Comparing Explodability Predictions from a Parameter-Optimized Semi-Analytic Model with Structure-Based Progenitor Criteria

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1. INTRODUCTION

Motivation: 3D is more reliable, but 1D remains the "large-sample workhorse."

3D realistic, but are computationally expensive. Difficult to cover large progenitors samples.

Practical applications (e.g. yield grids, galactic chemical evolution, and light-curve/spectral inference) still rely on **1D**

The key challenge:

Do traditional structure-based criteria truly represent the underlying explosion physics?

How can we use a limited set of 3D models to constrain the parameters of a fast 1D model?

Our strategy:

Calibrate a semi-analytic 1D model (**Muller et al., 2016**) using 3D results (**Burrows et al. 2024**).

Use calibrated model to rigorously compare structure-based criteria.

2. METHOD: MÜLLER MODEL

Semi-analytic 1D model: approximating multi-D effects with interpretable parameters

Framework:
Müller et al. (2016),
revisions by Takahashi et al. (2023).

Explosion criterion:
based on timescale ratio τ_{adv}/τ_{heat}

Outputs:
explodability, plus E_{expl}, M_{PNS}, and M_{Ni}.

TUNABLE PARAMETERS

α_{turb} Shock expansion factor due to turbulent stresses (Müller & Janka, 2015)

ζ Efficiency factor for conversion of accretion energy into luminosity (Müller & Janka, 2014)

$\tau_{1.5}$ Cooling time-scale for 1.5 $M_\odot$ neutron star (Hüdepohl, 2014)

α_{out} Volume fraction of outflows

β_{expl} Shock compression ratio during explosion phase

2. METHOD: PROGENITOR SETS

Calibration data: FORNAX 3D sample and fitting targets

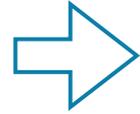| Progenitor set	Reference	Mass range $[M_\odot]$	Minimum mass spacing $[M_\odot]$
S16	Sukhbold et al. (2016)	9–120	0.1
S18	Sukhbold et al. (2018)	12–27	0.01
M16	Müller et al. (2016)	9.5–45	0.01
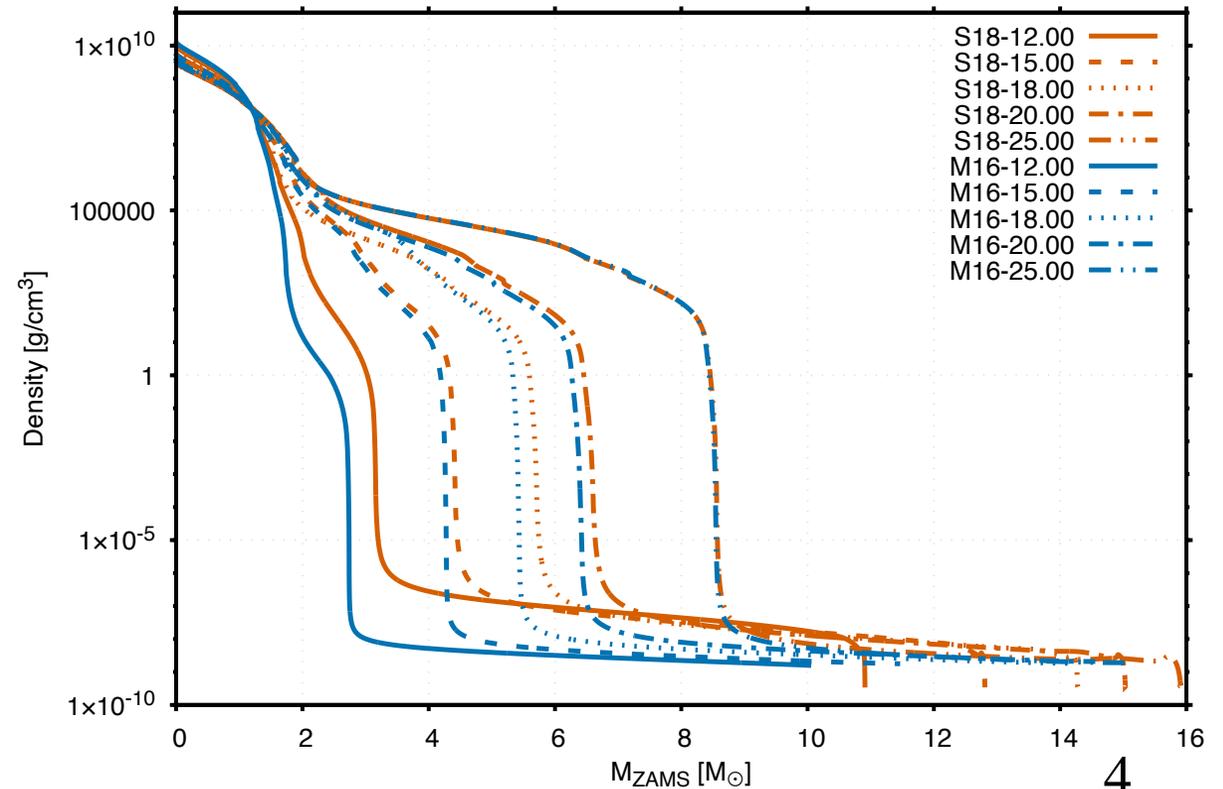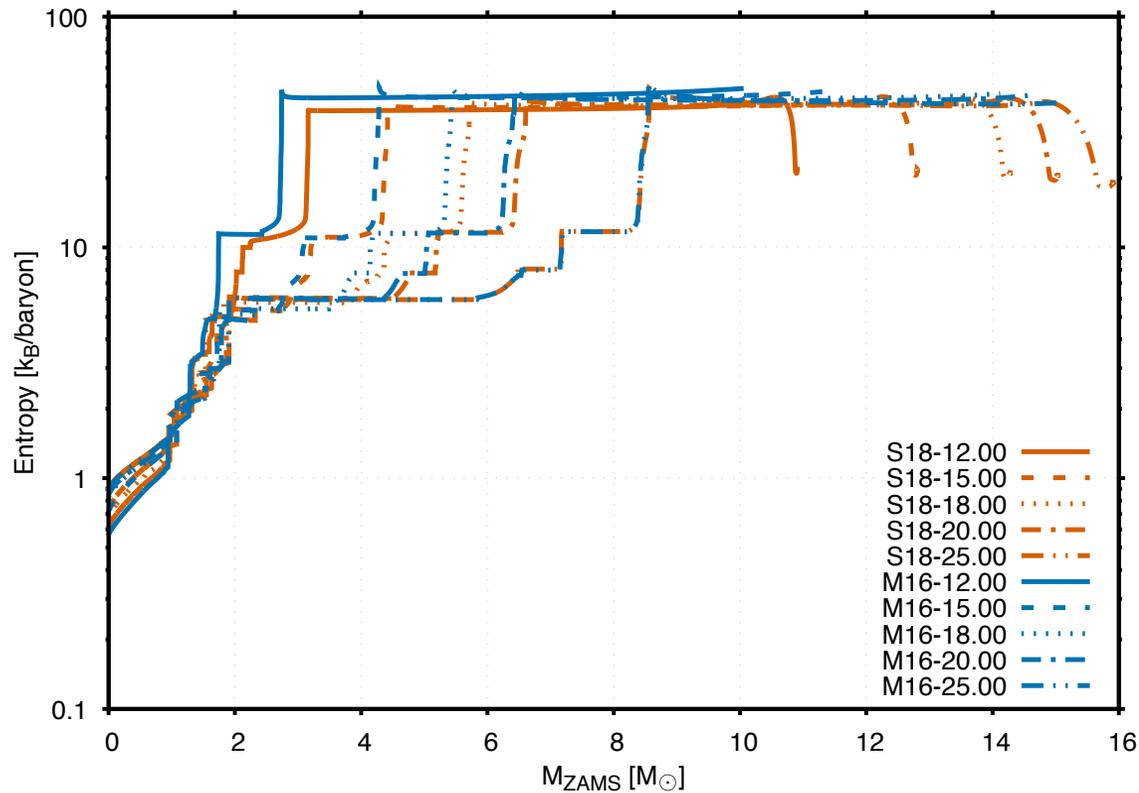
2. METHOD: OPTIMIZATION WORK

