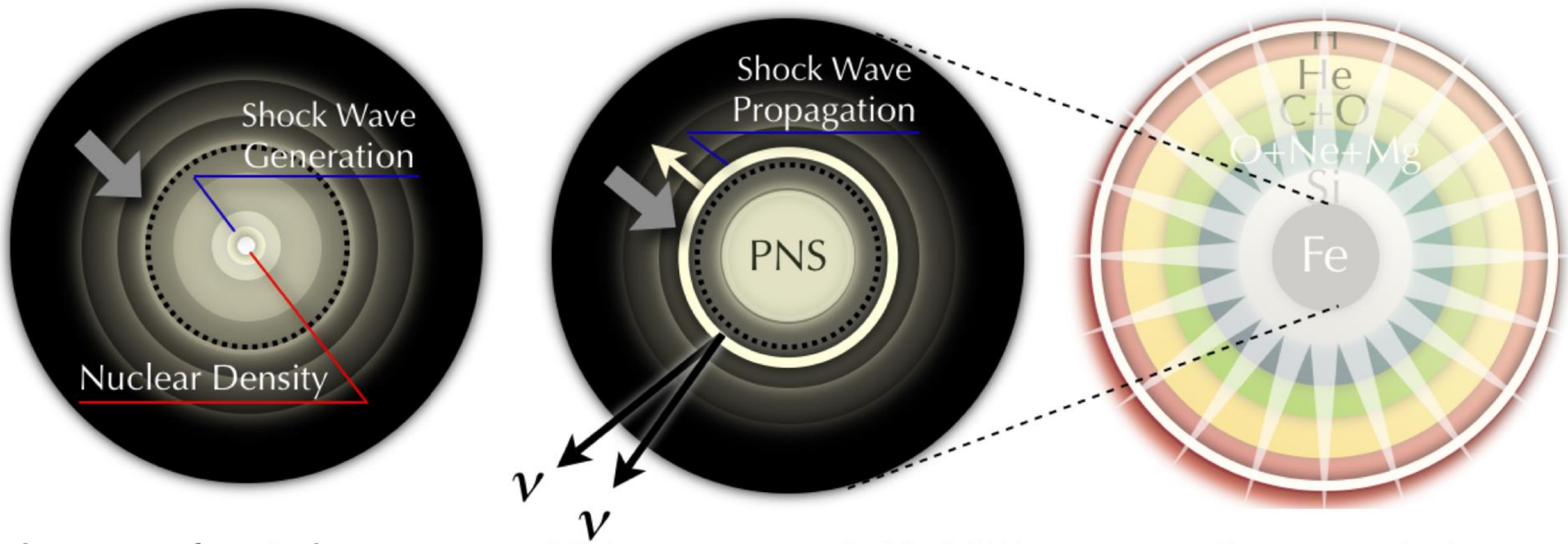
- **Central Density > 10^{14} g/cm^3 ⇒ Core bounce**

  Central region: collapse stop   Outer region: supersonic free fall

  Compression and bounce of the central part of the core

- **Generation of a spherical shock wave (SW)**

- **Propagation of a SW ⇒ Neutrino burst**



Increase of central pressure   High temperature behind SW   Prompt explosion

Delayed Explosion

- Shock Stalling

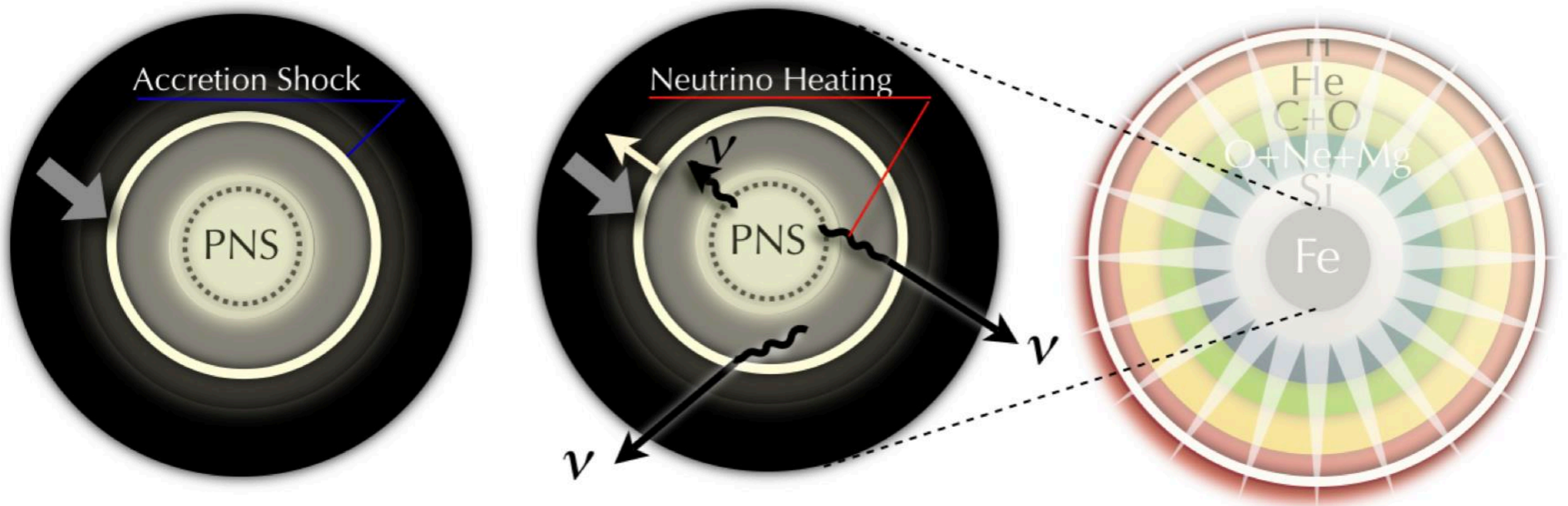
Photo disintegration reaction: **$^{56}Fe \rightarrow 13\ ^{4}He + 4\ n$**

