12th Supernova Neutrino Workshop

Proto-neutron star evolution with convection

Jinkun Liao[1], Chinami Kato[2], Hiroki Nagakura[3], Hideyuki Suzuki[1]

Affiliations: *1. Tokyo University of Science, 2. The University of Tokyo, 3. National Astronomical Observatory of Japan*

Introduction

- **Motivation**: A proto-neutron star (PNS) is the newborn compact remnant at the core of a supernova (SN) explosion. It is widely accepted that neutrino emission drives PNS cooling, making neutrinos critical to PNS evolution. Upcoming neutrino detectors, including upgraded water-Cherenkov observatories and the JUNO liquid scintillator detector, will extend the observable window for a Galactic supernova to tens of seconds. Long-term simulations of PNS cooling are therefore essential to accurately characterize the emitted neutrinos.
- **Limitation**: Given computational cost and complexity, the spherical symmetry assumption is a reasonable choice for systematic studies of PNS cooling, though multi-dimensional effects are known to play important roles in this process.
- **Purpose**: To advance the accurate understanding of PNS cooling and the associated neutrino emission from Supernovae, we employed convection with Mixing length theory.

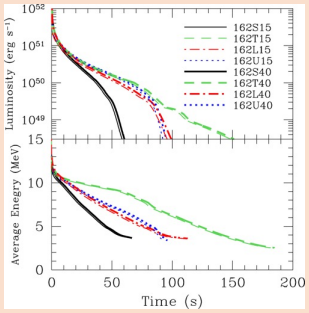

Figure 1. Luminosity and average energy of ν_e as a function of time after the bounce for PNS models with a baryon mass of M_b = 1.62 solar mass. (Ken'ichiro Nakazato *et al* 2022 *ApJ* **925** 98)

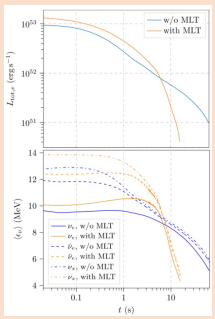

Figure 2. Total neutrino energy luminosity and mean energy of emitted neutrinos as a function of time, with and without convection (MLT). (A. Pascal, *MNRS*, Vol. 511, Issue 1, March 2022, Pages 356–370)

Method

- **Code:** quasi-static evolution code under spherical symmetry with general relativity.
- **Neutrino transfer:** multi-energy flux limited diffusion scheme
- **Flux limiter:** Mayle & Wilson (1987)
- **Nuclear equation state (EOS):** Togashi(2017)
- **Neutrino reactions:** Absorption and emission, Electron scattering, Thermal pair, Isoenergetic scattering off nucleon
- **Convection:** Mixing length theory **(MLT)**
  - Unstable criterion: Ledoux $C_L = \frac{\partial\rho}{\partial s}\Big|_{P,Y_e}\frac{ds}{dr} + \frac{\partial\rho}{\partial Y_e}\Big|_{P,s}\frac{dY_e}{dr} < 0$
  - Diffusion equations: $N_B\frac{dY_e}{dt} = \oint_{S'}(n_B D\nabla Y_e)\cdot d\boldsymbol{s}'$ $N_B\frac{ds}{dt} = \oint_{S'}(n_B D\nabla s)\cdot d\boldsymbol{s}'$

(D: diffusion coefficient, n_B: baryon number density, N_B: baryon number within the closed surface S', Y_e: electron fraction, s: entropy, $\boldsymbol{s}'$: infinitesimal surface element)

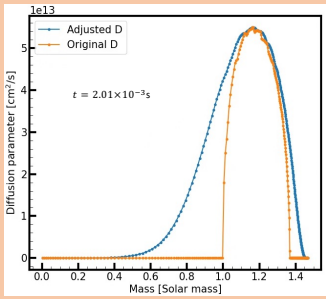

Figure 3. Overshooting of Diffusion parameter.

- Convective overshooting is physically mandatory, as the inertial principle and momentum-energy conservation forbid the non-physical instantaneous cessation of convective parcels at boundaries in classical MLT, requiring a finite penetration into the stable layer.
- When the convective boundary is treated with such a simple on/off threshold criterion, the solution can exhibit numerical oscillations.
- To address these issue, we adopted a Smoothing scheme for the diffusion coefficient D using a Gaussian function. The maximum value of $D'_{i\to k}$ as our final diffusion coefficient at mesh point k.

$$D'_{i\to k} = D_i \exp\left(-\frac{(r_k - r_i)^2}{\lambda_P^2}\right) \qquad i \in [1, i_{max}]$$

Results

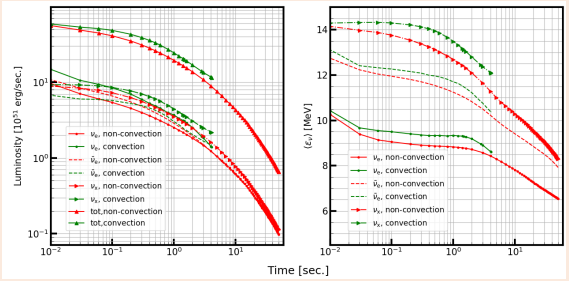

Figure 5. Mean energy of emitted neutrinos (right) and neutrino energy luminosity (left) as a function of time, for each neutrinos flavor, with and without convection.