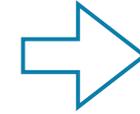Calibration workflow: fit α_turb(M_ff) first, then jointly scan the remaining parameters

STEP A

Scan α_{turb} and minimize the single-model misfit x_k^2

STEP B

Fit the best-fit α_{turb} values as a piecewise function of M_{ff} (free-fall mass coordinate)

STEP C

Scan $(\zeta, \iota_{1.5})$ within allowed ranges, then scan $(\beta_{\text{expl}}, \alpha_{\text{out}})$.

Misfit metric (χ^2-like): simultaneously constrains three observables,

$$\chi_k^2 = \left(\frac{E_{1\text{D},k} - E_{3\text{D},k}}{E_{3\text{D},k}} \right)^2$$

$$+ \left(\frac{M_{\text{PNS},1\text{D},k} - M_{\text{PNS},3\text{D},k}}{M_{\text{PNS},3\text{D},k}} \right)^2$$

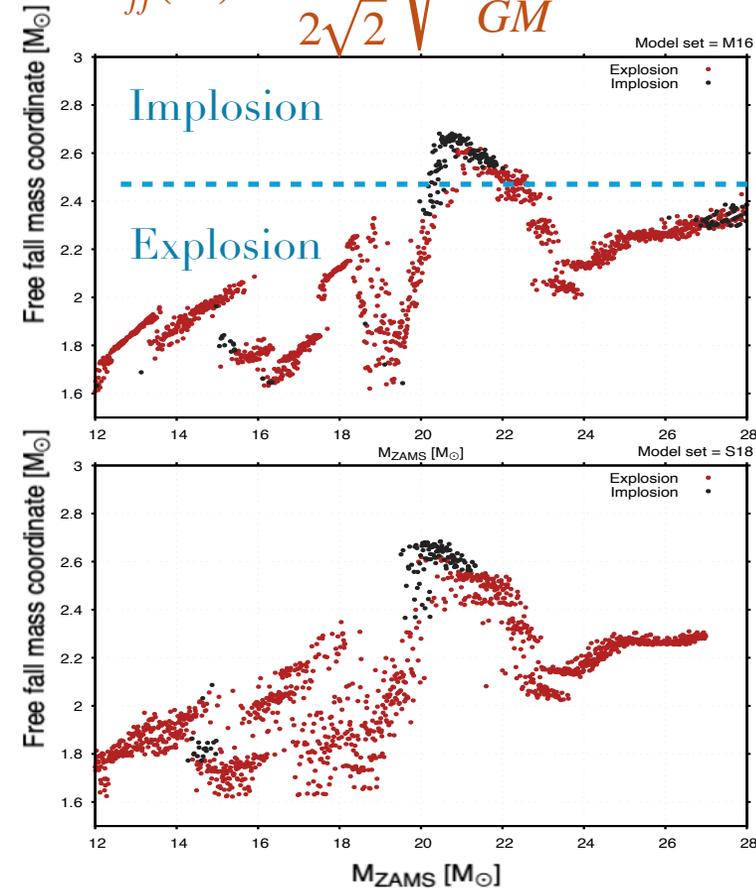$$+ \left(\frac{M_{\text{Ni},1\text{D},k} - M_{\text{Ni},3\text{D},k}}{M_{\text{Ni},3\text{D},k}} \right)^2,$$