- Neutrino heating for long time

Electron-type neutrino absorption on neutrons: $\nu_e + n \rightarrow p + e^-$

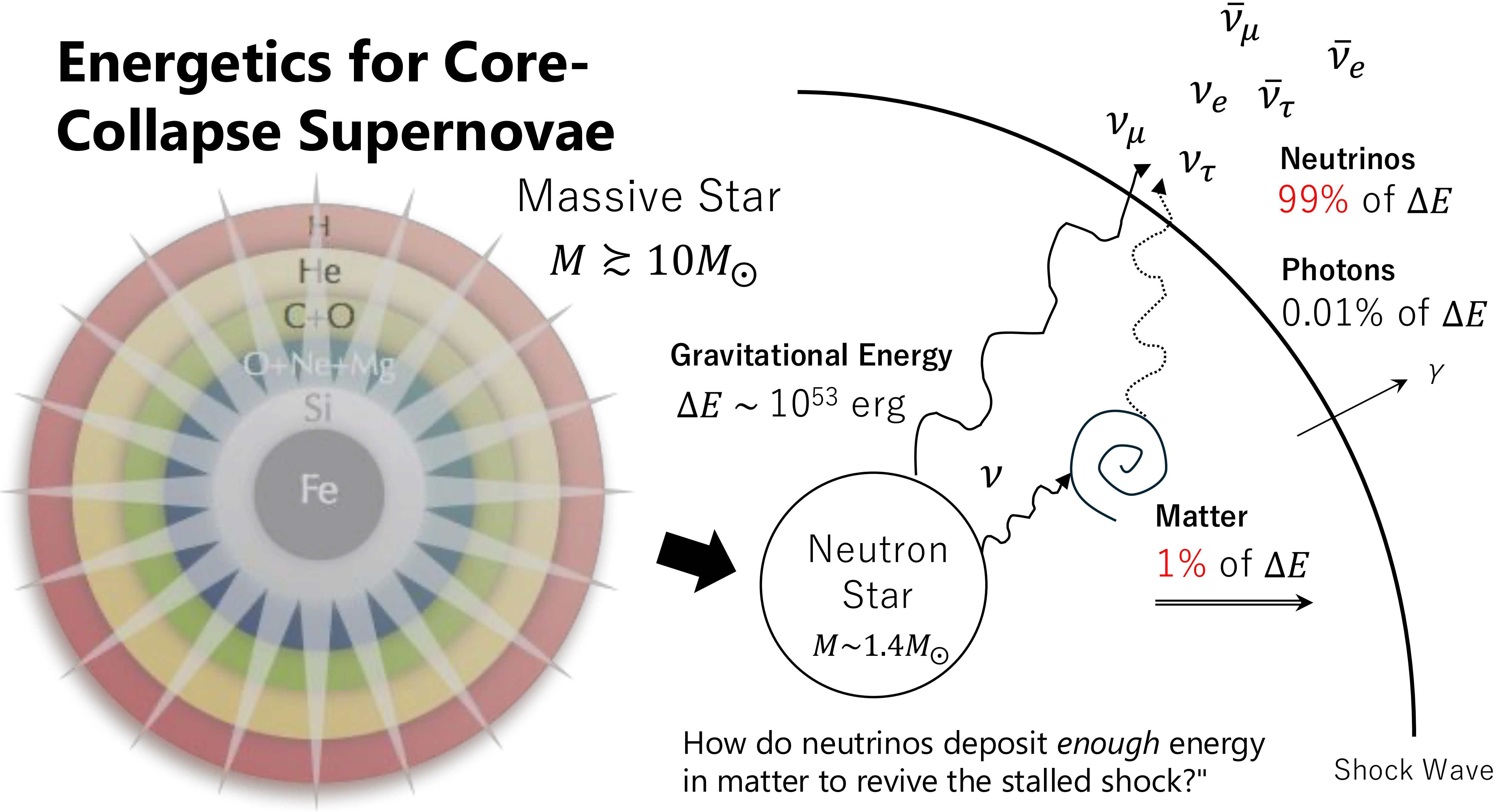
Electron-type antineutrino absorption on protons: $\bar{\nu}_e + p \rightarrow n + e^+$

- Shock revival



Shock Stall Shock Revival Delayed Explosion

Energetics for Core-Collapse Supernovae



How do neutrinos deposit *enough* energy in matter to revive the stalled shock?"

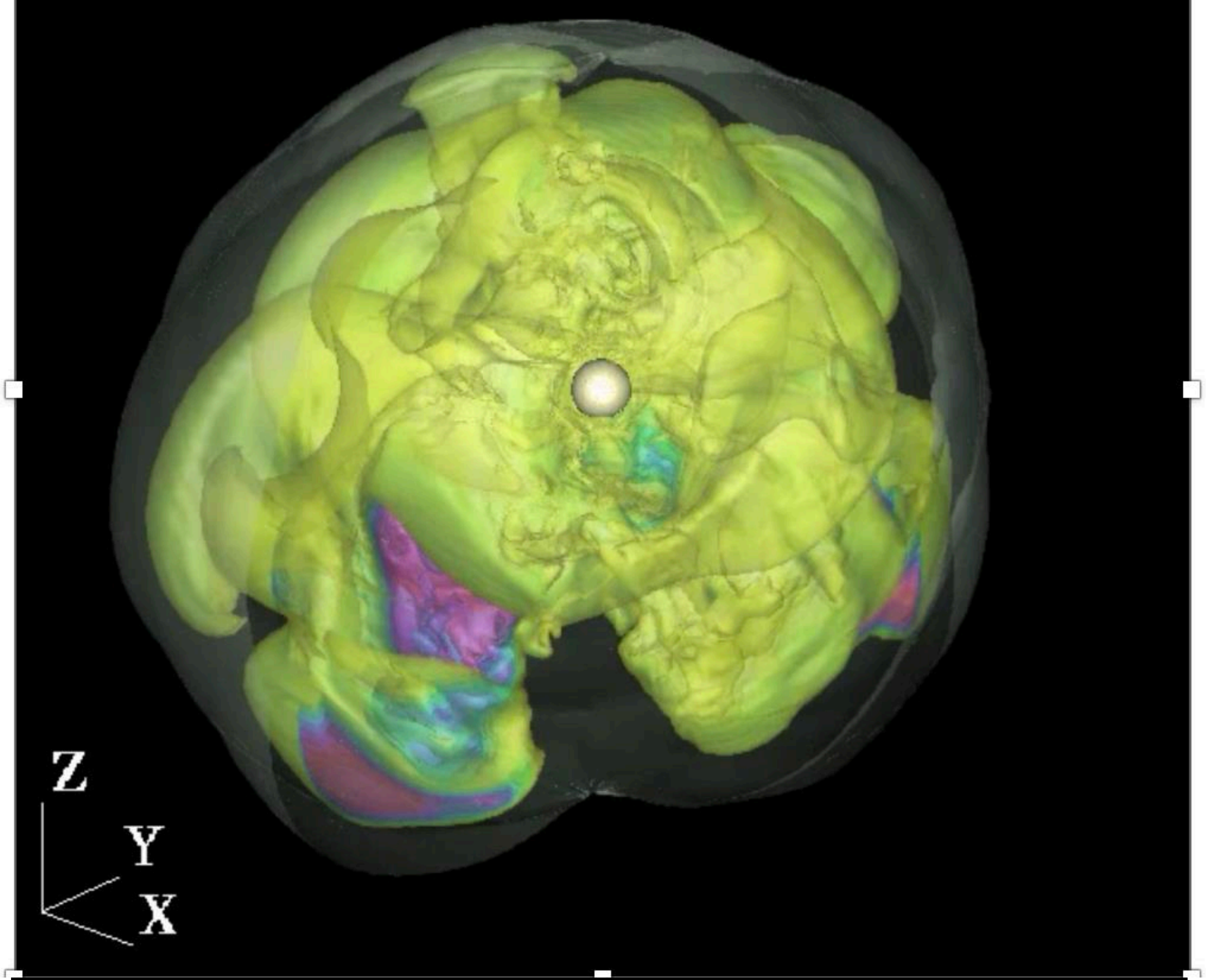
Movie of Entropy iso-surfaces

Red : high
Blue: middle
Yellow: low

The results of 3D simulations with Light-Bulb Approximation

(Iwakami 2008)