Convection increases the mean energy and luminosity of emitted neutrinos at the early phase.

Early-phase PNS evolution exhibits a dynamic convective region that flattens entropy s and Y_e profiles. This behavior aligns qualitatively with Pascal (2022), validating our multi-model integration approach.

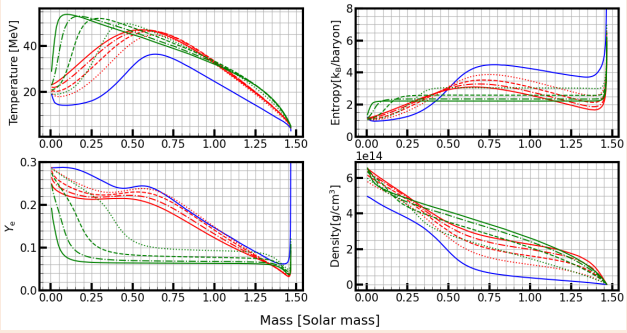

Figure 4. T, s, Y_e , ρ on mass. Dotted, dashed, dash dot, solid line represent time = 1s, 2s, 3s, 4s in the simulation. Red, green, blue lines are non-convection model, convection model and initial distributions.

Problems in long-term simulations remain

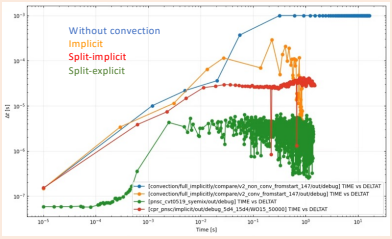

Figure 6. Variation of the time step size (Δt) over time

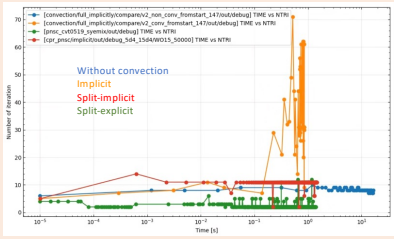

Figure 7. Convergence history in terms of iterations per time step

- **Computational Bottleneck**: Despite using various time integration methods, achieving long-term stability for multi-model systems remains challenging.
- **Performance Tade-off**: Prohibitive iteration counts or extremely small time step size increase the simulation cost duration to ~10 sec.

In long-term simulations (tens of seconds) with current scheme, the addition of MLT is posing a significant hurdle for real-time applications.

As the convection region(BVFQC < 0 in Fig.9) shrinks and approaches zero, spatial discontinuities emerge. These discontinuities may lead to severe convergence bottlenecks hindering stable long-term simulation.

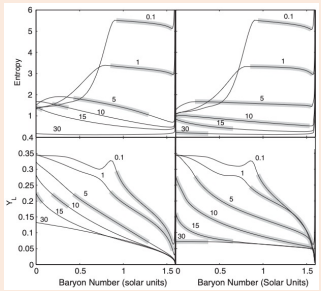

Figure 8. Evolution of the entropy and lepton fraction in a 1.6 $M_\odot$ rest mass PNS for 2 different EoS. The grayed regions are convectively unstable. The labels correspond to the model times in seconds (L. F. Roberts 2012 *ApJ* **755** 126)

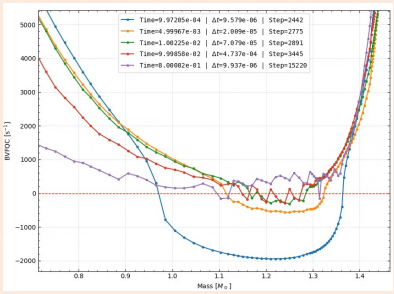

Figure 9. Evolution of Brunt-Väisälä frequency profiles across mass coordinates

Conclusion

A diffusion-based treatment of convection, inspired by Mixing-Length Theory (MLT), was implemented to examine its early influence on PNS evolution. Our results indicate that convection effectively transports internal energy from the inner core to the outer regions, elevating the temperature near the neutrino sphere and subsequently enhancing both neutrino luminosity and mean energy. These findings show qualitative agreement with previous studies (e.g., Pascal 2022). However, current simulations encounter significant numerical bottlenecks: as convective regions evolve, the emergence of spatial discontinuities leads to diminishing time steps (Δt) and prohibitive iteration counts. To facilitate robust long-term simulations across various progenitors and Equations of State (EOS), future work will focus on refining the numerical scheme to effectively enlarge stable time steps while suppressing iteration-related overhead, ensuring a more efficient exploration of supernova neutrino signals.

学術変革領域研究「地下稀」,"第12回超新星ニュートリノ研究会" @静岡県総合コンベンション施設 プラサ ヴェルデ; 2026. MAR. 9–10