Explodability: Structure-based progenitor criterion
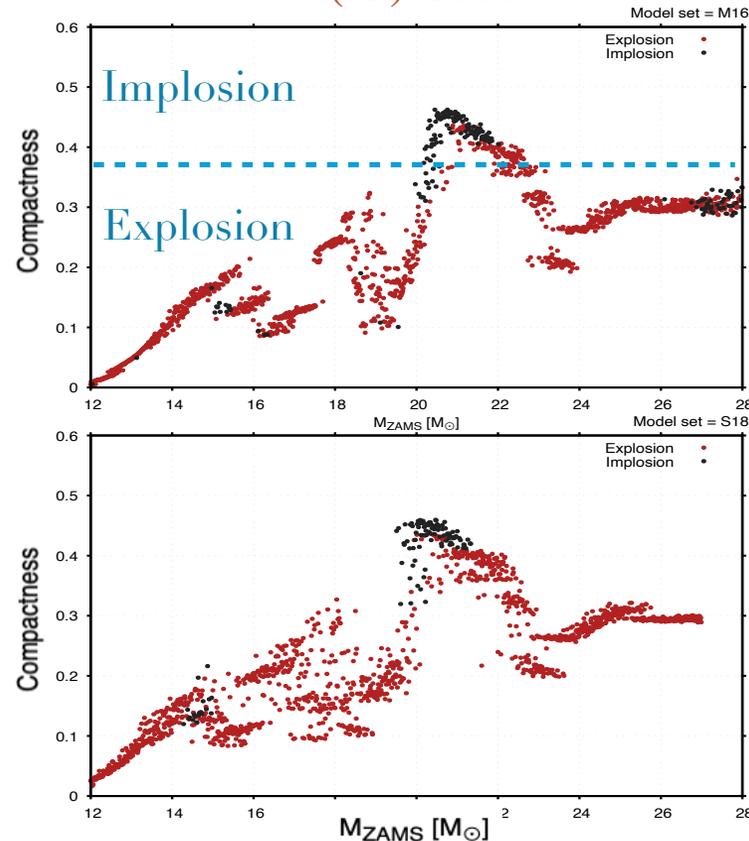
Free-fall Mass coordinate
K. Takahashi et al. (2023)

$$\tau_{ff}(M) = \frac{\pi}{2\sqrt{2}}\sqrt{\frac{R(M)^3}{GM}} = 1$$
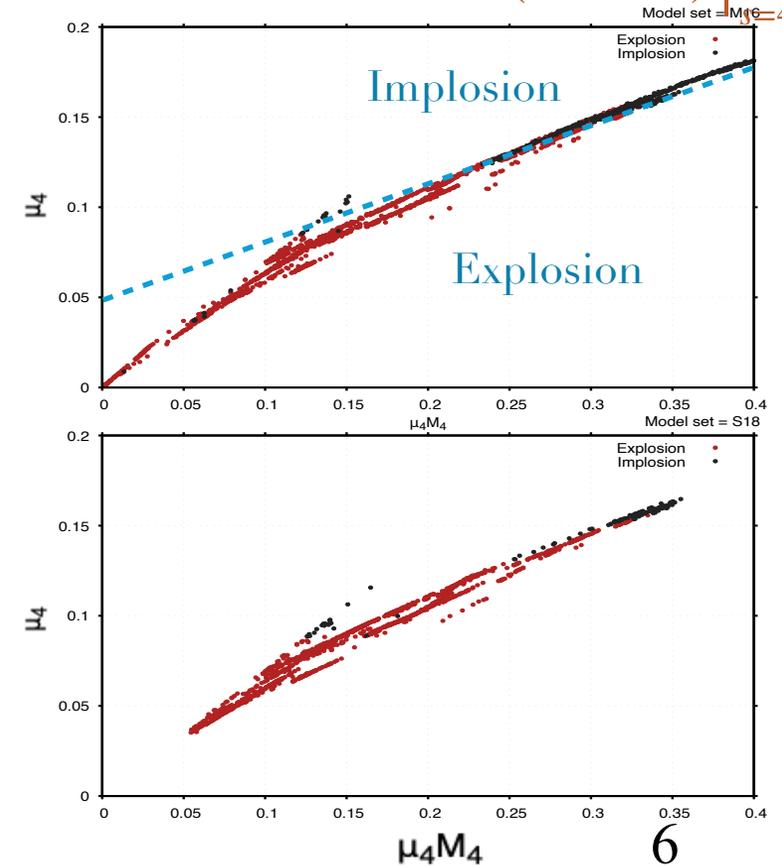
Compactness
E. O'Connor & C. D. Ott (2011)