Multi-Dimensional Effect



Hydrodynamical Instabilities

1) Convection

2) SASI

3) Other instabilities

Turbulent Flow

Formation of vortices with a wide range of scales

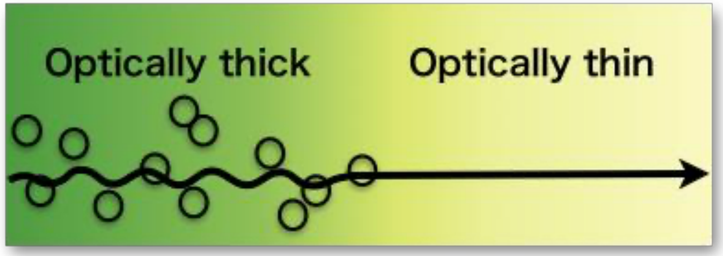
An increase of neutrino heating in the matter behind the shock wave.

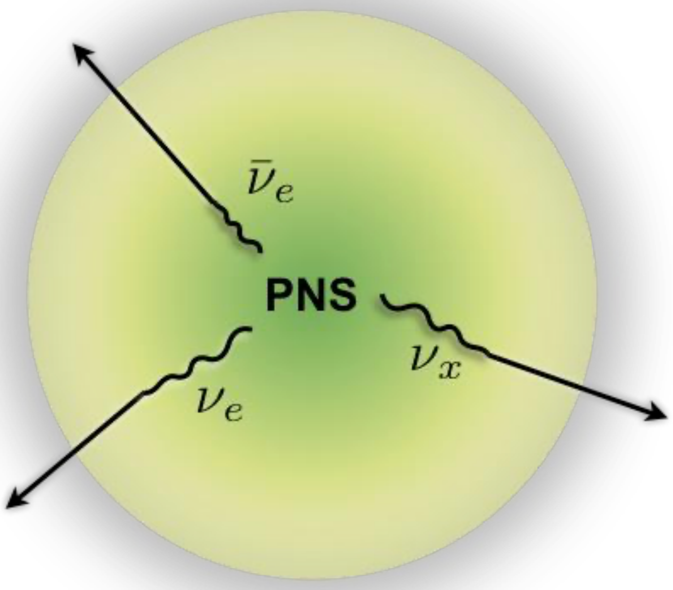
Neutrino Radiation Transport

Boltzmann Equation

$$\frac{dx^{\mu}}{d\lambda}\frac{\partial f}{\partial x^{\mu}}+\frac{dp^{i}}{d\lambda}\frac{\partial f}{\partial p^{i}}=\left(\frac{\delta f}{\delta\lambda}\right)_{\text{collision}}$$

$$f=f(x^{\mu},p^{i})\qquad(\mu=0,1,2,3;\quad i=1,2,3)$$





Approximation Method

- Leakage Scheme
- Light Bulb Approximation
- Ray-by-Ray Approach
- IDSA(Isotropic Diffusion Source Approximation)
- MGFLD(Multi-Group Flux Limited Diffusion) method
- Two-Moment method

OUR OBJECTIVE

- Development of the Boltzmann neutrino radiation hydrodynamical code for CCSNe simulations.
- Analysis of neutrino transport in the intermediate region between optically thick and thin
- Validation and improvement of approximation method
- Understanding the explosion mechanism through investigation of detailed microphysical processes

Conservation Form of the Boltzmann Equation in GR

(Nagakura et al. 2016, Akaho et al. 2021)

Boltzmann Equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\mu}}\bigg|_{q_i}\left[\left(e^{\mu}_{(0)}+\sum_{i=1}^{3}l_i e^{\mu}_{(i)}\right)\sqrt{-g}f\right] -\frac{1}{\epsilon^2}\frac{\partial}{\partial\epsilon}(\epsilon^3 f\omega_{(0)})+\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}(\sin\theta_\nu f\omega_{(\theta_\nu)}) -\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}(f\omega_{(\phi_\nu)})=S_{\mathrm{rad}},$$

Coefficients calculated with Ricci rotation coefficients

$$\omega_{(0)}\equiv\epsilon^{-2}p^{\mu}p_{\nu}\nabla_{\mu}e^{\mu}_{(0)},$$

$$\omega_{(\theta_\nu)}\equiv\sum_{i=1}^{3}\omega_{(i)}\frac{\partial l_{(i)}}{\partial\theta_\nu},$$

$$\omega_{(\phi_\nu)}\equiv\sum_{i=2}^{3}\omega_{(i)}\frac{\partial l_{(i)}}{\partial\phi_\nu},$$

$$\omega_i\equiv\epsilon^{-2}p^{\mu}p_{\nu}\nabla_{\mu}e^{\nu}_{(i)}.$$

Directional cosines for the direction of neutrino propagation

$$l_{(1)}=\cos\theta_\nu,$$
$$l_{(2)}=\sin\theta_\nu\cos\phi_\nu,$$
$$l_{(3)}=\sin\theta_\nu\sin\phi_\nu.$$

Spatial tetrad basis

$$e^{\mu}_{(0)}=n^{\mu},\qquad e^{\mu}_{(1)}=\gamma_{rr}^{-1/2}\left(\frac{\partial}{\partial r}\right)^{\mu},$$

$$e^{\mu}_{(2)}=-\frac{\gamma_{r\theta}}{\sqrt{\gamma_{rr}(\gamma_{rr}\gamma_{\theta\theta}-\gamma_{r\theta}^2)}}\left(\frac{\partial}{\partial r}\right)^{\mu}+\sqrt{\frac{\gamma_{rr}}{\gamma_{rr}\gamma_{\theta\theta}-\gamma_{r\theta}^2}}\left(\frac{\partial}{\partial\theta}\right)^{\mu}$$

$$e^{\mu}_{(3)}=\frac{\gamma^{r\phi}}{\sqrt{\gamma^{\phi\phi}}}\left(\frac{\partial}{\partial r}\right)^{\mu}+\frac{\gamma^{\theta\phi}}{\sqrt{\gamma^{\phi\phi}}}\left(\frac{\partial}{\partial\theta}\right)^{\mu}+\sqrt{\gamma^{\phi\phi}}\left(\frac{\partial}{\partial\phi}\right)^{\mu}$$

Collision Term

Weak Interaction

$$\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} = \left[\frac{\delta f}{\delta \tau}\right]_{\text{emis-abs (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{scat (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{pair (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{light (s)}}$$

Emission/Absorption

Electron Capture

$e^- + p \longleftrightarrow \nu_e + n$ [ecp]

Anti-Electron Capture

$e^+ + n \longleftrightarrow \bar{\nu}_e + p$ [aecp]

Electron Capture on nuclei

$e^- + A \longleftrightarrow \nu_e + A'$ [eca]

Scattering

Neutrino-Nucleon scattering

$\nu + N \longleftrightarrow \nu + N$ [nsc]

Neutrino-Nuclei scattering

$\nu + A \longleftrightarrow \nu + A$ [csc]

Neutrino-Electron scattering

$\nu + e \longleftrightarrow \nu + e$ [esc]

Pair Process

Electron-positron pair process

$e^- + e^+ \longleftrightarrow \nu_i + \bar{\nu}_i$ [pap]

Nucleon-nucleon bremsstrahlung

$N + N \longleftrightarrow N + N + \nu_i + \bar{\nu}_i$ [nbr]

Light Nuclei

$\nu_e + {}^2\mathrm{H} \longleftrightarrow e^- + p + p$

$\nu_e + n + n \longleftrightarrow e^- + {}^2\mathrm{H}$

$\nu_e + {}^3\mathrm{H} \longleftrightarrow e^- + {}^3\mathrm{He}$

$\bar{\nu}_e + {}^2\mathrm{H} \longleftrightarrow e^+ + n + n$

$\bar{\nu}_e + p + p \longleftrightarrow e^+ + {}^2\mathrm{H}$

$\bar{\nu}_e + {}^3\mathrm{He} \longleftrightarrow e^+ + {}^3\mathrm{H}$

Neutrino Number Density

s : species ($s = \nu_e,\ \bar{\nu}_e,\ \nu_x$)

$$\Gamma_s \equiv \int \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{\Gamma} \equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$$

Neutrino Energy Density (μ = 0)

Radiation Pressure (μ = 1, 2, 3)

$$G_s^\mu \equiv \int p_s^\mu \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{G^\mu} \equiv \sum_s G_s^\mu$$

Euler Equations

Hydrodynamics

$$\partial_t \boldsymbol{Q} + \partial_j \boldsymbol{U}^j = \boldsymbol{W_h} + \boldsymbol{W_i} \qquad \Delta\psi = 4\pi\rho$$

$$\boldsymbol{Q} = \begin{pmatrix} \sqrt{g}\rho \\ \sqrt{g}\rho v_r \\ \sqrt{g}\rho v_\theta \\ \sqrt{g}\rho v_\phi \\ \sqrt{g}(e + \frac{1}{2}\rho v^2) \\ \sqrt{g}\rho Y_e \end{pmatrix} \quad \boldsymbol{U}^j = \begin{pmatrix} \sqrt{g}\rho v^j \\ \sqrt{g}(\rho v_r v^j + p\delta_r^j) \\ \sqrt{g}(\rho v_\theta v^j + p\delta_\theta^j) \\ \sqrt{g}(\rho v_\phi v^j + p\delta_\phi^j) \\ \sqrt{g}(e + p + \frac{1}{2}\rho v^2)v^j \\ \sqrt{g}\rho Y_e v^j \end{pmatrix}$$

Continuity Equation

Equation of Motion

Energy Equation

Evolution Equation of Ye

$$\boldsymbol{W_h} = \begin{pmatrix} 0 \\ \sqrt{g}\rho\left(-\psi_{,r} + r(v^\theta)^2 + r\sin^2\theta(v^\phi)^2 + \frac{2p}{r\rho}\right) \\ \sqrt{g}\rho\left(-\psi_{,\theta}r^2 + \sin\theta\cos\theta(v^\phi)^2 + \frac{p\cos\theta}{\rho\sin\theta}\right) \\ -\sqrt{g}\rho\psi_{,\phi} \\ -\sqrt{g}\rho v^l\psi_{,l} \\ 0 \end{pmatrix} \quad \boldsymbol{W_i} = \begin{pmatrix} 0 \\ -\sqrt{g}G^r \\ -\sqrt{g}G^\theta \\ -\sqrt{g}G^\phi \\ -\sqrt{g}G^t \\ -\sqrt{g}\Gamma \end{pmatrix}$$

ρ : density, v : velocity, p : pressure, e : internal energy, ψ : gravitational potential,
Y_e : electron fraction, Γ : neutrino number density G^t : neutrino radiation energy,
G^i : neutrino radiation pressure, $\sqrt{g}$: volume factor in the spherical coordinates ($= r\sin^2\theta$)

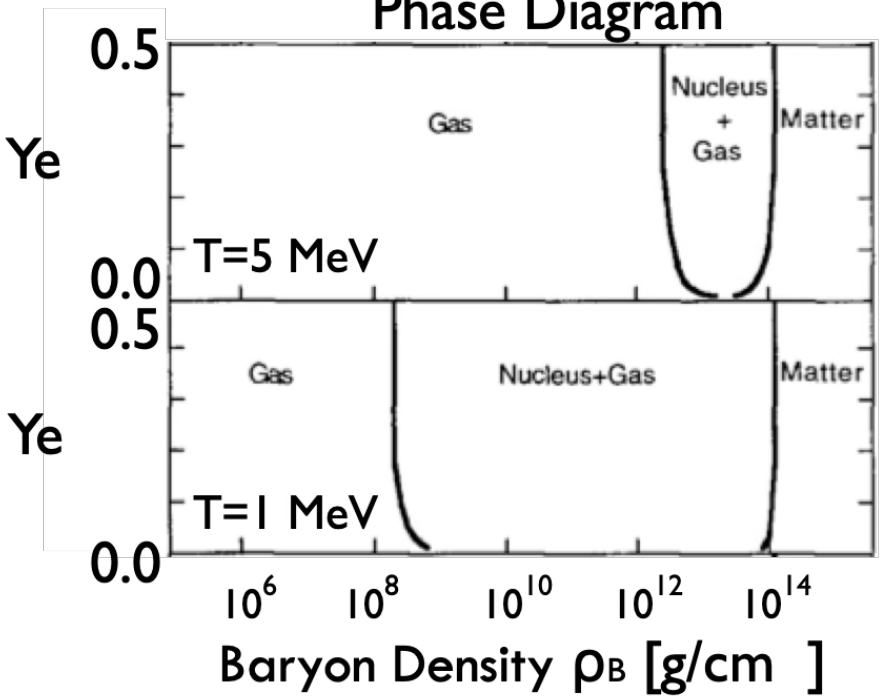
EOS of Nuclear Matter

Density ρ : 10^5 ~ $10^{15.5}$ g/cm^3

Temperature T : 0 ~ 50 MeV

Electron Fraction Ye : 0.01 ~ 0.5

Phase Diagram



Shen EOS (1998)

Gas : free nucleons gas + α particles

Nucleus + Gas: a single species of heavy nuclei forming Body-Centered-Cubic lattice with free nucleons gas

Matter: homogeneously distributed nuclear matter composed of nucleons (Relativistic Mean Field)

↓

Furusawa EOS (2011)

H^2 H^3 He^3 He^4

Light nuclei (Z<6)

Heavy nuclei (6≦Z)

↓

Togashi-Furusawa EOS (2017)

Variational method (Togashi et al. (2013))