$$\zeta_M = \frac{M/M_\odot}{r(M)/1000km}$$
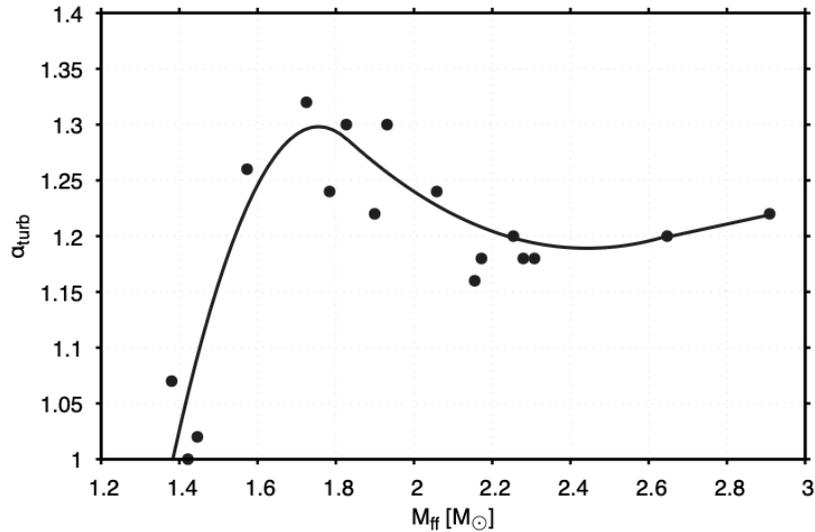
Two-parameter criterion
T. Ertl et al. (2016)

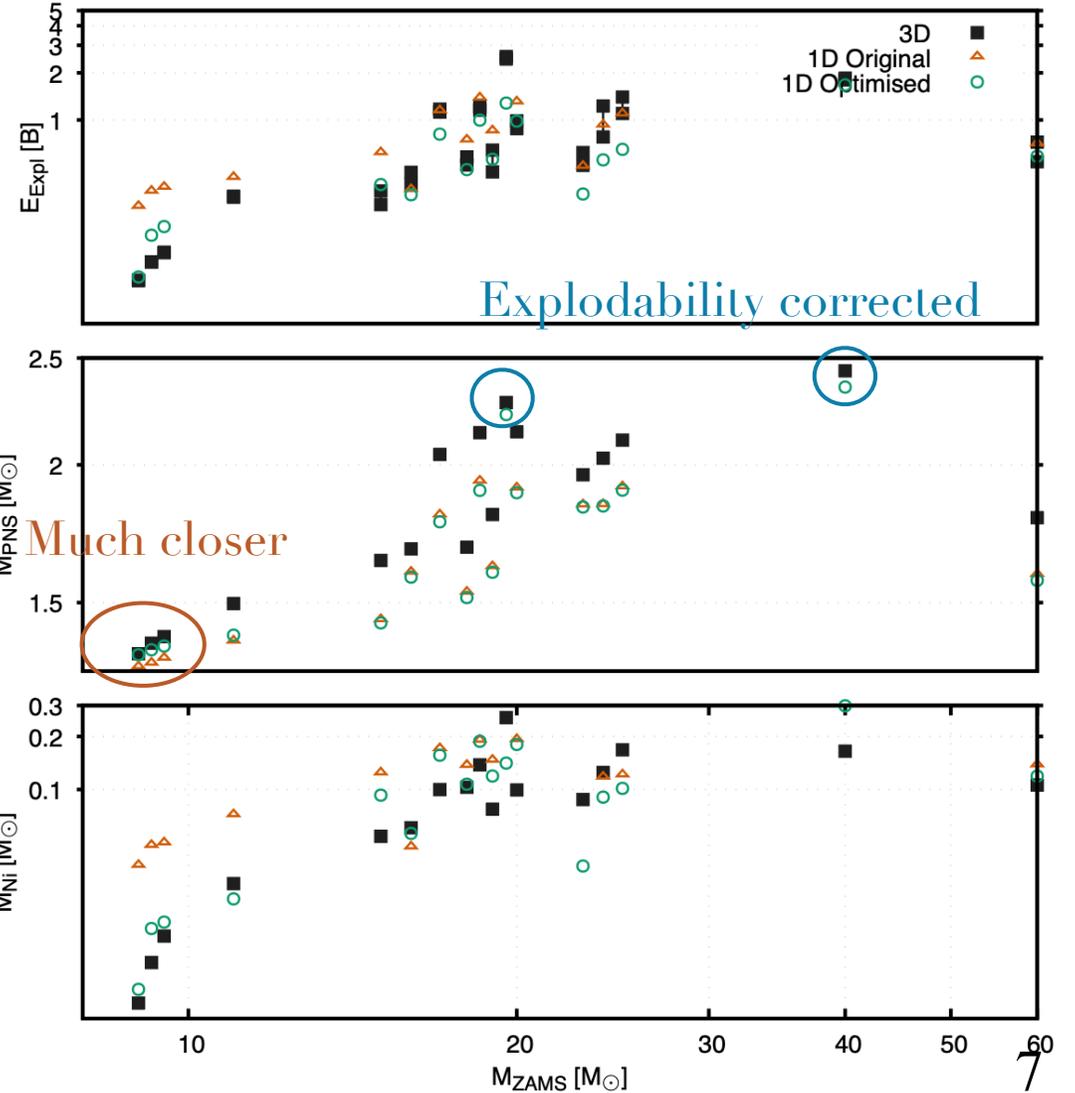$$M_4 = m(s = 4)/M_\odot, \mu_4 \equiv \frac{dm/M_\odot}{dr/(1000\ km)}\bigg|_{s=4}$$