VM Model

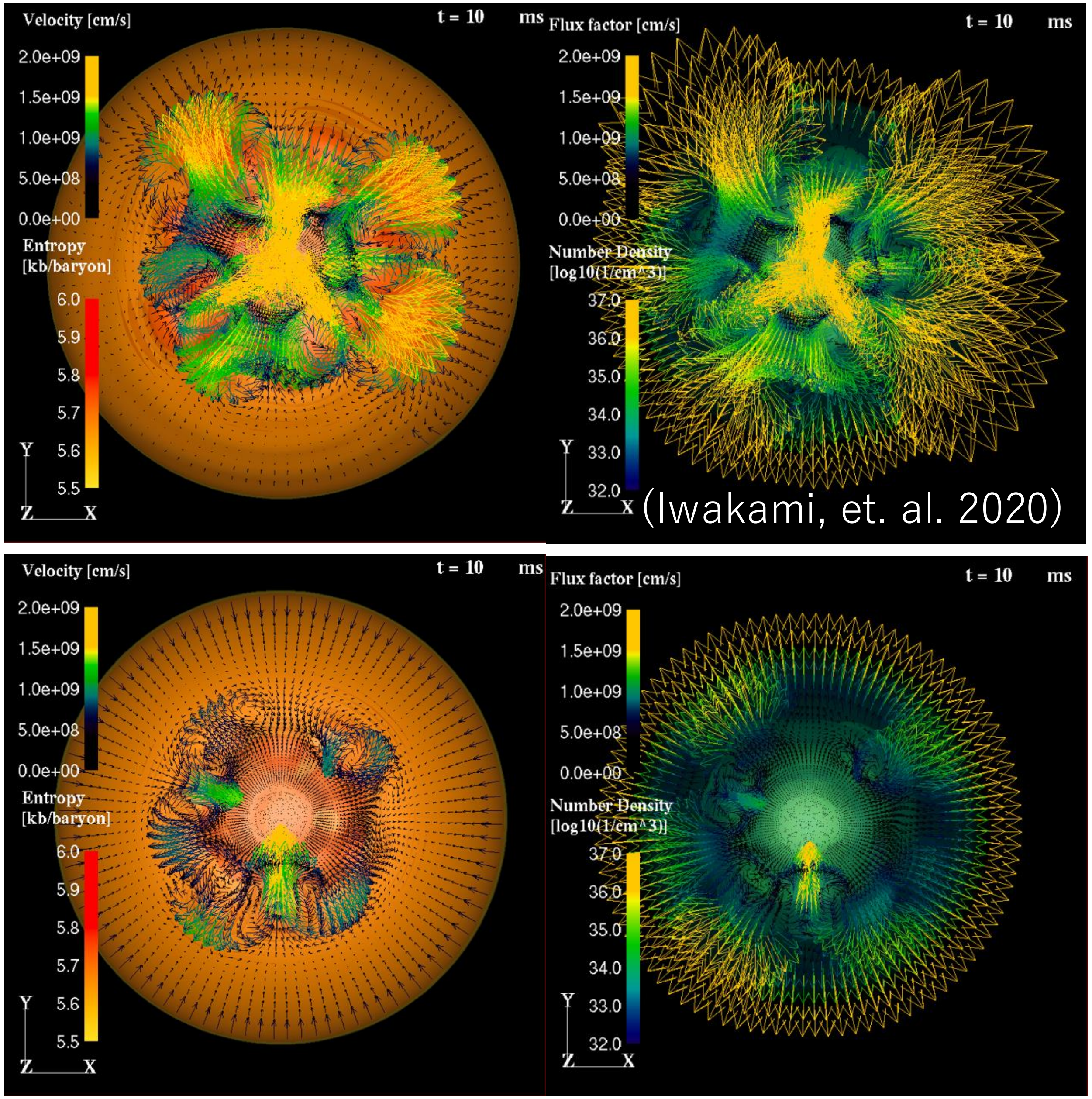
Prompt Convection at t=10ms（11.2Msol）

Newton

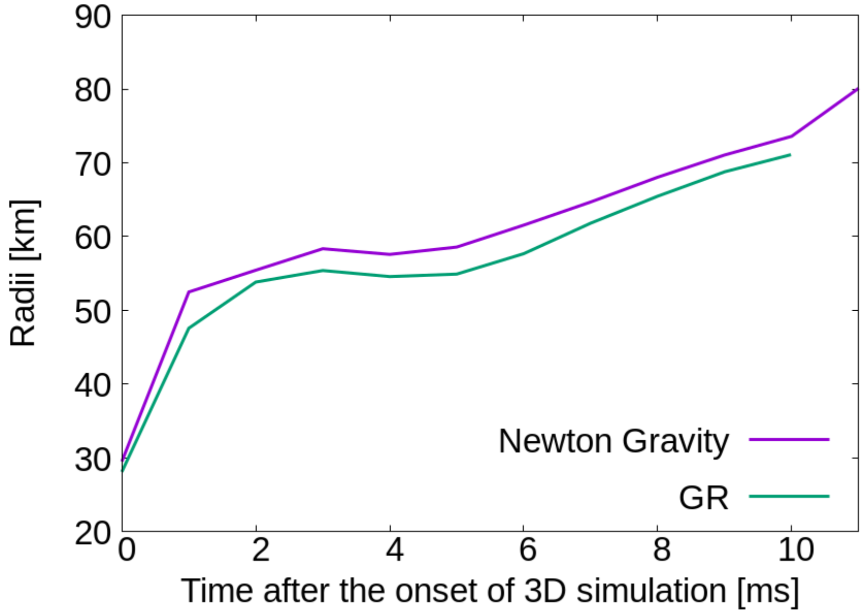
Spherically Symmetric Gravity

GR

Spherically Symmetric Gravity



(Iwakami, et. al. 2020)



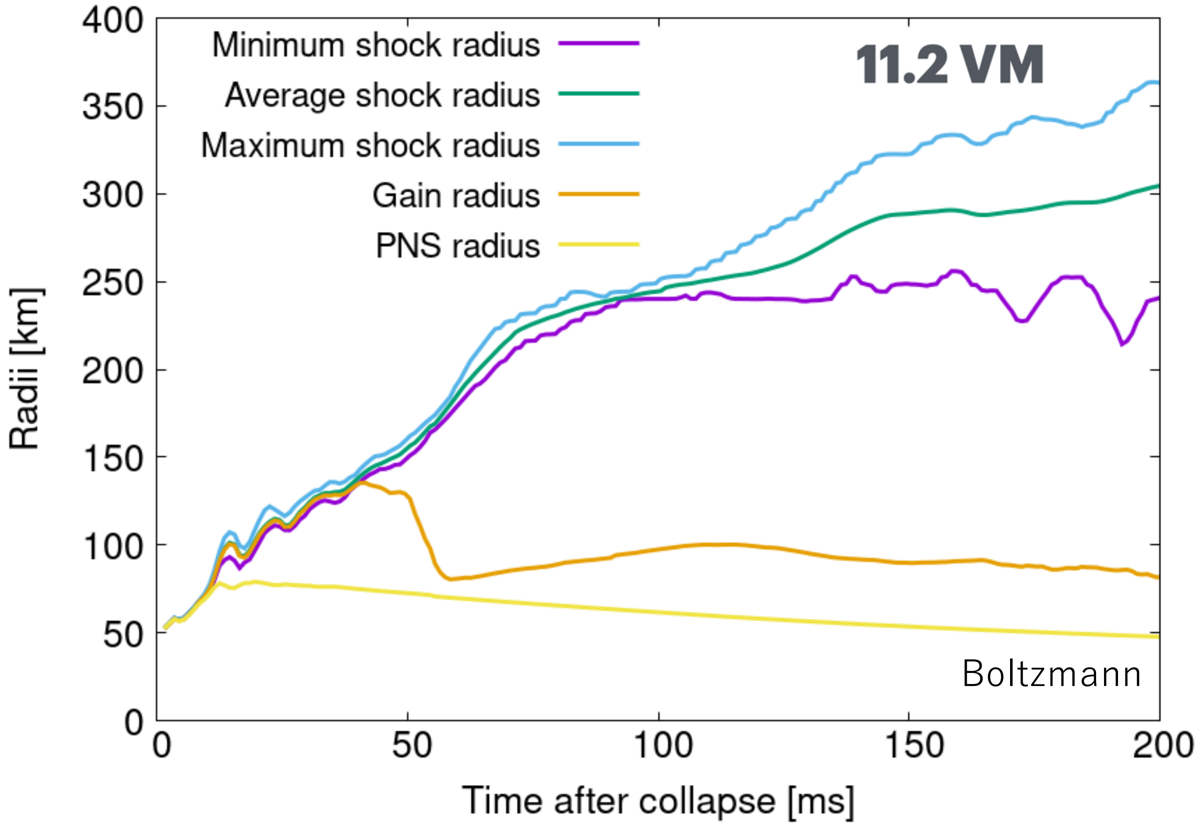
(the time corresponds to the emergence of the negative gradient in entropy profile.)

Comparing with them in Newton Gravity,

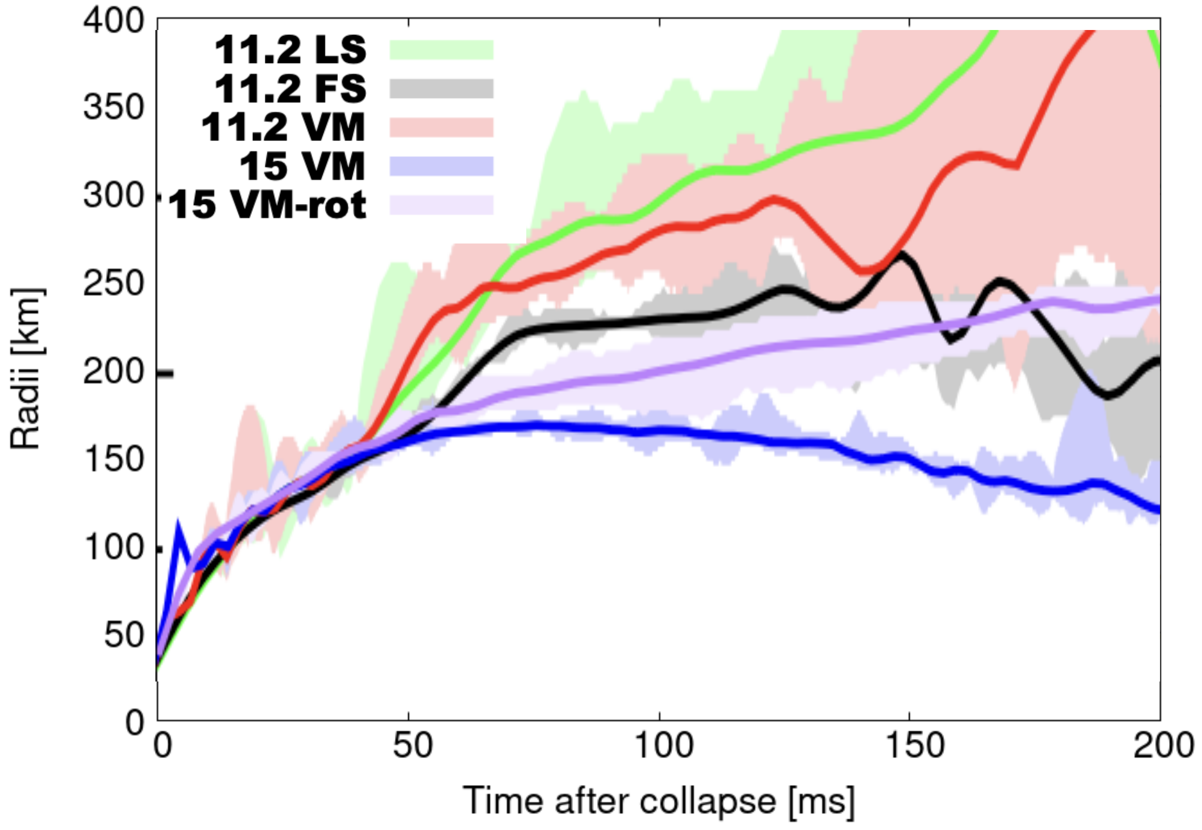
- The shock propagation is slower in GR.
- Convection is weaker in GR.

Shock Propagation (Newtonian 11.2Msol VMEOS)