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Result 4: Best-fit parameter set and improved overall agreement

Parameter	Original value	Optimized value
α_{turb}	1.18	Piecewise functions related to M_{ff} [†]
ζ	0.8	0.9
$\tau_{1.5}$	1.2	1.44
α_{out}	0.5	0.32
β_{expl}	4	7

Explodability corrected

Much closer

$\bar{\chi}^2_{\text{tot}}$ 4.2   ⟹   0.38

3.RESULTS & DISCUSSION

Result 3: α_turb must vary systematically with progenitor structure
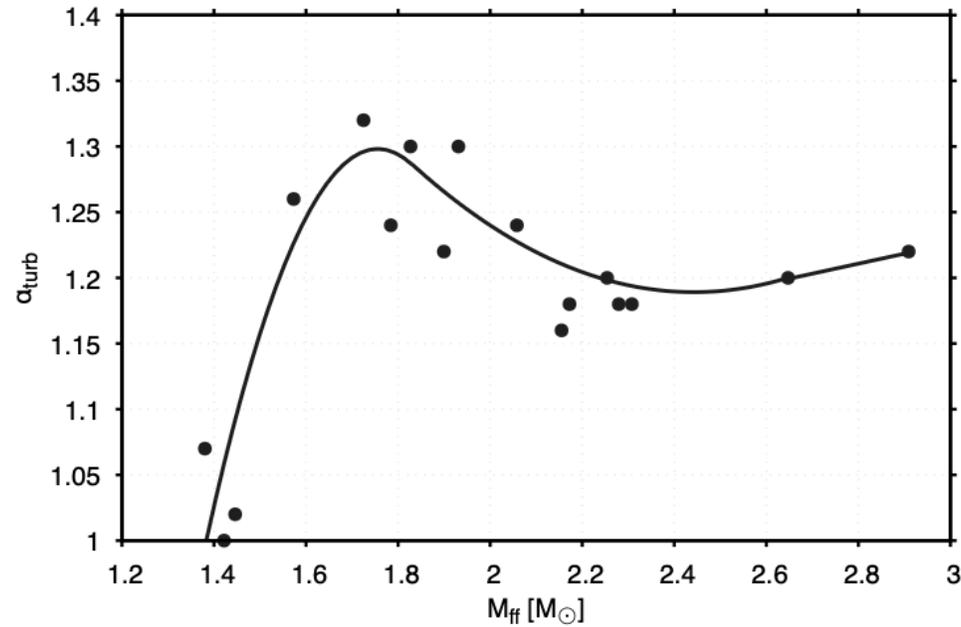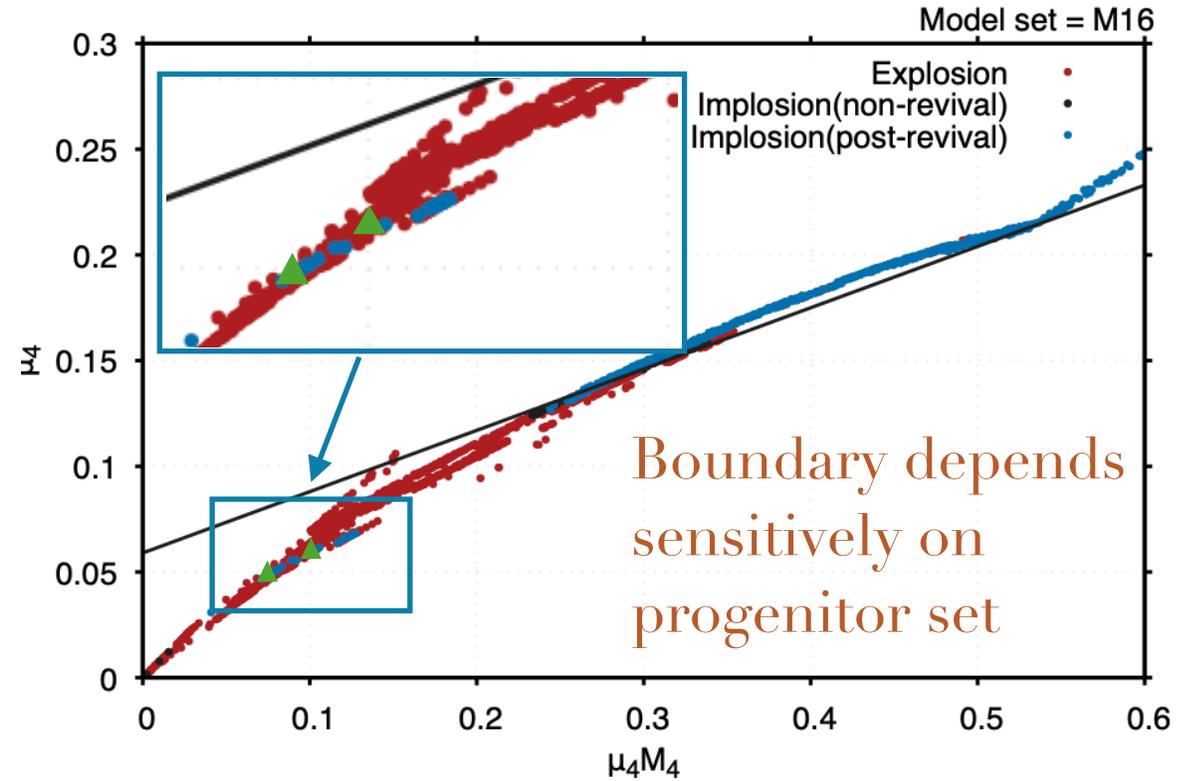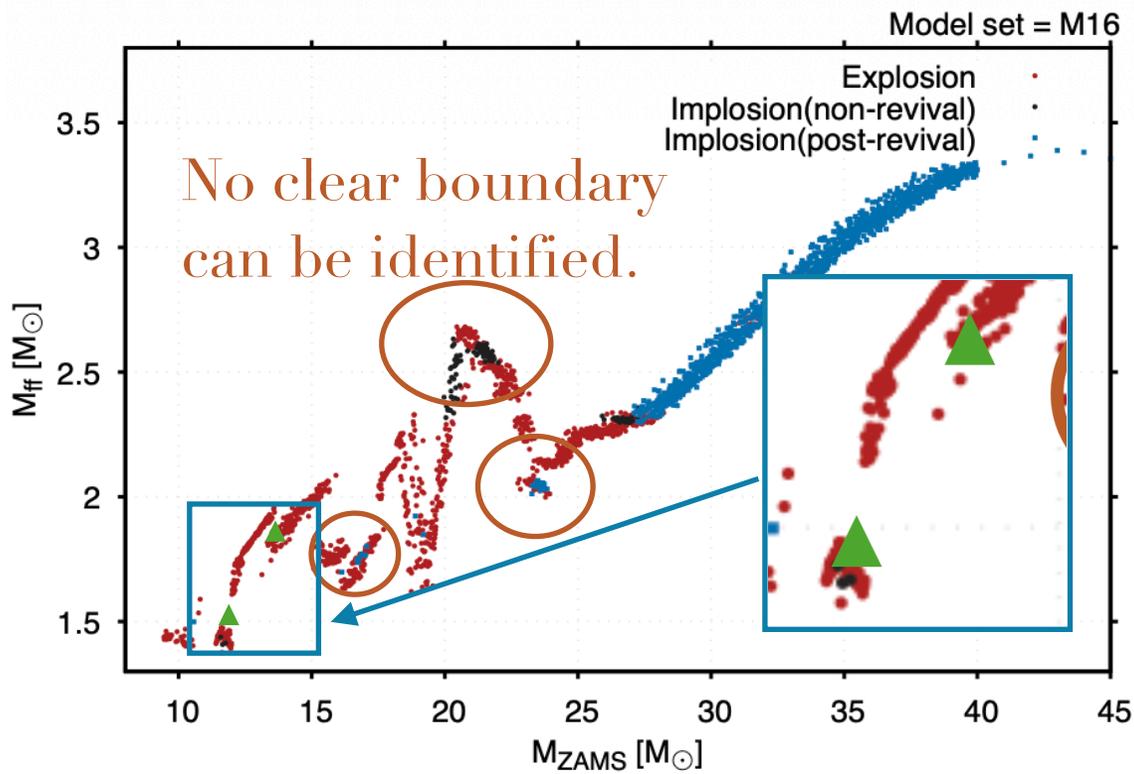