3D



2D

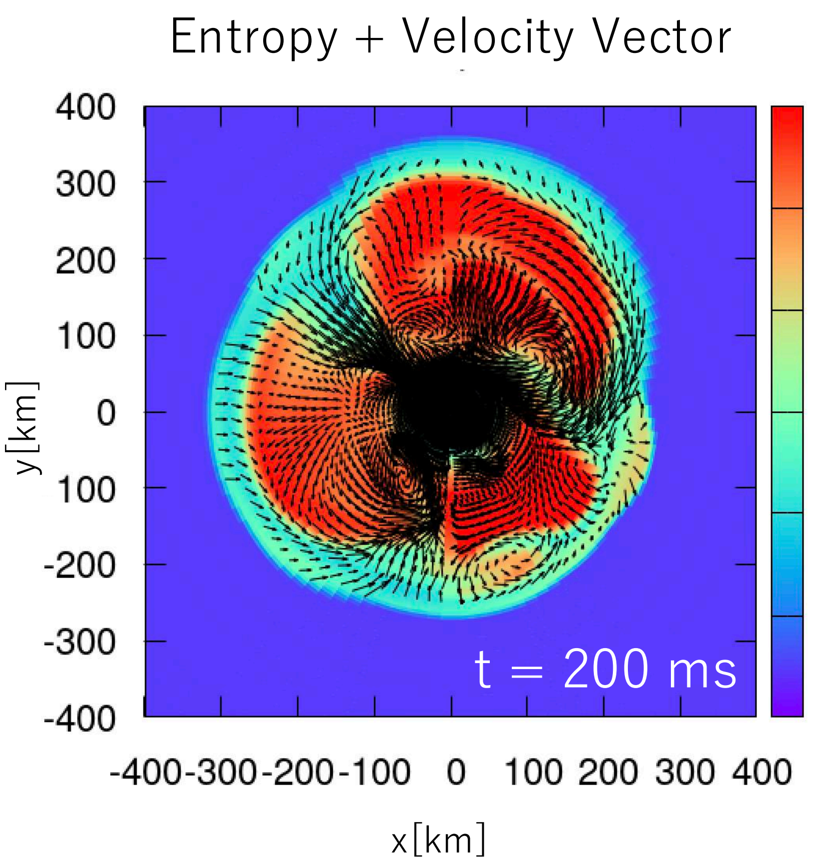


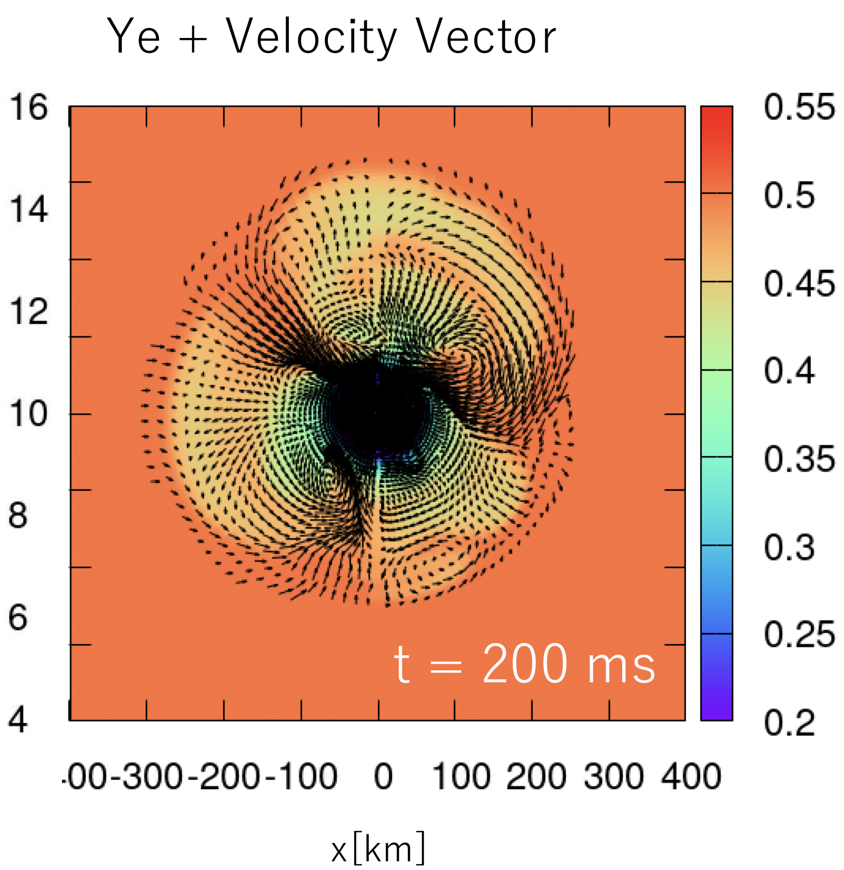
Time evolution of the average shock radius in **3D** (right, green line) is **slower** than that in 2D (left, red)

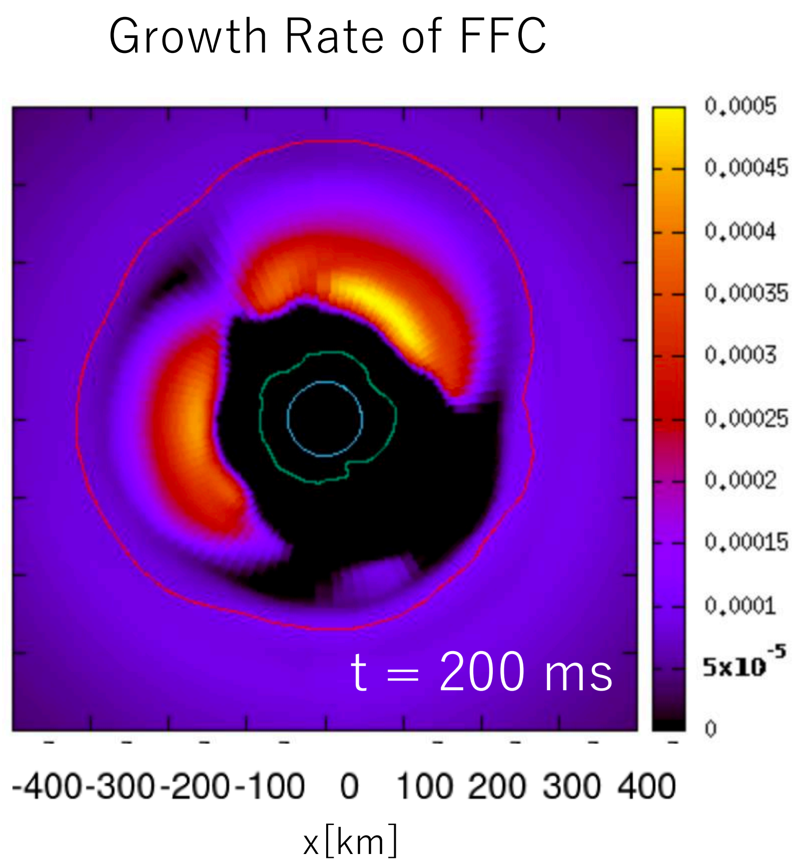
The shock wave propagates **almost spherically until 100ms.** (right, purple and blue lines agree with each other)

The shock wave **deforms after 100ms** (left, purple and blue line deviates from each other)

Convection and Neutrino First Flavor Conversion







FFC due to convection inside the shock wave can occur from t ≈ 130 ms, adding to the FFC originating from neutrino–heavy nuclei scattering outside the shock wave.

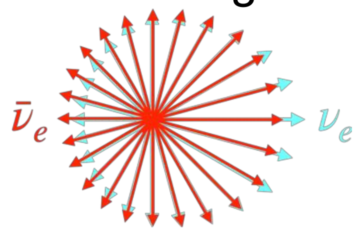
$$\sigma \sim \sqrt{-\left(\int_{\Delta G>0} \frac{d\Omega}{4\pi}\Delta G\right)\left(\int_{\Delta G<0} \frac{d\Omega}{4\pi}\Delta G\right)}$$