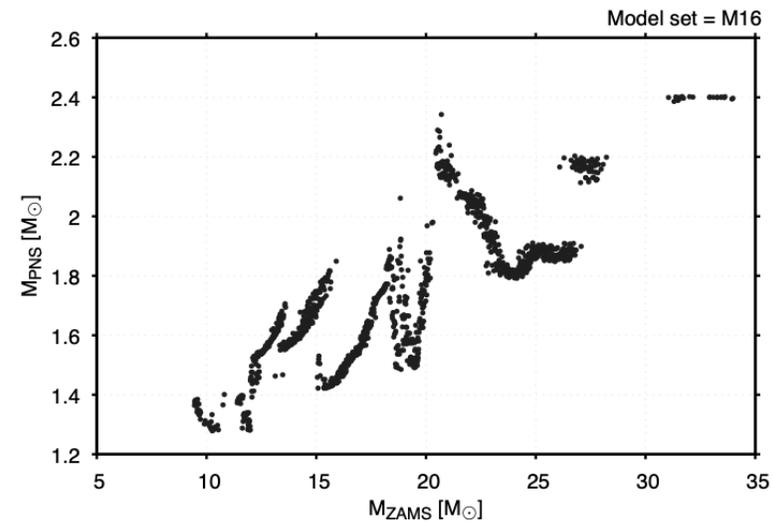Turbulence strength is governed by the balance between neutrino heating and accretion, varies systematically with progenitor structure.
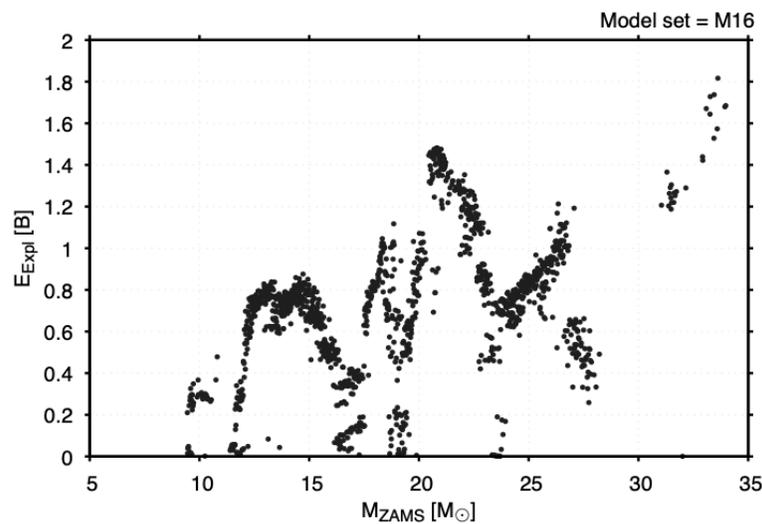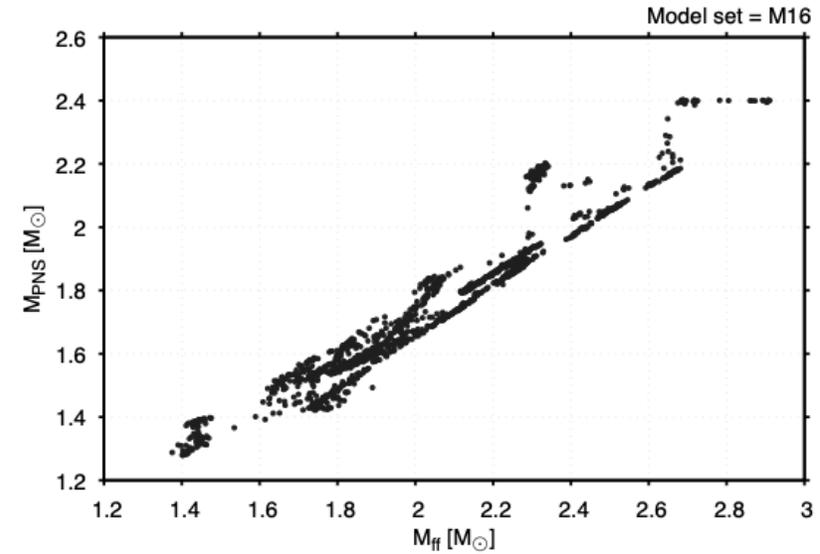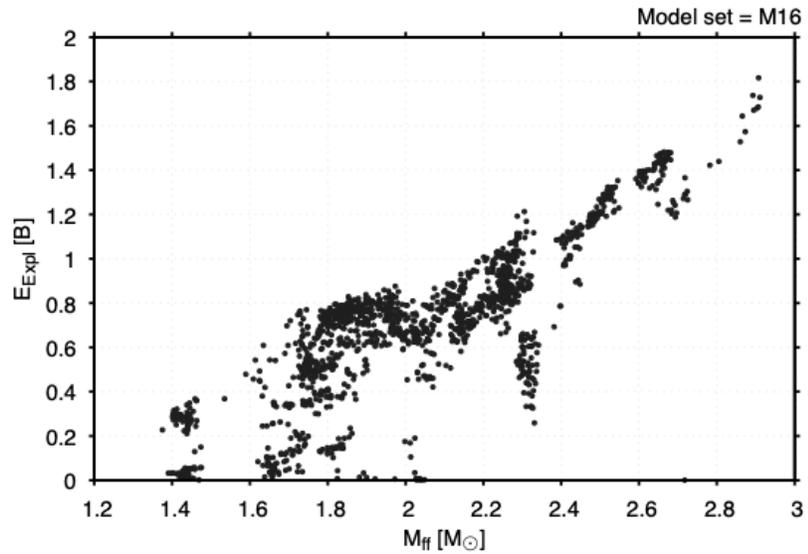
Result 1: Reshaping of the explodability map