Neutrino-Nuclei scattering

$$\nu + A \longleftrightarrow \nu + A \quad [\mathrm{csc}],$$

Anti-Electron Capture

$$e^{+} + n \longleftrightarrow \bar{\nu}_e + p \quad [\mathrm{aecp}],$$

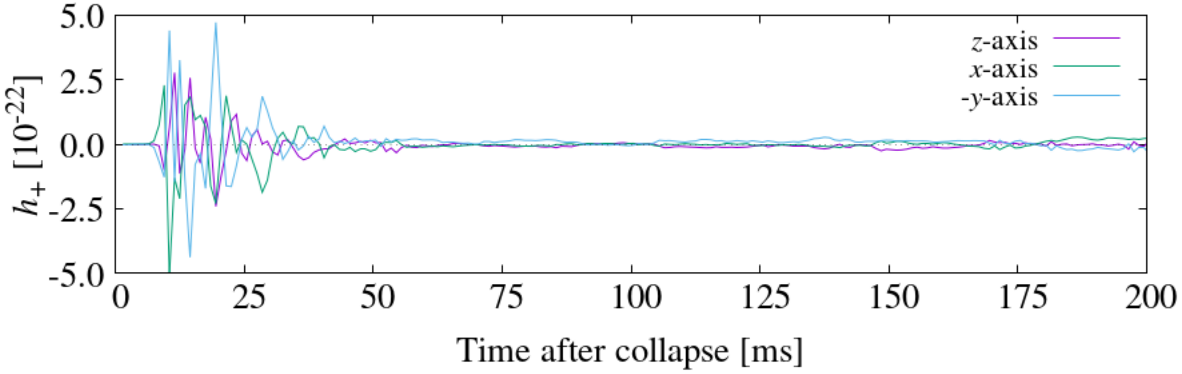


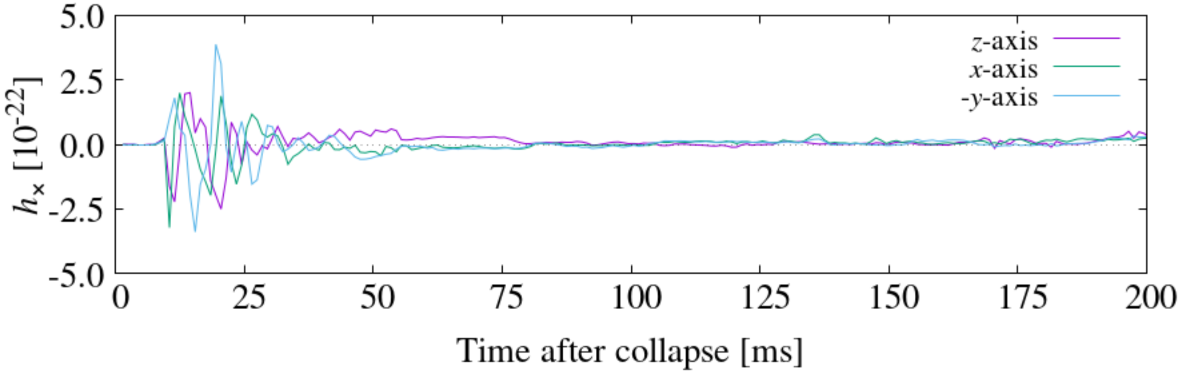
$$\Delta G = \frac{\sqrt{2}G_F}{2\pi^2}\int (f_{\nu_e} - f_{\bar{\nu}_e})\,\nu^2 d\nu$$

Gravitational Waves

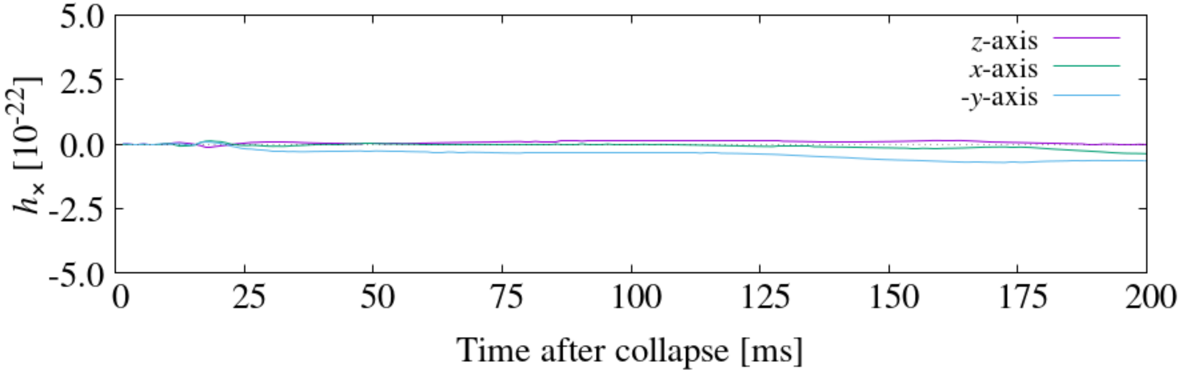
(generated within ~200km)

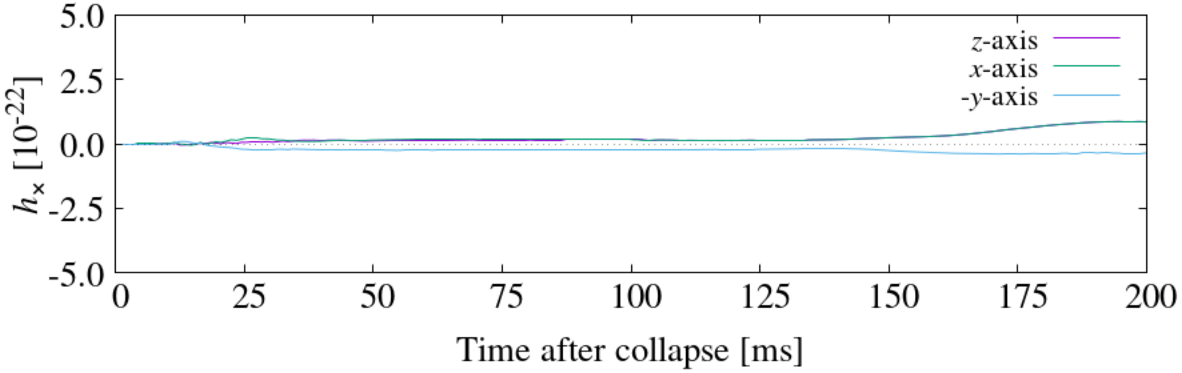
Matter origin





Neutrino origin





GW from matter is dominant in t ~ 10 - 50ms, while GW from neutrino increase with increasing time.

(GW calculation code of should be refined more.)

SUMMARY

We investigate the explosion mechanism of core-collapse supernovae.

- Our group focuses on understanding the detailed physics, such as neutrino radiation transport, EOS of nuclear matter, neutrino weak interactions, and neutrino oscillations.

- I have performed 3D simulations using Boltzmann neutrino radiation hydrodynamic code, until 200ms after core bounce (Newtonian gravity).

The speed of shock wave in 3D model is slower than it in 2D.

Neutrino-Nucleon scattering

$$\nu + N \longleftrightarrow \nu + N \quad [\mathrm{nsc}]$$

recoil, weak magnetism, strangeness effect should be implemented.

Fast neutrino flavor conversion can occur inside the spherical shock wave after t ~ 130ms.

- General relativistic 3D simulations are also started to investigate the gravitational wave from prompt convection for 10ms ~ 30ms.