They may no longer be sufficient as a standalone predictor of explodability.
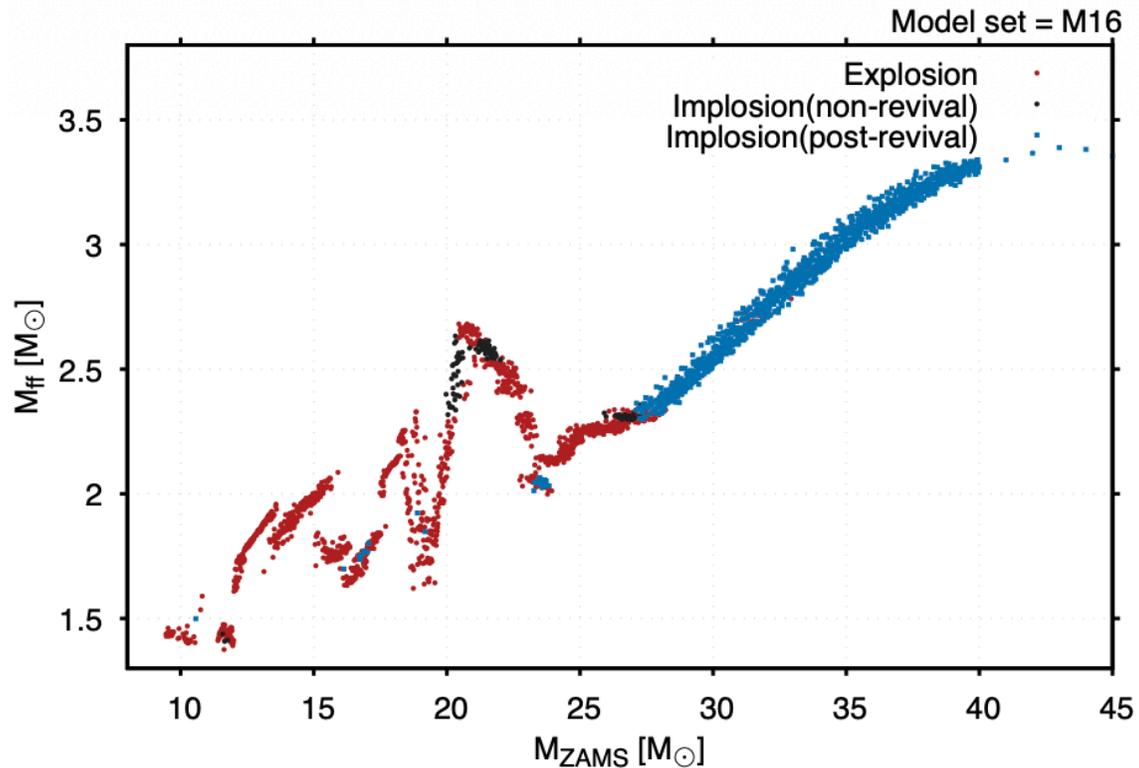
3.RESULTS & DISCUSSION

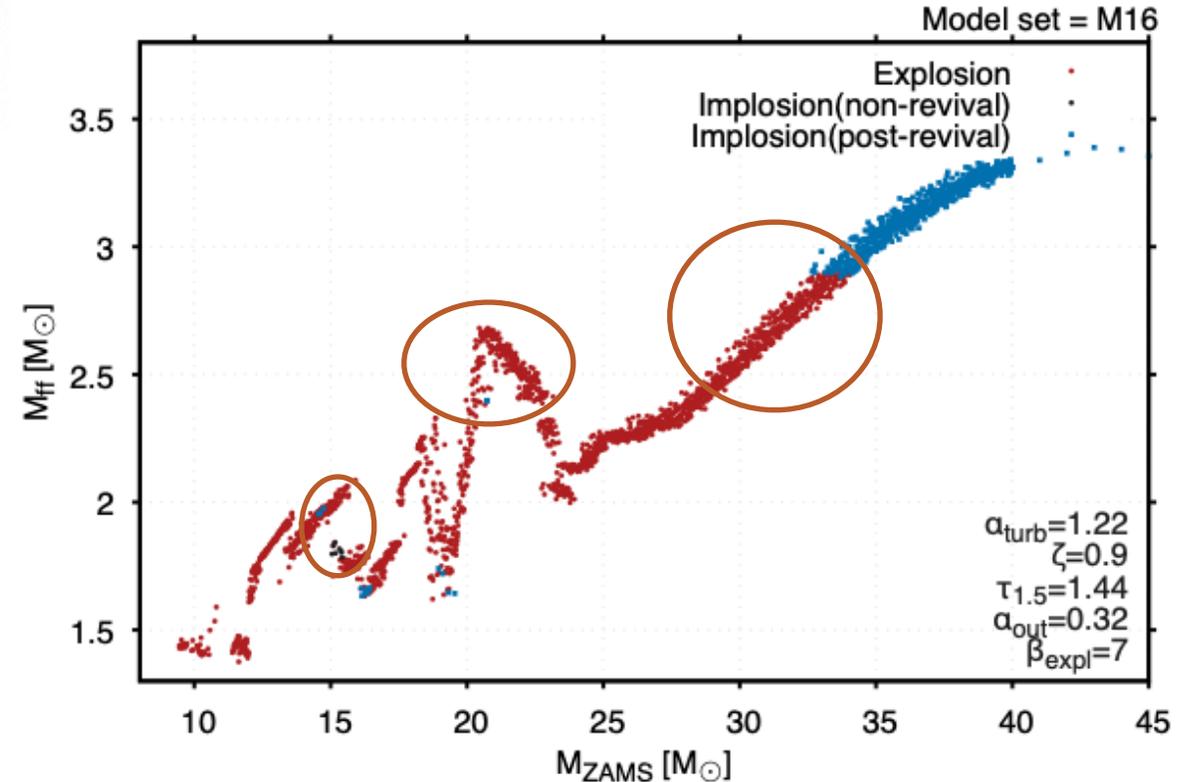Result 2: monotonic correlation between explosion outcome and M_ff

Result 3: α_turb must vary systematically with progenitor structure

If α_turb is fixed to a piecewise function

If α_turb is fixed to a constant value

3. SUMMARY

- A parameter-optimized semi-analytic 1D model can better reproduce 3D explosion trends then structure-based variables **while retaining computational speed**.

- Single-parameter structural diagnostics (M_{ff}) are better suited as **trend indicators** / **ranking variables**, rather than universal explosion boundaries.

- Next steps: incorporate **more 3D calibration cases** and a **more physical treatment of late-time evolution**, to reduce parameter degeneracies and extrapolation uncertainties.